Vibration Data Based Damage Detection Methods for a Population of Nominally Identical Structures

Ph.D. Thesis

Kyriakos J. Vamvoudakis–Stefanou

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Examination Committee

• Spilios Fassois, Professor Ph.D. (co–supervisor)
  Department of Mechanical Engineering & Aeronautics,
  University of Patras, Greece

• John Sakellariou, Assistant Professor Ph.D. (co–supervisor)
  Department of Mechanical Engineering & Aeronautics,
  University of Patras, Greece

• Dimitris Saravanos, Professor Ph.D. (advising committee member)
  Department of Mechanical Engineering & Aeronautics,
  University of Patras, Greece

• Constantinos Papadimitriou, Professor Ph.D.
  Department of Mechanical Engineering,
  University of Thessaly, Greece

• Dimitrios Giagopoulos, Assistant Professor Ph.D.
  Department of Mechanical Engineering,
  University of Western Macedonia, Greece

• Constantinos Berberidis, Professor Ph.D.
  Department of Computer Engineering & Informatics,
  University of Patras, Greece

• Evangelos Papatheou, Lecturer Ph.D.
  College of Engineering Mathematics and Physical Sciences,
  University of Exeter, United Kingdom
To my family and my beloved one.
Preface

The current thesis considers and tackles the problem of damage detection for a population of nominally identical structures, by exploring an original experimental study and postulating a number of novel methods. The thesis presents the experimental study in detail, explores the problem via conventional data based methods and postulates novel methods that significantly increase damage detection effectiveness.

The thesis is intended for readers familiar with statistics, probability theory, time series analysis, machine learning, and structural dynamics. Yet, effort has been invested to make this text friendly to the unfamiliar reader as well, by adding short explanations and/or proper references.

The main theory, experiments and methods description is organized into five chapters. Each one of these chapters is self–contained, in order to facilitate the reader, who is not required to read other chapters to understand the contents of a single chapter. Nevertheless, each chapter is logically connected to the previous one by tackling the latter’s problems or weaknesses. The main problem and the four chapters are concisely presented in the introductory chapter, overall conclusions are provided in the concluding chapter, while additional results, insights, and proofs are provided in the appendices chapter. The chapters contents, such as figures and photos, are intended for in–color projection (or print), yet most of them may be well–interpreted in gray scale as well.

You may contact the author at: kivast@gmail.com.

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Important Conventions

• Bold–face upper/lower case symbols designate matrix/column–vector quantities, respectively. Matrix transposition is indicated by the superscript $^T$.

• An overbar designates PCA transformation; for instance $\bar{\alpha}$ is the PCA–based transformation of $\alpha$.

• A functional argument in brackets designates function of an integer variable; for instance $x[t]$ is a function of normalized discrete time ($t = 1, 2, \ldots$). The conversion from discrete normalized time to analog time is based on $(t - 1)T_s$, with $T_s$ designating the sampling period.

• A hat designates estimator/estimate; for instance $\hat{\alpha}$ is an estimator/estimate of $\alpha$.

• The subscripts $o$ and $u$ designate quantities associated with the nominal (healthy) and current (unknown) state of the structure, respectively. The subscript $d$ designates damaged state. The superscript $^\star$ designates the U–HS–AR and U–cGM–AR methods hyper–parameters value selected upon completion of their training.
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The current thesis main topic of discussion is the problem of vibration data–based damage detection for a population of nominally identical structures. Vibration data–based damage detection as part of the broader Structural Health Monitoring family of methods, receives significant academic and industrial attention over the last several years, since random vibrations are typically naturally available during the structures normal operation, while the corresponding data acquisition and processing equipment is mature and of relatively low cost.

The respective vibration–based damage detection methods main concept is based on the fact that a damage changes the structural dynamics. Then, damage detection is based on tracking these changes via proper features that represent some of the modal characteristics of the structure. Nevertheless, such changes may occur due to a multitude of damage irrelevant factors, such as varying Environmental and Operational Conditions (EOCs), thus potentially “masking” the changes due to damage on the dynamics and jeopardizing effective vibration–based damage detection.

This issue is further amplified when the vibration–based damage detection problem is considered from an asset management viewpoint. In such a case, damage detection is not implemented on a single structure, but, rather, on a population of similar or nominally–identical structures. Of–course, this cannot be pursued by means of training using a single member of the population, as even nominally identical structures are not truly identical due to variability in the materials, manufacturing, assembly, boundary conditions, and so forth, thereby featuring variability or uncertainty in their dynamics that compounds the uncertainty originating from the varying EOCs.

The problem of damage detection for a population of nominally identical structures is effectively unexplored, with a very limited number of studies being available. Yet, the uncertainty in the population dynamics, may be simplistically treated, along with the EOCs stemming uncertainty, into a robustness setting using proper methods to account for this uncertainty. Such robust methods are typically of the machine–learning–type, with their main idea being the construction of a subspace, within a proper feature space, which contains the healthy state dynamics for all the population, under any EOC (Healthy Subspace). In this context, several methods have been proposed over the last two decades, exhibiting very good damage detection performance, yet they are subject to a number of limitations, including: (a) the requirements for relatively high numbers of signal records during the training phase; (b) the selection,
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often subjective, by the user of a number of hyper–parameters; (c) optimization procedures of significant complexity within high–dimensional spaces and/or non–convex problems; (d) assumptions regarding the Healthy Subspace distribution or geometry; (e) user expertise for the construction of the Healthy Subspace, which is not based on a simple and automated procedure.

Therefore, the problem is herein considered and for the first time systematically investigated through a proper benchmark experimental application study, employing a population sample of 31 nominally identical composite aerostructures that feature significant material, manufacturing, and assembly variability, affecting their dynamics. Each population member represents one of the tail booms of a commercial twin tail Unmanned Aerial Vehicle (UAV), while the considered invisible and barely visible damage scenarios are characterized by a combination of delamination, small cracks, and broken fibres, caused by impact at two distinct energy levels.

What is more, the above–mentioned limitations are addressed in the present thesis by postulating a number of novel robust damage detection methods, while using only a single vibration sensor and a limited frequency bandwidth. Toward this end, proper unsupervised Multiple Model (MM) methods, which increase detection performance and require limited vibration response signal records from the healthy structural state, are postulated and experimentally assessed by means of the benchmark application study in chapter 3. The damage detection results indicate that among the considered methods a PCA enhanced MM based method is the most prominent, achieving very good damage detection performance, at 96.3% correct damage detection rate for 3% false alarm rate, even under the significant uncertainty effects of the considered benchmark study.

Yet, this performance is pertinent to proper hyper–parameters selections, requiring user judgment and experience, in the method’s training phase. Toward addressing this limitation, a supervised version of the PCA–enhanced MM based method is postulated in chapter 4. This method employs vibration response signals from healthy as well damaged structures in its training phase, in order to automatically – that is without user intervention – determine its hyper–parameters. The respective damage detection results, for the herein considered population of nominally identical structures, indicate that the supervised method yields improved performance compared to most of its unsupervised counterparts, while it relieves the user from critical selections during its implementation.

However, damaged structures are typically not available during the methods training phase. Thus, a novel unsupervised method is postulated instead in chapter 5, which features automated training requiring only vibration response signals from healthy structures. This method constructs a Healthy Subspace representation, by means of the union of a number of deterministic hyper–spheres with distinct centres and radii, thereby providing a good approximation of any Healthy Subspace geometry. The postulated method’s damage detection performance is assessed by means of the benchmark application study, as well as an additional experimental case study considering damage detection on a single composite aerostructure under simulated local stiffness reduction type of damage and varying EOCs. The method is additionally compared with three well known robust damage detection methods, with the aggregate assessment results indicating the postulated method’s very good detection performance, which exceeds that of the alternative methods, while using limited signal records for its automated training and featuring robustness to any Healthy Subspace geometry.

The latter method’s performance and characteristics are achieved at the cost of increased computational complexity. This issue is addressed in chapter 6, where a novel crude Gaussian mixture model based damage detection method is postulated. The method is formulated within a probabilistic framework, featuring a very simple convex estimation procedure, thereby improving the computational complexity issues of the previous method. The crude Gaussian mixture based method approximates the Healthy Subspace, through the superposition of a proper set of isotropic Gaussian distributions, with distinct, properly defined, means and covariances. The latter method is assessed by means of the two experimental application studies of the previous chapter and through comparisons with its predecessors, as well as other powerful state–of–the–art methods. The assessment results indicate the crude Gaussian mixture
based method excellent performance, which is superior to all the other methods, given its unsupervised and automated operation under limited training signals records and any Healthy Subspace geometry.
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Introduction

Structural Health Monitoring (SHM) [Farrar et al. 2012] has received significant attention over the last two decades [Aravanis, J. Sakellariou, et al. 2020; Fassois and Kopsaftopoulos 2013; Montalvao et al. 2006; Peeters, Maecck, and Roeck 2001a; Sohn 2007; Zhou et al. 2014]. Its purpose is the development of proper methods for continuous, on-board, structural health assessment, including modules for damage detection, localization, and severity estimation. The advantages of such methods, compared to traditional maintenance strategies or Non-Destructive Evaluation (NDE) [Hellier 2012] techniques, are mainly related to service and maintenance costs reduction, as well as structural safety improvements.

These advantages may be further amplified, when a population of nominally identical structures is considered and population–based SHM [Bull et al. 2021; Worden, Bull, et al. 2020] is pursued. The term population of nominally identical structures refers to a set of structures manufactured with the same nominal geometric and material characteristics (for instance aircrafts of the same type, brand, and model), while the term population–based SHM refers to an SHM method trained using a subset of the population and implemented for structural health inference on every population member.

One of the most important families of SHM methods, is the one based on random vibration response signals, since such signals are easy to measure and naturally available, without interrupting the structure’s normal operation, or using specific excitation equipment [Fassois and Kopsaftopoulos 2013][Deraemaeker 2010, pp. 19-32]. The vibration–based SHM methods are divided into two main categories that are based on either: (a) detailed analytical models, such as Finite Element Models, or (b) data–based (inverse type) methods. In the present thesis, only the second category, of data–based methods, is of interest, as they do not require the computationally complex construction of analytical/physics based models for the representation of the often geometrically complex structures.

1.1 The Problem and its Importance

The response–only random vibration–based damage detection problem constitutes the first objective of the broader vibration–based SHM problem [Rytter 1993] and pertains to the detection of the presence of damage on a structure. This problem is of paramount importance, since a damage may be detected using solely vibration response signals from the structure, without interrupting its normal operation or requiring physics based modeling or other elaborate techniques. Once damage is detected, the remaining
SHM modules or other NDE techniques may be implemented, in order to diagnose the location and severity of the damage, as well as its effect on the structural integrity. Therefore, the effectiveness of the vibration–based damage detection module significantly affects the effectiveness of the overall SHM system.

The respective random vibration–based damage detection methods are typically trained in a baseline phase using a number of vibration signal records from the healthy state of the structure that needs to be monitored for the presence of damage. Once the training is completed, the methods are implemented in an inspection phase using new signals from the same structure, in order to deduce its health state. Yet, such an approach where each structure needs to be retained for vibration tests under various operational conditions before it becomes available for normal operation, yields a significant burden in terms of time and costs, especially when nominally identical structures are considered. The latter are manufactured in large numbers so as to have nominally identical geometrical and material characteristics, thus rendering the damage detection methods training per such structure a significant burden for the production procedure and the delivery times of the corresponding structures. Evidently, since the structures are nominally identical, the damage detection methods should be trained using signals from solely one of the structures and then be implemented on each one of the other nominally identical structures. The latter approach provides evident time and cost related advantages, yet its implementation poses a significant challenge that is explained in the sequel once the vibration–based damage detection methods concept is presented.

The random vibration–based damage detection methods use structural dynamics associated features, extracted from proper models of the vibration signals, and are based on the idea that these features change under the presence of damage [Chondros et al. 1989]. Of course, changes in the structural dynamics may additionally manifest due to a multitude of other factors such as varying environmental and operational conditions (EOCs), thereby leading to subsequent changes in the features that may be misinterpreted as relevant to damage [Peeters, Maeck, and Roeck 2001a; Sohn 2007; Zhou et al. 2014]. The aggregate effect of these damage irrelevant changes on the features, may be interpreted as uncertainty, since it yields a locus of all possible healthy state feature values that is not deterministically definable.

The uncertainty effects on the above–mentioned features may be further amplified in an asset management based damage detection context [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b], where damage detection for a population of nominally identical structures (for instance an aircraft fleet or a population of wind turbines) is tackled via methods inferring on the population structural dynamics using only 'sample' structures for their training. The challenge in this case stems from the fact that nominally identical structures depict deviations in their structural dynamics due to a combination of material, manufacturing, and assembly variability which compounds the variability associated with varying EOCs, thus causing a significant 'masking' effect on the damage associated features changes.

The resulting random vibration response–only damage detection for a population of nominally identical structures problem is thus very challenging, yet it is preferred over the alternative of custom training for each individual population member, due to the aforementioned time and cost associated advantages. Essentially, the problem may be practically understood as the construction of a proper representation for the locus of all possible healthy state feature values that is herein denoted as the Healthy Subspace.

1.2 State of the Art

1.2.1 Damage Detection for Population of Nominally Identical Structures

The problem of damage detection for a population of nominally identical structures has been considered in a limited collection of studies. Papatheou et al. [Papatheou, Barthorpe, et al. 2015; Papatheou, Rahman, et al. 2014] employ a small number of nominally identical aircraft wing structures and investigate the potential existence of frequency ranges where dynamical characteristics, such as Frequency Response Function (FRF) magnitude, natural frequencies, or mode shapes, may be common (‘invariant’).
for the considered healthy population, and as such they could be employed as characteristic quantities or features for damage detection. Obviously, such a procedure requires a thorough investigation of potential dynamical characteristics across the population, and subsequent, careful, selection of features that remain ‘invariant’ over it. In addition, it corresponds to a sort of information reduction\(^1\), and as such may not lead to the best possible detection performance. Bull \textit{et al.} [Bull et al. 2021] and Worden \textit{et al.} [Worden, Bull, et al. 2020] further investigate the problem by means of a simulated population of 8 degrees of freedom systems using the respective FRF magnitudes as damage detection features. A Gaussian Process regression model of the population FRF magnitudes is used in these studies to represent the healthy population dynamics and tackle damage detection by tracking deviations from this model, in terms of the Mahalanobis Squared Distance. Of course, the use of the Gaussian process model, may significantly affect the damage detection performance when the healthy population FRF magnitudes follow a different (possibly more complex) distribution. Studies not explicitly focused on populations of structures, yet addressing material uncertainty via simulation within a damage detection context, include [Chandrashekhar et al. 2016; Teimouri et al. 2016] which employ a fuzzy logic and an artificial neural network type system, respectively.

1.2.2 Damage Detection for a Single Structure Under Uncertainty

An alternative approach of addressing the problem is possible, by treating the population member–to–member variability as an additional source of uncertainty. Through this perspective, it is possible to employ a multitude of robust methods developed for tackling damage detection under uncertainty for a single structure, so as to address the present problem. The methods aiming at tackling the uncertainty problem –within a single structure context– may be distinguished into three broad classes: The 1\textit{st class} is based on the idea of ‘separating’ the effects of damage from those of uncertainty on the healthy dynamics, via explicit cause–and–effect type modeling between the uncertainty sources and their effects on the dynamics. Once such a model is obtained, the effects of uncertainty on the healthy dynamics may be ‘compensated for’ (‘removed’), with damage detection feature selection based on the ‘compensated’ dynamics [Comanducci et al. 2016; Hios et al. 2009a; Hios et al. 2009b; Hios et al. 2014; Hu et al. 2016; Ko et al. 2003; Lorenzoni et al. 2016; Peeters, Maeck, and De Roeck 2000; Peeters, Maeck, and Roeck 2001b; Worden and E. Cross 2018; Worden, Sohn, et al. 2002]. Of course, such methods are limited to measurable uncertainty sources (for instance temperature) and are not applicable in the case of a population of structures.

The 2\textit{nd class} is based on a somewhat similar idea, but relaxes the requirement for measurable uncertainty sources. This is achieved by using a multivariate feature space, which conveys structural dynamics related information, and selecting a subspace affected by uncertainty. Damage, is then assumed to (mainly) affect a ‘compensated’ subspace (unaffected by uncertainty), and damage detection is tackled by tracking changes of the feature projection on this ‘compensated’ subspace. Proper decomposition techniques are typically utilized to obtain such subspaces and the resulting methods may be classified into two main methodologies. The first methodology is based on the decomposition (often via Principal Components Analysis, PCA, and Auto–Associative Neural Networks) of the feature variables covariance matrix estimate. This results in a number of components (linear combinations of feature variables) that are ranked in terms of their contribution to the (inherent due to uncertainty) variability of the healthy feature. The components with the smallest contribution (unaffected by uncertainty) are selected to span the ‘compensated’ subspace [Bellino et al. 2010; Comanducci et al. 2016; Deraemaeker et al. 2008; Figueiredo, Park, et al. 2011; Figueiredo and A. Santos 2018; Giraldo 2006; Hu et al. 2016; Kojidi et al. 2014; Kullaa 2010; Manson 2002; Rama Mohan Rao et al. 2015; A. Santos, Silva, Sales, et al. 2015; Sen et al. 2019; Silva et al. 2019; Vanlanduit et al. 2005; Yan et al. 2005]. The second methodology is based on two premises: (a) the feature variables, employed for damage detection, exhibit non–stationary evolution (over long time periods) due to the acting uncertainty sources and (b) the feature variables are

\(^1\text{In the sense that the omitted frequency ranges may convey information related to the presence of potential damage.}\)
Chapter 1. Introduction

cointegrated (see definition and details in [E. J. Cross, Worden, et al. 2011]). A number of components (linear combinations of feature variables) is obtained via feature decomposition (often the Johansen procedure is used [E. J. Cross, Worden, et al. 2011]) and ranked in terms of their stationarity, with the ‘most stationary’ one (unaffected by uncertainty) being selected to span the ‘compensated’ subspace [E. J. Cross, Manson, et al. 2012; E. J. Cross, Worden, et al. 2011; Worden, Baldacchino, et al. 2016].

The limitation of this class of methods stems from the fact that information reduction is present, leading to potential loss of damage related information.

The 3rd class includes approaches that involve the probabilistic modeling of the dynamics for the entire healthy population, without requiring uncertainty source measurability. Such modelling features a Gaussian or non–Gaussian distribution, while the respective methods attempt to fully represent the healthy population dynamics under uncertainty without performing information reduction. The price to be paid for such methods is mainly associated with increased conceptual and computational complexity, as well as a requirement for sufficiently high numbers of vibration signal records during their training phase [Avendaño–Valencia and Fassois 2014; Avendaño–Valencia and Fassois 2015a; Avendaño–Valencia and Fassois 2015b; Avendaño–Valencia and Fassois 2015c; Chakraborty et al. 2015; Figueiredo, Radu, et al. 2014b; Kullaa 2014; Michaelides, Apostolellis, et al. 2011; Michaelides and Fassois 2013; Poulimenos et al. 2018; Vanik et al. 2000].

The aforementioned robust methods have been shown to achieve very good damage detection performance when a single structure is considered, yet these may be insufficient for the present problem of damage detection for a population of nominally identical structures as indicatively depicted for three state of the art and a conventional methods in Figure 1.1. The performance is therein presented in terms of Receiver Operating Characteristic (ROC) curves [Duda et al. 2000, pp. 34–35],[Fawcett 2006], each representing the true positive rate (percentage of correct damage detections) versus the false positive rate (percentage of false alarms) for varying decision threshold. The methods results for the particular problem indicate that their performance does not exceed the important performance rate of 90% correct damage detections, for false alarm rate as high as 5%. Nevertheless, these methods are additionally characterized by their own limitations, such as the requirements for:

1. Relatively high numbers of signal records for training (thousands of signal records may be required [Figueiredo and A. Santos 2018]).
2. The selection, often subjective, by the user of a number of hyper–parameters, as for instance illustrated in chapter 3 and chapter 4. The respective indicative effects of such selections on the damage detection performance of the well–known PCA–based method is showcased in Figure 1.2, via ROC curves (see ROC curves explanation in section 3.4). The respective curves depict three possible hyper–parameters user–based selections and the significant impact on the detection performance, which varies from good to mediocre.
3. Optimization procedures of significant complexity within high–dimensional spaces and/or non–convex problems [Figueiredo, Radu, et al. 2014a; Figueiredo and A. Santos 2018; Rogers et al. 2019]
4. Assumptions regarding the healthy features distribution [Bull et al. 2021; Yan et al. 2005]
5. user expertise for the methods training, which is not based on a simple and automated procedure
1.3 Thesis Goal, Objectives, and Assumptions

1.3.1 The Goal

The present thesis aims at systematically addressing the effectively unexplored problem of vibration-based damage detection for a population of nominally identical structures, by setting up a benchmark experimental application study that employs a collection of nominally identical composite aerostructures. Each structure corresponds to the half tail of an Unmanned Aerial Vehicle (UAV) and is subject to impact induced type of damages. This case study represents the first systematic attempt of experimentally addressing the problem and the respective vibration response signals constitute a valuable data-base.
that may be used as benchmark for future studies. The final thesis goal is then the postulation of proper methods that effectively tackle the problem using solely vibration response signals, do not require measurement of the uncertainty sources, and alleviate some or most of the above-mentioned limitations, while improving damage detection performance.

1.3.2 The Objectives

The main objectives of the present thesis are the following:

• The exploration of the vibration-based damage detection for a population of nominally identical structures problem, through vibration experiments with the collection of nominally identical composite aerostructures and investigation of their structural dynamics characteristics.
• The problem exploration through conventional and novel methods.
• The methods automated training, toward an easy implementation that does not require user judgment and expertise.
• The postulation of conceptually and computationally simple methods for damage detection under uncertainty.
• The achievement of high damage detection performance, given limited vibration response signals in the training (baseline) phase.

1.3.3 The Assumptions

The problem is addressed in the present thesis under the following assumptions:

1. The considered structures are characterized by linear and time invariant dynamics.
2. The excitation and response signals are assumed as zero mean, Gaussian random signals.
3. Only a single damage is considered in the structure.
4. The EOCs are non-measurable and constant during the measurement of a single signal.
5. The vibration excitation is not measured (response-only vibration-based damage detection) and its dynamics is incorporated in the damage detection problem as an extra source of uncertainty.
6. Vibration measurements from a single location on the structure are available.
7. A single type of signals such as vibration displacement, velocity, or acceleration is available for all the vibration measurements.
8. Sufficient vibration response signals under various conditions are available in the method's baseline (training) phase for representing the healthy state dynamics.
9. The damage detection problem is tackled in a batch mode, implying that the method operates on short duration batches of signal records collected periodically or on demand over time, with diagnostic decision making implemented at the end of a complete batch.
1.4 The Main Concept and Ideas

The ideas and the main concept developed by the present thesis in order to tackle the aforementioned problem, meet the respective objectives and alleviate the current state of the art limitations, are presented below.

The postulated damage detection framework is based on the idea that a number of vibration response signals from a sample collection of nominally identical structures affected by a sample set of EOCs is available in the baseline (training) phase of the damage detection methods. These signals are modeled via AutoRegressive (AR) models and the AR parameters, which convey structural dynamics relevant information are used as damage sensitive features. The idea is that the set of all these parameter vectors creates a swarm of points in the respective feature space, which may be used to define a subspace of the healthy state dynamics (healthy subspace) for the population of structures operating under varying EOCs. Then in the inspection (diagnostic) phase a new vibration response signal is obtained from a random member of the population of nominally identical structures and its AR model is identified. This structure is then characterized as damaged if and only if the respective parameter vector resides outside of the healthy subspace.

In order to define such a healthy subspace the current thesis, postulates four conceptually simple methods. The first method approximates the healthy subspace by means of a proper collection of healthy state AR models and the projection of their parameter vectors Gaussian distributions on a feature space of reduced dimensionality and increased sensitivity to damage, using the PCA technique. The second method automates the feature space dimensionality reduction procedure, by incorporating vibration response signals from damaged states of the structures, so as to relieve the user from critical selections during the baseline phase of the method. Yet, damaged structures are typically not available during the methods baseline phase, thus leading to the third method, which features automated operation, given limited healthy state vibration response signals in the baseline phase, and robust damage detection performance. The latter is achieved via the union of a number of hyper–spheres in the feature space, with proper centers and radii, which represent the healthy subspace. The last method attempts to further improve the damage detection performance and simplify the training procedure compared to the third method. Toward this end, the healthy subspace is approximated through the superposition of a proper collection of isotropic Gaussian distributions, while the method’s baseline and inspection phases are formulated within a probabilistic framework.

1.5 Chapter–wise Outline and Novel Contributions

The exploration of the benchmark experimental application study that considers the problem of vibration–based damage detection for a population of nominally identical structures, along with the respective experimental assessment of proper state–of–the–art methods, and the postulation and assessment of novel ones is herein organized in four main chapters. The chapters titles are summarized in Table 1.1, while their main contributions and limitations that connect each chapter with its predecessor are provided in Figure 1.3. Essentially, the chapters interconnection is based on the idea that the limitation of a chapter is the motivation for the next one. Each one of the four chapters is further outlined below.

1.5.1 Chapter 1: Vibration–Based Damage Detection for a Population of Nominally Identical Structures –Unsupervised Multiple Model (MM) Statistical Time Series Type Methods

In this chapter the problem of vibration–based damage detection for a population of nominally identical structures is explored using a sample population of nominally identical composite beams [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. Each beam is a small scale representation of an Unmanned Aerial Vehicle (UAV) tail boom, with the experimental set–up being properly configured to approximate the boom attachment to the aircraft fuselage and tail. The sample population consists of 23 healthy and 8 damaged beams with significant uncertainty in their dynamics due to material,
Chapter 1. Introduction

Table 1.1: Thesis main chapters.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vibration–based damage detection for a population of nominally identical structures – unsupervised multiple model (MM) statistical time series type methods</td>
</tr>
<tr>
<td>2</td>
<td>Vibration–based damage detection for a population of nominally identical structures – supervised PCA–enhanced parametric statistical time series type methods</td>
</tr>
<tr>
<td>3</td>
<td>An automated Hyper–Sphere based healthy subspace method for robust and unsupervised damage detection via random vibration response signals</td>
</tr>
<tr>
<td>4</td>
<td>A crude Gaussian mixture model based healthy subspace method for automated and unsupervised damage detection via random vibration response signals</td>
</tr>
</tbody>
</table>

Fig. 1.3. The thesis chapters main characteristics.

manufacturing, and assembly variability [Atlani et al. 2018; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. Each damaged beam is induced with a unique impact type damage scenario, characterized by the impact’s location and energy.


Of–course, such an observation is expected for these methods and therefore an alternative “robust” to uncertainty method is employed. This is a PCA based method that is frequently used in the literature and properly adapted for the current problem [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. The latter method creates a deterministic representation of the healthy state dynamics under uncertainty, using a subspace spanned by a subset of

\(^1\)Only the healthy vibration response characteristics are modeled for the method’s unsupervised training.
the principal components. The method yields clearly improved performance over the STSMs, yet this is still not within the acceptable performance range, while it requires user expertise and prior knowledge of the dynamics and the uncertainty to tune its hyper-parameter (i.e. number of principal components).

The solution is given by a proper adaptation of the Multiple Model (MM) framework [K. J. Vamvoudakis–Stefanou and Fassois 2014; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015b]. This uses a collection of simple AutoRegressive (AR) models from multiple sample structures of the population and under various conditions, so as to create a crude Gaussian Mixture (GM) model approximation for representing the healthy state dynamics. The method provides very good damage detection results and it is considered a viable solution for the problem. Unfortunately, this is not without limitations either, as it requires vibration response signals from a high number of structures under multiple conditions, so as to create a proper representation, while there is not a guideline for selecting this number.

The limitation of the MM based method is tackled by postulating a PCA–enhanced Multiple Model statistical time series method [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. This uses the parameters from multiple AR models as structural dynamics associated features and creates within the respective feature space a subspace (spanned by a subset of the principal components) that contains a significant part of the healthy state dynamics under uncertainty. The resulting subspace is a linear continuous (possibly high–dimensional) function (e.g. a line, a plane, etc.) and the MM representation is cast around it to model only the healthy dynamics that exceed this subspace, thereby requiring a lower number of signals.

The method is tested and systematically assessed using the population of nominally identical composite beams. The results indicate very good damage detection performance that exceeds the one achieved by the MM based method [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. Nevertheless, this is a performance that may deteriorate when the principal components spanning the healthy dynamics representing subspace are not properly selected. Due to the method’s unsupervised formulation this selection is based on user expertise and prior knowledge of the uncertainty effects on the AR model parameters, thereby yielding a burden for the potential user.


In order to relieve the user from the burden of properly tuning the PCA–enhanced MM based method, its supervised version that uses healthy and damaged vibration response characteristics during its training phase, is postulated in this chapter. The idea is that by incorporating signals from damaged structures of the population in the training phase, the principal components may be automatically selected as the ones yielding the best detection performance for these damages.

The supervised version is assessed using the population of nominally identical beams, depicting very good detection performance. This may be slightly decreased compared with its unsupervised counterpart, yet it is acquired automatically, thereby alleviating the limitation of the PCA–enhanced MM based method, while maintaining its advantages. The method is further assessed with respect to its sensitivity to the type of damages used for its training, showing that the type affects its performance, yet this always remains very good. The chapter’s only limitation stems from the fact that the requirement for damaged structures in the baseline phase is often difficultly satisfied, thus unsupervised damage detection alternatives, should be considered as well.

### Chapter 3: An Automated Hyper–Sphere Based Healthy Subspace Method for Robust and Unsupervised Damage Detection via Random Vibration Response Signals

An automated unsupervised damage detection method using the AR model parameters as features is postulated in this chapter to address the limitations of the previous chapters. This attempts to create a
deterministic approximation of the subspace containing all the healthy state dynamics under uncertainty (i.e. healthy subspace), via the union of a number of hyper–spheres, with properly determined centers and radii. Then, a parameter vector residing outside of the subspace is declares the existence of damaged. The number of centers, their position, and the radii (the method’s hyper–parameters) are automatically determined via a proper procedure that artificially creates parameter vectors for the healthy and damaged states. The hyper–parameters are then selected as the set yielding the best detection performance for the artificial vectors.

The method’s experimental assessment, using the population of nominally identical beams, as well as a single beam under significantly varying EOCs, indicates its very good damage detection performance and the effectiveness of its hyper–parameters automated selection. Compared with its predecessors the method achieves the “best” performance, given its unsupervised formulation, and effectively tackles most of their limitations. The chapter’s limitation is associated with increased computational complexity for the training and implementation of the postulated method.

1.5.4 Chapter 4: A Crude Gaussian Mixture Model Based Healthy Subspace Method for Automated and Unsupervised Damage Detection via Random Vibration Response Signals

In order to further improve the detection performance achieved in the previous chapter and alleviate the respective computational complexity issues, the problem is cast into a conceptually and computationally simple probabilistic framework. The resulting method is based on a representation of the healthy dynamics subspace that resembles a Gaussian mixture model (crude Gaussian mixture), yet its estimation is based on simple convex estimators and an automated training procedure.

The method is experimentally assessed and compared with powerful state–of–the–art Gaussian mixture, MM, and PCA based methods, using the population of nominally identical beams, as well as a single beam under significantly varying EOCs. Both experimental case studies indicate the excellent damage detection performance and robustness to uncertainty of the postulated method, as well as its superiority over the alternative methods.
The benchmark experimental study 

2.1 The composite beams 

2.2 The damage scenarios 

2.3 The experimental set-up and data acquisition 

2.4 Preliminary analysis: Uncertainty sources and effects of damage on the dynamics 

2.5 Structure of the benchmark study database
A paradigm application study, addressing the problem of damage detection for a population of nominally identical composite beams is presented in this chapter. This is a benchmark experimental study, employing a population of 31 nominally identical composite aero–structures with significant uncertainty in their dynamics, due to a multitude of uncertainty sources. The structures, the experimental set–up, the vibration experiments and the considered damage scenarios are presented in the sequel.

2.1 The Composite Beams

The study is based on 31 nominally identical sample composite beams consisting of several layers of woven and unidirectional (UD) fabric manufactured based on one shot Resin Transfer Molding (RTM) by Atard Defence & Aerospace Inc., Turkey. In further detail, 11 plies of twill (3K carbon fibre fabric of 200 gr/m²) and unidirectional (12K carbon fibre fabric of 300 gr/m²) carbon fibre weave are used following the pattern of Figure 2.1. The resin is injected under 6 bar pressure, by means of the Araldite LY 1564 system and using the Aradur 3486 hardener, while the curing is completed under 60°C for 10 hours and 100°C for 2 hours. The beams have square hollow cross section, uniform along their length, with nominal dimensions 600 × 65 × 65 mm (LxWxH), wall thickness of 3 mm, corner radius of 8 mm (see additional dimension details in Figure 2.2) and nominal mass 737 g. Each beam represents the topology of the main part of a commercial UAV boom and it is tightly clamped at one end simulating its connection to the UAV fuselage, while its free end is attached to an aluminium mass representing the aircraft tail (Figure 2.3).

2.2 The Damage Scenarios

The considered damage scenarios are induced via a pendulum type impact hammer by Fundacion Centro de Tecnologias Aeronauticas (CTA), Spain. These include a combination of delamination, small cracks and broken fibres, according to a thermography analysis implemented by CTA on each beam, with Figure 2.4 providing indicative thermography results for a specific beam. In particular, eight beams are damaged at different points (A, B, C, D) by using a distinct impact energy level of 5 J (Low – L) or 15 J (Higher – H). In addition, the damaged beams visual inspection indicates that the 5 J case leads to an
Fig. 2.1. Manufacturing details for the composite beams: the composite structure layers and the mandrel set-up. [Poulimenos et al. 2018].

Table 2.1: The damage scenarios and the experiments [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b].

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Impact Energy (J)</th>
<th>Damage Visual Characterisation</th>
<th>Impact Position</th>
<th>Number of beams</th>
<th>Number of experiments per beam</th>
<th>Total number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>—</td>
<td>—</td>
<td>23</td>
<td>7</td>
<td>161</td>
<td>7</td>
</tr>
<tr>
<td>Damage AL</td>
<td>5</td>
<td>Invisible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BL</td>
<td>5</td>
<td>Invisible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CL</td>
<td>5</td>
<td>Invisible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DL</td>
<td>5</td>
<td>Invisible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage AH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Each damage scenario is designated using two letters, the first indicating damage location (A,B,C,D; see Figure 2.6) and the second impact energy level (L: 5 J, H: 15 J).

invisible damage, while the 15 J to barely visible damage. Figure 2.5 provides indicative delamination sizes for the invisible (5 J) and barely visible (15 J) damage scenarios. More details on the damage scenarios are provided in Table 2.1 and Figure 2.6.

2.3 The Experimental Set-up and Data Acquisition

Each beam is excited at its free end with a random white Gaussian force applied vertically at Point X (see Figure 2.3) via an electromechanical shaker (LDS Model V406) equipped with a stinger and a vibration controller (LDS COMET USB COM–200). The exerted force, although not used in any of the
2.4 Preliminary Analysis: Uncertainty Sources and Effects of Damage on the Dynamics

Fig. 2.2. Sketch of the half tail with the basic dimensions (in mm) of the composite beam, the mass (horizontal stabilizer) and the clamping, along with the impact locations (Points A, B, C, D), the measurement Point Y1, and the excitation Point X: (a) top view, (b) side view.

damage detection methods of this study, is measured via an impedance head (PCB 288D01, sensitivity 98.41 mV/lb), while the vibration acceleration response signal is measured at point Y1 via a lightweight accelerometer (PCB ICP 352C22, 0.5 g, bandwidth 0.003 – 10 kHz, sensitivity ~ 1.052 mV/m/s²) as shown in Figure 2.3 and Figure 2.6. The measured signals are driven through a signal conditioner (PCB F482A20) into the data acquisition system that is a National Instruments 9234 module featuring four 24–bit simultaneously sampled A/D channels, anti–aliasing filters and specialized software provided by the Advances & Innovation in Science and Engineering CO EE (ADVISE), Greece. 161 experiments are carried out at the Stochastic Mechanical Systems & Automation Laboratory of the University of Patras, Greece, with 23 healthy beams (7 experiments per beam), and 56 additional experiments with 8 damaged beams. Each measured vibration signal is sample mean corrected and normalized by its own standard deviation. Full details about the experiments, the signals and their non–parametric and parametric modeling are provided in Table 2.1 and Table 2.2. Note that all the models used in the following sections are estimated according to the details of Table 2.2.

2.4 Preliminary Analysis: Uncertainty Sources and Effects of Damage on the Dynamics

Figure 2.7 depicts the rectangular cross–section, the thickness and the mass of the 23 healthy beams as well as the temperature under which the experiments were carried out. Except for the experiments with the third beam, the temperature during the experiments may be considered constant. Yet, it is obvious that the other three reported characteristics of the healthy beams are significantly varying due to manufacturing and materials variability. This, compounded with the (unmeasurable) operational variability due to the
Chapter 2. The Benchmark Experimental Study

Fig. 2.3. Experimental set-up: Point X represents the force excitation position, Point Y1 the vibration acceleration measurement position [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015a].

assembly of the aluminium tail on each beam as well as each beam’s mounting on the clamping indicate that there are several uncertainty sources that significantly affect the dynamics of the healthy beams.

This fact is confirmed through indicative Welch–based FRF magnitude estimates [Ljung 1999, pp. 173–187] for four healthy beams in Figure 2.8(a), in which the discrepancies in their dynamics are evident in the ranges of 600–900 Hz and 1 700 – 2 100 Hz. In addition, FRF magnitude estimates for the 23

Fig. 2.4. Indicative thermography analysis results for a specific composite beam subject to 15 J impact energy (barely visible) damage, revealing: (a) delamination, (b) fibres breakage, and (c)(d) a wide area with voids [Poulimenos et al. 2018].
2.4 Preliminary Analysis: Uncertainty Sources and Effects of Damage on the Dynamics

Fig. 2.5. Indicative delamination size for a beam subject to: (a) 5 J impact energy, (b) 15 J impact energy. [The white lined area indicates the damage area under the upper layer of the composite material.]

Fig. 2.6. Sketch (top view) of the experimental set–up with the beam, the tail mass, the clamping, the damage locations and the measurement Point Y1 (dimensions in mm).

Table 2.2: Vibration signal and estimation details.

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling frequency:</td>
<td>$f_s = 4654.5$ Hz, Signal bandwidth: $[5 - 2327.25]$ Hz</td>
</tr>
<tr>
<td>Signal normalized via sample mean removal and division by its sample standard deviation</td>
<td></td>
</tr>
<tr>
<td>Signal length used in non–parametric representations: $N = 112000$ samples ($24.06$ s)</td>
<td>Welch based estimation details: Hamming window; $8192$ samples long segment; $90%$ overlap; Matlab function $tfestimate.m$</td>
</tr>
<tr>
<td>Signal length used for AR modeling: $N = 10000$ samples ($2.15$ s)</td>
<td>AR model estimation details: Ordinary Least Squares (OLS), QR implementation, Matlab func.: $arx.m$</td>
</tr>
</tbody>
</table>

healthy beams as well as for the damaged ones under low (invisible damage) and higher (barely visible damage) impact energy are presented in Figure 2.8(b),(c), respectively, while zooms of the most affected frequency ranges (per impact energy) are presented in Figure 2.9. From these it is obvious that the significant variability of the healthy beams dynamics completely 'masks' the effects of the low magnitude damages (Figure 2.9(a),(b)), thus pinpointing a highly challenging damage detection problem. On the other hand, small deviations between the healthy and damaged dynamics are exhibited in Figure 2.9(c),(d) for the considered scenarios with the barely visible damage. This effect is in further detail depicted
in Figure 2.10 where the PSD estimates [Ljung 1999, pp. 173–187] envelope for all the healthy state experiments is compared with its counterpart corresponding to each distinct damage scenario. It is evident that the healthy state envelope significantly overlaps with most of its counterparts corresponding to the distinct damage scenarios. Exception to this observation are damage scenarios AH, BH and CH where discrepancies between the healthy and damaged state envelopes are depicted in the high frequency range.

2.5 Structure of the Benchmark Study Database

The vibration signals obtained from the composite beams during the vibration experiments for the benchmark application study described above, are organised in a database. The database constitutes a directory tree, the structure of which is shown in Figure 2.11. This consists of two levels according to the health state and the beam index. Each level of the database contains the folders of the next level. The data files themselves are txt files of the form ‘Experiment name _realization number -vibr.txt’.

The ‘experiment name’ tag stands for the beam number, while the ‘realization number’ for the number of the experiment with the specific beam, for instance ‘Beam 1_Experiment 1-vibr.txt’. These files contain the non–normalized excitation–vibration acceleration response signals in a column wise set–up, with the first column being the excitation signal (in Volts, needs to be divided by 0.0224 to obtain force measurements in N), and the second column corresponding to the measurement Point Y1 acceleration signal (in g).
2.5 Structure of the Benchmark Study Database

Fig. 2.8. Preliminary analysis: Welch–based FRF magnitude estimates for (a) four indicative healthy beams, (b) all healthy and the four beams subject to invisible damage and, (c) all healthy and the four beams subject to barely visible damage (single experiment per beam).

Fig. 2.9. Preliminary analysis: Zooms on the Welch–based FRF magnitude estimates for different frequency ranges: (a),(b) all healthy and the four damaged beams (invisible damage); (c),(d) all healthy and the four beams damaged beams (barely visible damage). [Single experiment per beam.]
Fig. 2.10. Effects of uncertainty on the population of nominally identical beams dynamics and its ‘masking’ effect on each damage scenario (all experiments per healthy/damaged beam). [Each colored zone represents an envelope containing the Welch–based PSD estimates for all the experiments and beams corresponding to a particular structural health state; the purple arrows indicate areas of great discrepancy between the healthy and damaged PSD estimate envelopes.].
2.5 Structure of the Benchmark Study Database

Fig. 2.11. The structure of the database containing the vibration signals from the benchmark experimental study.
# Unsupervised Multiple Model type methods

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<th>Page</th>
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Vibration–Based Damage Detection for a Population of Nominally Identical Structures: Unsupervised Multiple Model (MM) Statistical Time Series Type Methods

The problem of vibration–based damage detection for a population of nominally identical structures is considered in this chapter via unsupervised statistical time series type methods. For this purpose a population sample comprising 31 nominally identical composite beams with significant beam–to–beam variability in the dynamics is employed, with impact–induced damage at various positions and two distinct energy levels. Two Multiple Model, MM, based statistical time series type methods are postulated, assessed, and compared with two ‘conventional’ methods. The assessment is based on a comprehensive and systematic procedure, making use of thousands of test cases via a ‘rotation’ procedure, with the results presented in the form of Receiver Operating Characteristic, ROC, curves. These indicate that ‘conventional’ methods are mostly ineffective, especially with low impact energy damages. On the other hand, the postulated Multiple Model parameter based methods achieve significantly improved performance, characterized as very good and providing overall correct damage detection rates approaching 100% for false alarm rates at or above 5%.

3.1 Introduction

Vibration–based damage detection, as part of the broader Structural Health Monitoring (SHM) problem, is practically important as vibration signals are easy to measure, the measurement and acquisition technology is mature and at reasonable cost, and the signals are often naturally available without interrupting the structure’s normal operation or requiring specific excitation equipment [Fassois and Kopsaftopoulos 2013][Deraemaeker 2010, pp. 19–32].

The main premise on which vibration–based damage detection is based is that damage affects the structural dynamics, and these effects manifest themselves as changes in the vibration response characteristics[Chondros et al. 1989]. In most cases a baseline (training) phase is required in which the healthy (unsupervised case) or healthy and damaged (supervised case) vibration response characteristics (typically in the form of characteristic quantity or feature) are modeled, so that deviations may be, in the method’s inspection (operational) phase, subsequently detected [Barthorpe et al. 2012; Fassois and

1The challenging vibration–response–only case is currently considered; the extension to the excitation–response case is straightforward.
A current technology barrier is that the baseline modeling accuracy is inadequate under varying operating (loading and boundary) and/or environmental (temperature, humidity, and so on) conditions, and this may have serious detrimental effects on damage detection performance due to the fact that the changes in the vibration response signal characteristics may be so significant as to limit or completely 'mask' those caused by damage [Sohn 2007]. A considerable body of work has been devoted to overcoming or alleviating this problem, thereby strengthening the methods’ robustness —more details in the sequel.

Another limitation, which is in the focus of the present study, is that current vibration–based methods are assumed to operate on a single and particular structure to which they are ‘tuned’/‘trained’ in the baseline phase. Yet, from an asset management viewpoint, one is oftentimes interested in monitoring and performing damage or fault detection not on a single, but, rather, on a population of similar or nominally–identical structures [Papatheou, Barhorpe, et al. 2015]. Typical cases may include aircraft fleets, wind turbine facilities, rail vehicles, and many more. It is clear —and is re–confirmed in section 3.4 of the present study—that this cannot be effectively achieved based on ‘training’ using a single member of the population. The reason is associated with the fact that even nominally identical structures are not truly identical due to variability in the materials, manufacturing, assembly, boundary conditions, and so forth, which cause corresponding variability or uncertainty in the dynamics and the vibration response characteristics. On the other hand, the alternative of performing separate ‘training’ for each population member is not desirable due to the required time and cost.

The problem has been considered by Papatheou et al. [Papatheou, Barhorpe, et al. 2015; Papatheou, Rahman, et al. 2014] who employ a limited number of nominally identical aircraft wing structures and investigate the potential existence of frequency ranges where dynamical characteristics, such as Frequency Response Function (FRF) magnitude, natural frequencies, or mode shapes, may be common (‘invariant’) for the considered healthy population, and as such they could be employed as characteristic quantities or features for damage detection. Obviously, such a procedure requires a thorough investigation of potential dynamical characteristics across the population, and subsequent, careful, selection of features that remain ‘invariant’ over it. In addition, it corresponds to a sort of information reduction², and as such may not lead to the best possible detection performance. Studies not explicitly focused on populations of structures, yet addressing material uncertainty via simulation within a damage detection context, include [Chandrashekhar et al. 2016; Teimouri et al. 2016] which employ a fuzzy logic and an artificial neural network type system, respectively.

Another route, arguably somewhat simplistic and perhaps suitable under limited population member–to–member variability, would be to attempt casting the problem into a ‘robustness’ setting³, meaning to view the population member–to–member variability simply as additional uncertainty⁴. The methods aiming at tackling the uncertainty problem —within a single structure context—may be distinguished into three distinct, broad, classes: The 1st class is based on the idea of ’separating’ the effects of damage from those of uncertainty on the healthy dynamics by explicit cause–and–effect type modeling of the latter. Once such a model is available, the effects of uncertainty on the healthy dynamics may be ‘compensated for’ (‘removed’), with damage detection characteristic quantity or feature selection based on the ’compensated’ dynamics [Comanducci et al. 2016; Hios et al. 2009a; Hios et al. 2009b; Hios et al. 2014; Hu et al. 2016; Ko et al. 2003; Lorenzoni et al. 2016; Peeters, Maeck, and De Roeck 2000; Peeters, Maeck, and Roeck 2001b; Worden and E. Cross 2018; Worden, Sohn, et al. 2002]. Such an approach is limited to measurable uncertainty sources (causes), while requires the availability of experiments with various uncertainty source levels in the baseline (learning) phase of the healthy structure for enabling proper modeling. This may be possible for certain uncertainty sources (for instance temperature), but not

---

²In the sense that the omitted frequency ranges may convey information related to the presence of potential damage.

³The robustness setting may, indeed, not be entirely suitable as it does not attempt to model the natural variability present in the population.

⁴Additional to the uncertainty stemming from varying operational and environmental conditions.
3.1 Introduction

necessarily for others.

The 2nd class employs a somewhat similar idea, but relaxes the requirement for uncertainty source measurability. This is achieved by using a multivariate feature space, which conveys structural dynamics related information, and selecting a subspace affected by uncertainty. Damage, is then assumed to (mainly) affect a ‘compensated’ subspace (unaffected by uncertainty), orthogonal to the selected one, and damage detection is tackled by tracking changes of the feature projection on this ‘compensated’ subspace. Proper decomposition techniques are typically utilized to obtain such subspaces and the resulting methods may be classified into two main methodologies. The first methodology is based on the decomposition (often via Singular Value Decomposition, SVD, and related methods) of the feature variables covariance matrix estimate. This results in a number of components (linear combinations of feature variables) that are ranked in terms of their contribution to the (inherent due to uncertainty) variability of the healthy feature or characteristic quantity. The components with the smallest contribution (unaffected by uncertainty) are selected to span the ‘compensated’ subspace [Bellino et al. 2010; Comanducci et al. 2016; Deraemaeker et al. 2008; Figueiredo, Park, et al. 2011; Giraldo 2006; Hu et al. 2016; Kojidi et al. 2014; Kullaa 2010; Manson 2002; Rama Mohan Rao et al. 2015; A. Santos, Silva, Sales, et al. 2015; Vanlanduit et al. 2005; Yan et al. 2005]. The second methodology is based on two premisses: (a) the characteristic quantity or feature variables, employed for damage detection, exhibit non–stationary evolution (over long time periods) due to the acting uncertainty sources and (b) the feature variables are cointegrated (see definition and details in [E. J. Cross, Worden, et al. 2011]). A number of components (linear combinations of feature variables) is obtained via feature decomposition (often the Johansen procedure is used [E. J. Cross, Worden, et al. 2011]) and ranked in terms of their stationarity, with the ‘most stationary’ one (unaffected by uncertainty) being selected to span the ‘compensated’ subspace [E. J. Cross, Manson, et al. 2012; E. J. Cross, Worden, et al. 2011; Worden, Baldacchino, et al. 2016]. The limitation of this class of methods stems from the fact that information reduction is present, leading to potential loss of damage related information.

The 3rd class includes approaches that involve the probabilistic modeling of the dynamics for the entire healthy population, also without requiring uncertainty source measurability. This would involve a Random Coefficient (RC) type model of the healthy dynamics, that is a model with parameter vector following a Gaussian or non–Gaussian distribution. Methods in this class may, in principle, be claimed to be ‘most appropriate’ for the problem, in the sense that they attempt to fully represent the dynamics of the healthy population without information truncation. The price to be paid for such a method mainly is that of increased complexity and the need for a sufficiently high number of vibration signal records [Avendaño–Valencia and Fassois 2014; Avendaño–Valencia and Fassois 2015a; Avendaño–Valencia and Fassois 2015b; Avendaño–Valencia and Fassois 2015c; Chakraborty et al. 2015; Figueiredo, Radu, et al. 2014b; Kullaa 2014; Michaelides, Apostolellis, et al. 2011; Michaelides and Fassois 2013; K. J. Vamvoudakis–Stefanou and Fassois 2014; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015b; Vanik et al. 2000].

The present study is motivated by an industrial problem in which vibration–based damage detection is of interest for a population of nominally identical composite beams, each one representing the topology of the main part of a commercial Unmanned Aerial Vehicle (UAV) boom. The study is based on a population sample consisting of 31 beams. Each damage scenario is characterized by a combination of delamination, small cracks, and broken fibres caused by impact at two distinct (Low, 5 J, and Higher, 15 J) energy levels. The problem is highly challenging due to considerable beam–to–beam variability in the dynamics and the additional uncertainty associated with operational and environmental conditions. The resulting overall variability is in fact so significant as to nearly ‘mask’ that caused by damage.

The problem has been tackled (in various forms and with different population samples) by the present author in preliminary studies reported in a series of conference papers. The use of conventional

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Note that the population may, theoretically, include infinite members (for instance future production), while the term ‘sample’ is employed to designate the currently available members based on which inference on the population is to be made.

The main goals of the present study are:

(i) A systematic study of vibration–based damage detection in a population of nominally identical structures;

(ii) The postulation of two MM–based methods;

(iii) Their thorough assessment, including comparisons with two ’conventional’ methods.

All methods employed are of the model parameter based form, which has been confirmed as offering high detection performance [Kopsaftopoulos et al. 2011; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2014b], and presently use AutoRegressive (AR) representations of the dynamics. They are also of the unsupervised form; supervised counterparts, which may achieve increased detection performance at the price of including vibration signals from damaged sample structures in the baseline phase, are to be treated in 4.

The Multiple Model (MM) based methods postulated in this work are conceptually simple, and are based on representing the (variable) healthy structural dynamics via a set of estimated conventional models, specifically via their parameter vector Gaussian probability density functions (also see [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015b]). The characteristic quantity or feature is then obtained from this MM representation. The methods may be thought of as RC model based, in the sense that a MM representation of the healthy dynamics may be interpreted as a crude non–Gaussian Random Coefficient (RC) model, with the set of models defining a Gaussian Mixture (GM) type representation of the parameter density function [Avendaño–Valencia and S. D.Fassois 2017a; Avendaño–Valencia and S. D.Fassois 2017b]. The advantage of MM representations over proper RC counterparts is that they are much simpler to estimate, and this may be accomplished based on a limited number of signal records. On the other hand, their approximation of actual RC models is only crude.

The study offers a number of unique characteristics and innovative contributions, which may be summarized as follows:

1. The problem of vibration–based damage detection for a population of nominally identical structures is, for the first time, systematically addressed within an unsupervised framework using 31 nominally identical composite beams with significant beam–to–beam variability in the dynamics.

2. The damage scenarios include delamination, small cracks, and broken fibres caused by impact at various positions and at two distinct energy levels. The overall variability in the dynamics of the healthy population nearly ‘masks’ that caused by damage.

3. Additional problem challenges include the vibration–response–only setting, the use of a single vibration sensor, and a limited (5 to 2 327 Hz) frequency bandwidth.


5. The MM–based methods are comprehensively assessed via a high number (5 600 to 8 400) of test

---

The methods are based on MM representations of the healthy dynamics and should not be confused with Multiple Model methods used for fault isolation in which each single model represents a distinct faulty state of a system [Zhao et al. 2015].

The use of the minimal possible number of sensors obviously is highly desirable.
3.2 Precise Problem Statement

cases (inspection experiments) employing a proper ‘rotation’ procedure warranting the reliability of the results which are presented in terms of ROC curves. Comparisons with two ‘conventional’ statistical time series type methods are also made.

6. A sensitivity analysis is performed in order to reveal the effects of user selected parameters for each method.

The study aims at providing answers to the following broad questions:

Q1. To what extent may ‘conventional’ unsupervised methods tackle the problem?

Q2. Are the unsupervised MM–based methods capable of effectively tackling the problem?

Q3. What performance levels are achievable under the previously stated conditions?

The rest of this chapter is organized as follows: The precise problem statement is presented in section 3.2. The paradigm application problem treated is presented in section 3.3. Damage detection based on two ‘conventional’ statistical time series type methods is presented in section 3.4. The Multiple Model based methods are presented in section 3.5, while their experimental assessment is presented in section section 3.6. Concluding remarks are finally summarized in section 3.7.

3.2 Precise Problem Statement – the Unsupervised Detection Problem

<table>
<thead>
<tr>
<th>Given:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Baseline (Training) Phase:</strong></td>
</tr>
<tr>
<td>A set of ( p ) sampled scalar vibration–response signals, each represented as ( y[t] ), with ( t = 1,2,\ldots,N ) designating normalized (by the sampling period) discrete time and ( N ) the signal length. These signals are obtained from ( \nu ) (( \leq p )) members (sample structures) of the considered population.</td>
</tr>
<tr>
<td>2. <strong>Inspection (Operational) Phase:</strong></td>
</tr>
<tr>
<td>A vibration response signal ( y_u[t] ), with ( t = 1,2,\ldots,N ), is obtained from any member (sample structure) of the population.</td>
</tr>
<tr>
<td>Determine:</td>
</tr>
</tbody>
</table>

\( ^a \)The subscript ‘u’ is used to designate a structure in currently unknown health state.

Remarks:

1. Certain conventional methods, such as the U–AR (see subsection 3.4.1) may only use a single (instead of \( p \)) scalar vibration–response signal(s) in the baseline phase.

2. Vibration displacement, velocity, or acceleration signals may be used.

3. The problem is tackled in a batch, that is non–sequential, operational mode, meaning that the decision is made after the entire vibration response signal \( y_u[t] \) \( (t = 1,2,\ldots,N) \) has been obtained.

4. The case of scalar vibration response signal is considered for simplicity. Extensions to the case of vector (multiple) vibration response signals are possible. The case of measured excitation signal(s) may be easily accommodated as well.

5. The vibration response signal is, for all structures and in both the baseline and inspection phases, measured at the same (fixed) location and specified direction.

6. In general, the structure being tested for damage in the inspection phase may or may not be part of the sample structures employed in the baseline phase. For the purposes of the present study, and for fully revealing the limitations of the methods, the latter (stricter) case is treated (details in subsection 3.6.1).

\( ^8 \)The structures selected for use in the baseline phase are ‘rotated’ within the available sample structures; a procedure aiming at warranting ‘consistency’ of the results, in the sense of eliminating the dependence of the detection performance on the sample structures used in the baseline phase.

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Chapter 3. Unsupervised Multiple Model Type Methods

Table 3.1: The damage scenarios and the experiments [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b].

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Impact Energy (J)</th>
<th>Damage Visual Characterisation</th>
<th>Impact Position</th>
<th>Number of Total number of experiments of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>—</td>
<td>—</td>
<td>23</td>
<td>7</td>
</tr>
<tr>
<td>Damage AL</td>
<td>5</td>
<td>Invisible</td>
<td>Point A</td>
<td>1</td>
</tr>
<tr>
<td>Damage BL</td>
<td>5</td>
<td>Invisible</td>
<td>Point B</td>
<td>1</td>
</tr>
<tr>
<td>Damage CL</td>
<td>5</td>
<td>Invisible</td>
<td>Point C</td>
<td>1</td>
</tr>
<tr>
<td>Damage DL</td>
<td>5</td>
<td>Invisible</td>
<td>Point D</td>
<td>1</td>
</tr>
<tr>
<td>Damage AH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point A</td>
<td>1</td>
</tr>
<tr>
<td>Damage BH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point B</td>
<td>1</td>
</tr>
<tr>
<td>Damage CH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point C</td>
<td>1</td>
</tr>
<tr>
<td>Damage DH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point D</td>
<td>1</td>
</tr>
</tbody>
</table>

Each damage scenario is designated using two letters, the first indicating damage location (A,B,C,D; see Figure 2.6) and the second impact energy level (L: 5 J, H: 15 J).

Fig. 3.1. Sketch (top view) of the experimental set–up with the beam, the tail mass, the clamping, the damage locations and the measurement Point Y1 (dimensions in mm).

3.3 Experimental Set–up and Damage Scenarios

The comparative assessment of the damage detection methodology is based on the population of 23 healthy and 8 damaged nominally identical composite beams presented in chapter 2. These represent the topology of the main part of a commercial Unmanned Aerial Vehicle (UAV) and exhibit significant variability in their dynamics due to a combination of uncertainty sources, including manufacturing, environmental, and operational conditions. Eight of the composite beams are damaged via a pendulum type impact hammer, at one of the points A, B, C, D (Figure 3.1) by using either a low (L) or a higher (H) impact energy. Each combination of damage position and impact energy represent a specific damage scenario designated via two letters as shown in Table 3.1. The experimental set–up is shown in Figure 3.1, while details regarding the experiments, the signals, and their processing are provided in Table 3.1 and Table 3.2. Further information is available in chapter 2.

3.4 Damage Detection via Conventional Statistical Time Series Type Methods

Two ‘conventional’ unsupervised statistical time series type methods, one based on the parameter vector of a single AutoRegressive (AR) Model [Fassois and J. S. Sakellariou 2007; Fassois and J. S. Sakellariou 2009], referred to as U–AR, and the other being a PCA–based version motivated by [Yan et al.
3.4 Damage Detection via Conventional Statistical Time Series Type Methods

Table 3.2: Vibration signal and estimation details.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling frequency:</td>
<td>$f_s = 4654.5$ Hz, Signal bandwidth: $[5 - 2327.25]$ Hz</td>
</tr>
<tr>
<td>Signal normalized via sample mean removal and division by its sample standard deviation.</td>
<td></td>
</tr>
<tr>
<td>Signal length used in non–parametric representations:</td>
<td>$N = 112000$ samples (24.06 s).</td>
</tr>
<tr>
<td>Welch based estimation details:</td>
<td>Hamming window; 8192 samples long segment; 90% overlap; Matlab function tfestimate.m.</td>
</tr>
<tr>
<td>Signal length used for AR modeling:</td>
<td>$N = 10000$ samples (2.15 s).</td>
</tr>
<tr>
<td>AR model estimation details:</td>
<td>Ordinary Least Squares (OLS), QR implementation, Matlab func.: arx.m</td>
</tr>
</tbody>
</table>

and referred to as U–PCA–AR, are employed for damage detection for the population of composite beams. The methods’ damage detection performance is assessed via a systematic procedure including numerous test cases (see details in subsection subsection 3.6.1) and Receiver Operating Characteristic (ROC) curves [Duda et al. 2000, pp. 34–35], [Fawcett 2006], each representing the true positive rate (percentage of correct damage detections) versus the false positive rate (percentage of false alarms) for varying decision threshold. The Area Under the ROC Curve (AUC) which may range from 0 to 1 is also used, with values approaching 1 indicating excellent performance and values close to 0.5 poor performance (see also [Fawcett 2006]).

3.4.1 Damage Detection via a Conventional Parameter–based (U–AR) Method

The parameter–based U–AR method is chosen as it has achieved the best performance in damage detection for a population of composite beams in a previous experimental assessment of various conventional methods [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2014b]. The operational phases of the method including the baseline (training) phase as well as the inspection (operational) phase are presented below (its characteristic quantity and test statistic are summarized in Table 3.3).

Baseline phase. A single vibration response signal from the healthy structure is used for the identification of an AR model of the form [Fassois 2001]:

$$\begin{align*}
    y[t] + \sum_{i=1}^{n} a_i \cdot y[t-i] &= e[t], \\
    e[t] &\sim \text{NID}(0, \sigma_e^2)
\end{align*}$$

with $t = 1, \ldots, N$ designating normalized (by the sampling period) discrete time, $y[t]$ the vibration response signal, $n$ the model order, $a_i$ the $i$–th AR parameter, and $e[t]$ the model residual assumed to be a white Gaussian zero–mean sequence with variance $\sigma_e^2$. NID stands for Normally Independently Distributed with the indicated mean and variance. The AR model parameter vector:

$$\alpha := [a_1 a_2 \ldots a_n]^T$$

constitutes the method’s characteristic quantity or feature.

In the baseline phase an estimate of the AR model parameter vector, say $\alpha_o$, is obtained based on the available vibration signal, along with its covariance matrix $\Sigma_o$, expressing estimation uncertainty. Estimation is based on standard procedures employing Ordinary Least Squares with QR implementation [Ljung 1999, pp. 318–320], [Fassois 2001], [Söderström et al. 1989, pp. 205–207]. Model order selection is achieved based on the Bayesian Information Criterion (BIC) and the Residual Sum of Squares / Signal Sum of Squares (RSS/SSS).

9Bold–face capital/lower letters designate matrices/vectors, respectively.
10Notice that for simplicity of notation no distinction is presently made between a quantity and its estimate.
11The subscript o designates the healthy structural state.
12Covariances representing estimation uncertainty (estimation based on a single signal) are designated by the symbol $\Sigma$, while sample covariances (estimation based on multiple signals) are designated by the symbol $P$. 37
**Chapter 3. Unsupervised Multiple Model Type Methods**

**Inspection phase.** In this phase a vibration response signal is obtained from a structure in currently unknown state. Then a fresh AR model of the same order with that of the baseline phase is estimated and its parameter vector \( \alpha_u \) is obtained. Damage detection is then accomplished by comparing \( \alpha_u \) to its baseline counterpart \( \alpha_o \) based on the following hypothesis testing (\( a \) indicates risk level) [Fassois and J. S. Sakellariou 2007; Fassois and J. S. Sakellariou 2009]:

\[
H_0 : \chi^2_\alpha := (\alpha_o - \alpha_u)^T \cdot 2\Sigma_o^{-1} \cdot (\alpha_o - \alpha_u) \leq \chi^2_{1-a}(n) \rightarrow \text{null hypothesis – healthy structure}
\]

\[
H_1 : \text{Else} \rightarrow \text{alternative hypothesis – damaged structure}
\]

with \( \chi^2_{1-a}(n) \) designating the \( \chi^2 \) distribution’s \( 1 - a \) critical limit and \( \chi^2_\alpha \) the method’s test statistic.

**Results.** The method’s best performance is achieved for AR model order \( n = 77 \) (estimation details in Table 2.2). Figure 3.2 (a),(c) and (d) depict the method’s performance in terms of the test statistic \( \chi^2_\alpha \), the ROC curve, and the AUC measure, respectively, based on which inadequate performance is observed. It is, indeed, obvious from Figure 3.2(a) that the method has difficulty in separating the healthy from the damaged states for most of the damage scenarios. This is also evident from the corresponding ROC curve and the AUC. The ROC curve specifically shows inadequate detection performance (less than 50% successful detection) for a low (< 2%) false alarm rate.

**3.4.2 Damage Detection via a Conventional PCA–based (U–PCA–AR) Method**

This is a class 2 method motivated by [Yan et al. 2005] that employs the (centered) PCA [Cadima et al. 2009; Jackson 1991][Rencher et al. 2012, pp. 405–432] for defining an \( n \)–dimensional orthogonal...
3.4 Damage Detection via Conventional Statistical Time Series Type Methods

Table 3.3: The methods with their characteristic quantities (features) and test pseudo–statistic.

<table>
<thead>
<tr>
<th>Method</th>
<th>Characteristic quantity (feature)</th>
<th>Test pseudo–statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>U–AR</td>
<td>AR parameter vector ( \alpha )</td>
<td>( \chi^2_\alpha = (\alpha - \alpha_o)^T \cdot 2 \Sigma_0^{-1} \cdot (\alpha - \alpha_o) )</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>set of AR parameter vectors ( { \alpha } )</td>
<td>( D = | \bar{\alpha}_m |_2 )</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>set of AR parameter vectors ( { \alpha } )</td>
<td>( D = \sum_{k=1}^{p} d(m_{o,k}, \bar{m}_o) )</td>
</tr>
<tr>
<td>U–PCA–MM–AR</td>
<td>set of AR parameter vectors ( { \alpha } )</td>
<td>( D = \sum_{k=1}^{p} d(\bar{m}_{o,k}, \bar{m}_o) )</td>
</tr>
</tbody>
</table>

\( d(\cdot, \cdot) \) designates Kullback–Leibler divergence between two models, \( \bar{\alpha} \) designates PCA transformed and truncated model. \( \{ \alpha \} \) designates a set of AR model parameter vectors, that is \( \{ \alpha \} := \{ \alpha_1, \ldots, \alpha_p \} \).

coordinate system on which a random (due to estimation error) healthy AR parameter vector may be projected, with the projections forming a set of mutually uncorrelated random variables. This is achieved by performing a PCA decomposition of the sample covariance matrix of the \( p \) estimated (in the baseline phase) AR parameter vectors. The decomposition provides the eigenvalues (arranged in decreasing order) and the corresponding eigenvectors, with the latter defining the aforementioned transformed \( n \)–dimensional orthogonal coordinate system (principal coordinates). The idea then is to base damage detection on AR parameter vector projections into a subspace of the transformed coordinate system defined by \( m \) (properly selected, \( m < n \)) eigenvectors corresponding to the \( m \) 'smallest' eigenvalues\(^{13}\). Parameter vector projections into this subspace are characterized by relatively 'low' (natural) variability in the healthy state, thus expected to exhibit discrepancies mainly under the presence of damage.

The method’s characteristic quantity and test pseudo–statistic are also provided in Table 3.3, while the method’s operational phases are summarized below.

**Baseline phase.** In this phase a total of \( p (\gg n) \) vibration signals are obtained from corresponding experiments with the \( n \) sample healthy structures. Based on these, \( p \) conventional AR\((n) \) models, say \( m_{o,k} \) \((k = 1, \ldots, p)\), and their parameter vectors, \( \{ \alpha_o \} = \{ \alpha_{o,1}, \ldots, \alpha_{o,p} \} \), are obtained. The sample mean and covariance matrix (corresponding to the obtained models) are then computed, and each scalar parameter of each obtained vector is centered by subtracting its sample mean [Yan et al. 2005].

All parameter vectors below are assumed centered (sample–mean–corrected).

Then PCA is performed to the \( n \times n \) sample covariance matrix:

\[
\mathbf{P} := \frac{1}{p-1} \sum_{k=1}^{p} \mathbf{a}_{o,k} \mathbf{a}_{o,k}^T
\]

obtained from the (centered) parameter vectors \( \mathbf{a}_{o,i} \) \((i = 1, \ldots, p)\). This leads to the decomposition:

\[
\mathbf{P} = \mathbf{U} \mathbf{S}^2 \mathbf{U}^T
\]

with:

\[
\mathbf{S}^2 := \text{diag}(s_1^2, s_2^2, \ldots, s_n^2) \quad (n \times n), \quad \mathbf{U} := [\mathbf{u}_1 \ \mathbf{u}_2 \ \ldots \ \mathbf{u}_n] \quad (n \times n)
\]

with \( \text{diag}(s_1^2, s_2^2, \ldots, s_n^2) \) designating a diagonal matrix containing the (positive) eigenvalues (set in decreasing order), \( s_i \), of \( \mathbf{P} \), and \( \mathbf{U} \) the matrix containing the respective eigenvectors, \( \mathbf{u}_i \), of \( \mathbf{P} \). The eigenvectors are orthonormal, such that \( \mathbf{u}_i^T \mathbf{u}_j = \delta_{ij} \), with \( \delta_{ij} \) designating the Kronecker delta. Thus \( \mathbf{U} \) is orthonormal, that is \( \mathbf{U} \mathbf{U}^T = \mathbf{I}_n \), with \( \mathbf{I}_n \) designating the \( n \)–dimensional identity matrix. Thus from Equation 3.4 one obtains:

\[
\mathbf{S}^2 = \mathbf{U}^T \mathbf{P} \mathbf{U}
\]

\(^{13}\)The remaining eigenvalues being \( q \) in number, with \( q = n - m \).
which indicates that PCA diagonalizes the covariance matrix $P$. The (centered) parameter vector $\alpha$ may be then transformed into the PCA coordinates without loss of information as follows:

$$\alpha = \sum_{j=1}^{n} \bar{\alpha}_j u_j = U \bar{\alpha} \iff \bar{\alpha} = U^T \alpha$$

(3.7)

with $\bar{\alpha}$ ($n \times 1$) designating the transformed version of $\alpha$ and $\bar{\alpha}_j$ its $j$-th element (all centered). Due to the diagonal form of $S^2$, the transformed components $\bar{\alpha}_j = u_j^T \alpha$ ($j = 1, \ldots, n$) are mutually uncorrelated and their variances are equal to $s^2_j$, that is:

$$E\{\bar{\alpha}_j \cdot \bar{\alpha}_i\} = s^2_j \cdot \delta_{ij}$$

(3.8)

with $\delta_{ij}$ designating the Kronecker delta.

Relating the first $q$ columns (principal components) of $U$ – which correspond to the largest eigenvalues that explain a certain high fraction, say $\gamma$ ($\%$)$^{14}$, of the total parameter vector variability – to uncertainty sources acting under the healthy structural condition and the remaining $m = n - q$ principal components to damage (orthonormality between features affected by damage and those by uncertainty is tacitly assumed), the parameter vector $\alpha$ may be written as:

$$\alpha = \sum_{j=1}^{q} \bar{\alpha}_j u_j + \sum_{j=q+1}^{n} \bar{\alpha}_j u_j = \begin{bmatrix} U_q & \cdots & U_m \end{bmatrix} \begin{bmatrix} \bar{\alpha}_q \\ \ldots \\ \bar{\alpha}_m \end{bmatrix} = U_q \bar{\alpha}_q + U_m \bar{\alpha}_m$$

(3.9a)

where obviously:

$$\bar{\alpha}_q = U_q^T \alpha \quad \text{and} \quad \bar{\alpha}_m = U_m^T \alpha$$

(3.9b)

with $U_q$ designating the so-called loading matrix that includes the first $q$ columns of $U$, $U_m$ designating the matrix composed of the remaining $m$ columns of $U$, and $\bar{\alpha}_q, \bar{\alpha}_m$ designating the projections of $\alpha$ into the $q$-dimensional and $m$-dimensional transformed subspaces, respectively.

A key point is the selection of the reduced dimensionality $m$ of the transformed subspace within which damage detection is to be performed. Since the eigenvalues have been set in decreasing order, the question essentially is selecting the first $q$ of them (major ones), which are to be dropped, while keeping the last $m$ (minor ones), with the eigenvectors of the latter ones defining the transformed subspace within which damage detection is to be performed. The approach followed for this selection is quite simple: One chooses $q$ as the minimum number of eigenvalues that (cumulatively) succeed in explaining a pre-specified fraction $\gamma$ of the parameter vector variability$^{15}$, that is:

$$q := \min_l \left\{ l \in [1, \ldots, n] \mid \gamma \leq \frac{\sum_{j=1}^{l} s^2_j}{\sum_{j=1}^{n} s^2_j} \times 100 \% \right\}$$

(3.10)

By virtue of Equation 3.8 the numerator of the fraction above equals the sum of the variances of the $l$ (transformed) components $\bar{\alpha}_j$, while the denominator equals the sum of the variances of all (transformed) components$^{16}$.

**Inspection phase.** Once a fresh vibration response signal is obtained from a sample structure, an AR model $m_a$ of the same order as the models in the baseline phase is estimated and its parameters are

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$^{14}$For instance around 90%.

$^{15}$Parameter vector variability may be defined in various ways; presently the trace of its covariance matrix is used.

$^{16}$It is also well known that the sum of all variances remains unchanged under the transformation, that is $tr(P) = tr(S^2)$ [Jolliffe 2002, p. 112]; $tr(\cdot)$ designating trace.
centered by subtracting the sample mean values obtained in the baseline phase, yielding the parameter vector \( \alpha_u \). This is subsequently transformed, based on Equation 3.9b, into the reduced \( m \)-dimensional PCA space as:

\[
(\bar{\alpha}_m)_u = U_m^T \alpha_u
\]  

(3.11)

Damage detection may be then based on detecting significant deviations in a measure of \( (\bar{\alpha}_m)_u \), presently selected as its \( l_2 \) norm. Thus damage detection is based on the following decision making mechanism:

\[
D = |(\bar{\alpha}_m)_u|_{l_2} \leq l_{lim} \rightarrow \text{Healthy structure}
\]

otherwise \( \rightarrow \text{Damaged structure} \)  

(3.12)

with \( D \) designating the method’s test pseudo–statistic and \( l_{lim} \) a user–specified limit selected in accordance with the results of the baseline phase.

Results. Based on the assessment procedure of subsection subsection 3.6.1 the best performance for the U–PCA–AR method is achieved for \( \gamma = 90\% \), \( n = 57 \) and \( \nu = 15 \) (\( p = 105 \)). It is noted that, as with all PCA–based methods, variations in \( \gamma \) generally affect performance due to removal of damage related components or retainment of uncertainty related ones, respectively (see also subsection 3.6.1). The method’s performance, in terms of the test pseudo–statistic \( D \), ROC curve, and AUC is presented in Figure 3.2(b),(c) and (d), respectively. Based on these, it is evident that damage detection has been significantly improved over the U–AR method. Yet, the achieved performance is, still, not satisfactory as the true positive rate does not exceed 75\% for false alarm rates lower than 2\% (see Figure 3.2(c)).

3.5 Two Multiple Model (MM) Based Methods

In contrast to ‘standard’ statistical time series methods where a single model is typically employed for representing the (variable) healthy dynamics [Fassois and J. S. Sakellariou 2009; Fassois and Kopsaftopoulos 2013], Multiple Model (MM) based methods employ a MM representation for modeling the population healthy structural dynamics under uncertainty. This consists a set of estimated conventional models along with the Gaussian probability density functions of their estimated parameter vectors (also see [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2016b; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015b]). The characteristic quantity or feature is then obtained from this MM representation. Two such methods, both based on AR representations, are presently postulated: (a) a MM–AR method and (b) a PCA–enhanced, U–PCA–MM–AR, method.

3.5.1 The Multiple Model AR (U–MM–AR) Method

Baseline phase. As with the U–PCA–AR method, \( p \) vibration signals (constant number of signals per sample structure) are obtained from \( \nu \) nominally identical healthy structures. Based on these, a Multiple Model representation of the healthy structural dynamics \( m_o = \{m_{o,1}, \ldots, m_{o,p}\} \) is determined via the estimation of a set of \( p \) conventional AR(\( n \)) models. Each estimated parameter vector is (asymptotically, that is as the signal length \( N \rightarrow \infty \)) associated with a Gaussian probability density function, with mean equal to each point estimate \( \alpha_{o,i} \) and estimated covariance \( \Sigma_{o,i} \).

Inspection phase. Given a fresh vibration response signal from a current sample structure, the objective is to decide whether or not its dynamics is adequately represented by the set of healthy AR models included in the MM representation \( m_o \), in which case the structure is declared as healthy; otherwise it is declared as damaged. Toward this end a fresh AR model, \( m_u \), of the same order as those in \( m_o \), is identified. Then a proper test pseudo–statistic, say \( D(m_o, m_u) \), expressing the distance between the current model \( m_o \) and the MM representation \( m_o \) is obtained in the space defined by the method’s characteristic quantity (AR parameter vector); also see Table 3.3.

Damage detection is then declared if and only if the test pseudo–statistic \( D(m_o, m_u) \) is greater than a user specified threshold \( l_{lim} \), implying that the current model \( m_u \) does not belong to the MM representation.
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$m_o$; otherwise the structure is declared as healthy:

$$D(m_o, m_u) \leq l_{lim} \rightarrow \text{Healthy structure}$$
$$\text{otherwise} \rightarrow \text{Damaged structure} \quad (3.13)$$

The test pseudo–statistic $D(m_o, m_u)$ is presently defined as the sum of the distances of $m_u$ from all elements (individual models) of $m_o$, that is:

$$D(m_o, m_u) := \sum_{k=1}^{p} d(m_{o,k}, m_u) \quad (3.14)$$

with $d(m_{o,k}, m_u)$ indicating distance between two individual models (see the schematic diagram of Figure 3.3). This is selected to be the Kullback–Leibler (KL) divergence (pseudo–distance) [Press et al. 2007, pp. 756–758] between the two models. As the two model parameter vectors asymptotically $(N \to \infty)$ follow Gaussian distributions, the KL divergence may be expressed as:

$$d(m_{o,k}, m_u) := \frac{1}{2} \left[ tr \left( \Sigma_{m_{o,k}}^{-1} \Sigma_{m_u} \right) + (\alpha_{o,k} - \alpha_u)^T \Sigma_{o,k}^{-1} (\alpha_{o,k} - \alpha_u) - 1 - \ln \left( \frac{det \Sigma_u}{det \Sigma_{o,k}} \right) \right] \quad (3.15)$$

with $tr(\cdot)$ designating trace and $det(\cdot)$ determinant of the indicated quantity. As the KL–divergence is a non–symmetric pseudo–distance, it should be used as provided above (without interchanging the two models).

Fig. 3.3. A schematic representation of the Multiple Model concept: The set of models $m_o = \{m_{o,1}, \ldots, m_{o,p}\}$ represent the healthy dynamics, while $m_u$ represents the current dynamics that is to be classified as healthy or damaged; the decision is based on the distance $D$ between $m_u$ and $m_o$.

3.5.2 The PCA–based Multiple Model AR (U–PCA–MM–AR) Method

This may be thought of as a version of the previous, U–MM–AR, method in which the detection process is taking place in a PCA–transformed and truncated space; in a sense it is a combination of the U–PCA–AR and U–MM–AR methods. The aim is obvious, that is basing the detection test pseudo–statistic on the transformed and truncated space for potentially improved detection performance; yet there is always the danger of compromise due to truncation.

In the context of this method the non–centered (no sample mean subtraction) parameter vector $\alpha$, along with the non–centered PCA, are used; the latter implying that PCA is based on the non–centered
second order sample moment of the baseline–obtained parameter vectors instead of their centered counterpart (that is the covariance matrix) [Cadima et al. 2009; Jackson 1991]17. The present PCA–based MM method may be also viewed as a combined class 2 and class 3 method, with such a combination being dictated by the fact that the MM approach is only a crude approximation to a genuine class 3 method (that is to an RC model based method).

In the rest of this subsection the parameter vectors \( \alpha \) are not sample–mean–corrected.

**Baseline phase.** In this phase the MM representation \( m_o \) of the healthy structural dynamics is obtained based on \( p \) (\( \gg n \)) vibration signals from corresponding experiments with \( n \) sample healthy structures. Then the sample non–central second moment of the AR parameter vector is obtained as:

\[
P := \frac{1}{p} \sum_{k=1}^{p} \alpha_{o,k}^T \alpha_{o,k} \quad (n \times n)
\]

Applying the SVD, \( P \) is written as:

\[
P = US^2U^T
\]

with \( U \ (n \times n) \) designating a real unitary matrix whose columns are the set of all available principal components and \( S^2 \) the \( n \times n \) diagonal matrix containing the eigenvalues of \( P \) (squared singular values).

The idea is analogous to that of the U–PCA–AR method, that is to base damage detection on AR parameter vector projections into a subspace of the transformed coordinate system defined by \( m^s \) (properly selected, \( m^s < n \)) eigenvectors. These correspond to the single largest and the \( m \) smallest eigenvalues (\( m^s := m + 1 \)). The reason for maintaining, in this case, the eigenvector corresponding to the largest eigenvalue is that this is quite affected by the parameter vector sample mean [Jackson 1991, p. 73], which should be accounted for. Again, parameter vector projections into this \( m^s \)–dimensional subspace are characterized by relatively low (natural) variability in the healthy state, thus expected to exhibit discrepancies mainly under the presence of damage.

The loading matrix \( L \) is obtained as:

\[
L = [u_1 \ u_{n-m+1} \ldots u_n] \quad (n \times m^s)
\]

where \( u_j \) is the \( j \)–th column of \( U \). Then the \( \alpha_{o,k} \) vectors and their covariances \( \Sigma_{o,k} \) are transformed into the truncated \( m^s \)–dimensional space through the expressions:

\[
(\alpha_m)_{o,k} = L^T \alpha_{o,k} \quad (m^s \times 1), \quad (\Sigma_m)_{o,k} = L^T \Sigma_{o,k} L \quad (m^s \times m^s), \quad \text{for} \quad k = 1, \ldots, p
\]

The selection of the reduced dimensionality \( m^s \) is done similarly to the U–PCA–AR method, by selecting \( q (= n-m) \) as the minimum number of the largest eigenvalues that (cumulatively) succeed in explaining a pre–specified fraction \( \gamma \) of the parameter vector variability, and then keeping the eigenvectors corresponding to the single largest eigenvalue and those of the \( m \) smallest (totally \( m^s \)). Notice that although the present \( \gamma \) has the same meaning as that used in the U–PCA–AR method, it will generally take quite higher values for a given \( q \) due to the effect of the parameter vector sample mean on the first eigenvalue [Jackson 1991, p. 73]. The value of \( \gamma \) within the present U–PCA–MM–AR method is thus selected quite higher than its U–PCA–AR counterpart, presently in the vicinity of 99% (also see Table 3.5 in the next section).

**Inspection phase.** Given a fresh vibration response signal from a current sample structure in unknown state, an AR model \( m_o \) of the same order as those in the baseline phase is estimated. Then its parameter vector \( \alpha_u \) and its associated covariance matrix \( \Sigma_u \) are transformed into the \( m^s \)–dimensional PCA space as:

\[
(\alpha_{m^s})_u = L^T \alpha_u \quad (m^s \times 1), \quad (\Sigma_{m^s})_u = L^T \Sigma_u L \quad (m^s \times m^s)
\]

17There is an on–going discussion on the use of the non–centered PCA [Honeine 2014]; it is presently employed in order for the parameter vector sample mean to be accounted for.
Damage detection is then based on the decision making procedure of Equation 3.13, where the pseudo–distances in Equation 3.14 are computed using the transformed and truncated \((m^{*} – \text{dimensional})\) parameter vectors and their covariances (see also Table 3.3).

### 3.6 Experimental Assessment of the MM–based Methods

In this section the U–MM–AR and U–PCA–MM–AR methods are assessed, while comparisons with the 'conventional' methods of section 3.4 are also made. This is based on a systematic procedure which uses 'rotation' to warrant 'consistency' of the results; that is eliminating the dependence of detection performance on the sample structures selected for use within the baseline phase. As previously indicated, the sample structures selected for use in the inspection phase, thus in assessing detection performance, do not presently coincide with any of those used in the baseline phase. The sensitivity of each method on user specified parameters is also studied.

#### 3.6.1 The Assessment Procedure

The methods are assessed based on a systematic procedure that consists of the following steps.

**Step 1:** A set of \(\nu\) beams among the 23 healthy sample beams is selected, referred to as **baseline beams**, for use in the methods’ baseline (training) phase. The remaining \((23 – \nu)\) healthy sample beams along with the eight damaged sample beams are referred to as **inspection beams** –considered as of ‘unknown’ structural state –and are reserved for the methods’ assessment in the inspection (operational) phase. Experiments (and signals) obtained by the baseline beams are designated as **baseline experiments**, while those obtained by the inspection beams are designated as **inspection experiments**.

**Step 2:** All possible combinations of \(\nu\) \((\nu = 7, 9, 11, 13, 15)\) baseline beams among the available 23 healthy sample beams are considered. If the total number of combinations is practically too high, a subset of them is employed in a way that each healthy sample beam is used in at least one combination. The employed combinations are designated as **employed sets of baseline beams** and lead to numerous sets of baseline beams.

**Step 3:** The damage detection procedure of each method is carried out for all employed sets of baseline beams and the corresponding inspection experiments. This leads to a high number of test cases on which damage detection assessment is to be based. The total number of test cases obtained via this procedure is designated as **number of aggregate inspection experiments**.

**Step 4:** Steps 2 and 3 are repeated for all considered numbers of baseline beams \(\nu\).

For each number \(\nu\) of baseline beams considered, the number of **inspection beams, baseline and inspection experiments, employed sets of baseline beams** and **aggregate inspection experiments** are summarized in Table 3.4.

Remark: It is important to note that the test pseudo–statistic \(D\) may scale differently for each set of \(\nu\) baseline beams, due to differences in the selected principal components. Thus, in order to assess the U–PCA–AR and U–PCA–MM–AR methods based on all the employed sets of baseline beams, using a common threshold (distinct for each method, but common for all the employed sets of baseline beams), a proper normalization is needed\(^{18}\). This is presently achieved by dividing each \(D\) value with the maximum, say \(D^{*}\), that is obtained per method, per \(\nu\) baseline beams, and per employed set of baseline beams as it arises by the corresponding inspection experiments. Thus the normalized test pseudo–statistic takes always values between 0 and 1. This normalization is required only for the assessment procedure employed in this study which employs more than one employed sets of baseline beams.

18Alternatively, averaging of the ROC curves per set of \(\nu\) baseline beams may be attempted based on 'vertical' curve averaging or threshold averaging [Fawcett 2006]. The former uses averaging of the ROC curves per false positive rate, while the latter averaging out the various ROC curves for a set of common thresholds. Both approaches are not presently applicable as common false alarm rates are not available for vertical averaging, while common thresholds cannot be adopted due to scaling issues.
3.6 Experimental Assessment of the MM–based Methods

Table 3.4: Inspection beams and aggregate inspection experiments for various values of $\nu$.

<table>
<thead>
<tr>
<th>No. of baseline beams ((\nu))</th>
<th>No. of inspection beams** ((i))</th>
<th>No. of baseline experiments ((p = 7 \cdot \nu))</th>
<th>No. of inspection experiments (per set of baseline beams; (z = 7 \cdot i))</th>
<th>No. of employed sets of baseline beams*** ((h))</th>
<th>No. of aggregate inspection experiments **** ((i = h \cdot z))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>30</td>
<td>7</td>
<td>210</td>
<td>23</td>
<td>4 830</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
<td>49</td>
<td>168</td>
<td>50</td>
<td>8 400</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
<td>63</td>
<td>154</td>
<td>50</td>
<td>7 700</td>
</tr>
<tr>
<td>11</td>
<td>20</td>
<td>77</td>
<td>140</td>
<td>50</td>
<td>7 000</td>
</tr>
<tr>
<td>13</td>
<td>18</td>
<td>91</td>
<td>126</td>
<td>50</td>
<td>6 300</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>105</td>
<td>112</td>
<td>50</td>
<td>5 600</td>
</tr>
</tbody>
</table>

* This case is used only for the U–AR method.
** Inspection beams: all beams (healthy and damaged) not used as baseline beams.
*** Set of baseline beams: a set consisting of \(\nu\) (healthy) baseline beams.
**** Aggregate inspection experiments: all inspection experiments corresponding to all employed sets of baseline beams.

Table 3.5: Selected design parameters, no of inspection experiments, and ROC curve construction details for each method.

<table>
<thead>
<tr>
<th>Method</th>
<th>No. of used thresholds (points on the ROC)</th>
<th>No. of aggregate inspection experiments</th>
<th>(n)</th>
<th>(\gamma) (%)</th>
<th>(\nu)</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U–AR</td>
<td>4831</td>
<td>4 830</td>
<td>77</td>
<td>–</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>5552</td>
<td>5 600</td>
<td>57</td>
<td>90.0</td>
<td>15</td>
<td>105</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>5552</td>
<td>5 600</td>
<td>77</td>
<td>–</td>
<td>15</td>
<td>105</td>
</tr>
<tr>
<td>U–PCA–MM–AR</td>
<td>5552</td>
<td>5 600</td>
<td>57</td>
<td>99.4</td>
<td>15</td>
<td>105</td>
</tr>
</tbody>
</table>

3.6.2 Preliminaries & Method Design Parameter Selection

'Good' values of the design parameters are empirically selected, see Table 3.5, for each method via the sensitivity analysis of subsection 3.6.4. Yet a number of remarks are now in order.

A good number of baseline beams is needed for properly capturing beam–related variability of the healthy dynamics; \(\nu = 15\) selected for all three methods employing multiple beams in the baseline phase. With 7 baseline experiments per beam, the corresponding number of baseline experiments for these methods is \(p = 105\). The numbers of aggregate inspection experiments and details on the construction of the ROC curves are indicated in Table 3.5.

AR model estimation details are presented in Table 2.2 (see also subsection 3.4.1). The PCA–based methods achieve 'best' damage detection performance for \(n = 57\), while the other methods for \(n = 77\). This may be associated with the fact that obtaining reliable parameter vector sample covariance matrices in the PCA–based methods requires \(p >> n\). This, in essence, limits the eligible AR orders to somewhat lower values. In any case, both orders are considered as sufficient from a modeling point of view (see next paragraph), with \(n = 77\) representing the highest appropriate order for the healthy set of beams, and \(n = 57\) an 'average' adequate order. Representative AR(57) and AR(77) based Power Spectral Density estimates for a healthy beam are compared in Figure 3.4. Evidently, the quite good agreement between the two (with only minor discrepancies) further attests to the capacity of both models in properly representing the dynamics.

Details on the PCA analysis performed in the baseline phase of the PCA–based methods are, for each method, provided in Figure 3.5, in which the eigenvalues of \(\mathbf{P}\) are depicted along with the contribution of
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![Graph showing Parametric PSD (dB) for AR(57) versus AR(77) models](image)

**Fig. 3.4.** AR modeling of the vibration response signal: AR(57) versus AR(77) model based PSD estimates for an indicative healthy beam.

Each eigenvalue to the parameter vector total variability, that is:

\[ \frac{s_j^2}{\sum_{i=1}^{n} s_i^2} \times 100 \% \]

It may be observed that the first principal component in the U–PCA–MM–AR method (non–centred PCA) is much larger than the remaining ones and explains a very high fraction of the variability, 98.24%, while the first component in the U–PCA–AR method (using centred PCA) is close to the remaining ones and explains only 29.46% of the variability.

These observations are also reflected in the selection of the principal components employed in each method. Indeed, the U–PCA–AR method almost never uses the first 15 principal components (sometimes not even the 16th) for achieving its best damage detection performance; see Figure 3.6 in which the frequency of selection of each principal component in all 50 employed sets of baseline beams (see Table 3.4) is presented for the selected design parameters (Table 3.5) of each method. On the other hand the U–PCA–MM–AR method uses most of the principal components, with very few being always left out; thus it exploits most of the information included in the model parameter vector. Thus 15–16 components explain a selected \( \gamma = 90\% \) (Table 3.5) of the model parameter vector variability in the U–PCA–AR method (centered PCA–based), while only 6 components explain a selected \( \gamma = 99.4\% \) (Table 3.5) of the variability in the U–PCA–MM–AR method (non–centered PCA–based). This discrepancy between the two methods is due to various factors, most notably to the fact that the U–PCA–AR uses the centered PCA and the U–PCA–MM–AR the non–centered one\(^{19}\), thus the high value of \( \gamma \) in the latter case is due to the inclusion of the largest eigenvalue in it.

3.6.3 Damage Detection Results and Comparisons

The obtained results for all methods and all considered aggregate inspection experiments (Table 3.4) are provided in Figure 3.7 in terms of each method’s test pseudo–statistic, that is the quantity directly

\(^{19}\)Non–centered PCA performs a rotation of the original \( n \)–dimensional coordinate system with respect to its origin so as to maximize the non–central second moments of the model parameter projections on the resulting axes. On the other hand, the centered PCA translates the origin to the parameter vector sample mean and rotates the coordinate system so as to maximize the variance of the model parameter projections on the resulting axes [Cadima et al. 2009].
3.6 Experimental Assessment of the MM–based Methods

employed in the decision making mechanism. Evidently, the two MM–based methods achieve better ‘separation’ of the healthy from the damaged sample structures.

The same test pseudo–statistics are also presented via box plots in Figure 3.8. From Figure 3.8(a),(b) it is evident that the U–PCA–MM–AR and U–MM–AR methods may separate, in most cases and better than the ‘conventional’ methods, the healthy from the damaged sample structures, independently of the damage severity and location. On the other hand, between the ‘conventional’ methods themselves the U–AR may detect only the BL, BH and CH damage scenarios, while the performance of the U–PCA–AR is better, detecting all scenarios except for the slightest (low impact energy) ones CL and DL (Figure 3.8(c),(d)). It is worth noting that the outliers (red crosses) that correspond to the healthy structural state and exceed the threshold separating (black continuous) the healthy from the damaged structural states, thus implying false alarms, correspond to a small percentage (≤ 3%) of all considered aggregate inspection experiments.

The performance of all four methods, distinctly for the low (invisible damage) and higher (barely visible damage) impact energy damage scenarios, is presented via ROC curves in Figure 3.9. It is evident that the detection of invisible damages (left subplot) is inadequate for ‘conventional’ methods (correct detection rate lower than 75% at an already significant false alarm rate of 5%), but significantly improved for the MM–based methods (detection rate approaching 100% at false alarm rate of 5%). The situation is generally better for barely visible damages (right subplot), in which case the ‘conventional’ U–PCA–AR method’s performance is very close (even slightly better) to those of the MM–based methods (detection rate approaching 100% for false alarm rates of 2.5% or lower; the ‘conventional’ U–AR method still

Fig. 3.5. Log–eigenvalue diagrams (a),(b) and fraction of the total parameter vector variability explained by each principal component (c),(d) for the U–PCA–AR (upper row) and U–PCA–MM–AR (lower row) methods. [The values are provided for all 50 sets of employed baseline beams in the form of error bars; the central point of each error bar designates sample mean and the whiskers are a sample standard deviation above/below the sample mean.]
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Fig. 3.6. Principal component frequency of use: (a) U–PCA–AR (γ = 90%, n = 57, ν = 15), and (b) U–PCA–MM–AR (γ = 99.4%, n = 57, ν = 15). [50 sets of employed baseline beams; refer to Table 3.4.]

Fig. 3.7. Experimental assessment of all methods for each health scenario and all aggregate inspection experiments via the test pseudo–statistic (normalized $D$ or $\chi^2$): (a) U–PCA–MM–AR, (b) U–MM–AR, (c) U–AR, and (d) U–PCA–AR methods.
Fig. 3.8. Experimental assessment of all methods for each health scenario and all aggregate inspection experiments via box plots of the test pseudo–statistic (normalized $D$ or $\chi^2_{\alpha}$): (a) U–PCA–MM–AR, (b) U–MM–AR, (c) U–AR and (d) U–PCA–AR methods. [The top and bottom of each box are the 25th and 75th percentiles, while the distance between the top and bottom is the interquartile range. The red line in the middle of each box is the sample median and the lines extending above and below each box are the whiskers. These are drawn from the ends of the interquartile range and their length is 1.5 times the interquartile range. The red crosses represent observations beyond the whiskers (outliers). The black horizontal line is an indicative separation between the healthy and damaged states, being defined by the top whisker of the healthy state.]
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Table 3.6: AUC based detection performance of each method for the various damage scenarios.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage AL</td>
<td>0.9256</td>
<td>0.9861</td>
<td><strong>0.9996</strong></td>
<td>0.9955</td>
</tr>
<tr>
<td>Damage BL</td>
<td>0.9786</td>
<td>0.9991</td>
<td>1</td>
<td>0.9996</td>
</tr>
<tr>
<td>Damage CL</td>
<td>0.8186</td>
<td>0.9414</td>
<td><strong>0.9934</strong></td>
<td>0.9828</td>
</tr>
<tr>
<td>Damage DL</td>
<td>0.7074</td>
<td>0.8599</td>
<td>0.9699</td>
<td><strong>0.9772</strong></td>
</tr>
<tr>
<td>Damage AH</td>
<td>0.9124</td>
<td><strong>0.9997</strong></td>
<td>0.9906</td>
<td>0.9995</td>
</tr>
<tr>
<td>Damage BH</td>
<td>0.9959</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Damage CH</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Damage DH</td>
<td>0.8646</td>
<td><strong>0.9962</strong></td>
<td>0.9914</td>
<td>0.9951</td>
</tr>
<tr>
<td>All damages</td>
<td>0.9006</td>
<td>0.9728</td>
<td>0.9931</td>
<td><strong>0.9937</strong></td>
</tr>
</tbody>
</table>

†The best performance for each damage scenario is shown in bold face.

Fig. 3.9. Experimental assessment of all methods in terms of ROC curves for invisible (a) and barely visible (b) damage scenarios. ['Optimal’ design parameters used –details in Table 3.5.]

yielding inadequate performance). The performance of all four methods, distinctly for each damage scenario, is provided in Table 3.6 in terms of the AUC, with the U–PCA–MM–AR method achieving the best overall performance (although this is not translated to uniformly, that is for all damage scenarios, best performance).

Corresponding results, that cumulatively refer to both the invisible and barely visible damage scenarios (all aggregate inspection experiments; Table 3.4), are presented in Figure 3.10. Evidently the two MM–based methods perform best, with the U–PCA–MM–AR method exhibiting a slight lead for most false alarm rates. For both, the correct detection rate approaches 100% for false alarm rates above 5%, which is quite good for this detection problem. The performance of the ‘conventional’ methods is much less impressive: The U–PCA–AR method approaches a correct detection rate of 100% for significantly higher false alarm rates (above 40%), while the U–AR method is clearly inadequate.

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20ROC construction details in Table 3.5.
3.6 Experimental Assessment of the MM–based Methods

Fig. 3.10. Experimental assessment of all methods cumulatively for all damage scenarios via ROC curves (a) and AUC values (b). ['Optimal’ design parameters used –details in Table 3.5.]

3.6.4 Sensitivity Analysis of the Methods

Fig. 3.11. Sensitivity analysis for the U–MM–AR method. AUC for varying AR order \( n \) and number of baseline beams \( \nu \). [50 baseline sets; the ’optimal’ performance point is indicated by a magenta circle and an arrow; points with AUC values close to the ’optimal’ are indicated by black arrows.]
The sensitivity of the U–MM–AR and U–PCA–MM–AR methods to their design parameters is examined via the AUC in Figure 3.11 and Figure 3.12, respectively.

For the U–MM–AR method, it is evident (Figure 3.11) that increasing the number of baseline beams and the AR model order leads to improved performance (AUC values), with the best performance thus achieved for \( \nu = 15 \) and \( n = 77 \). It is also evident that very good performance is achieved for other, neighboring to the ’optimal’ model order of \( n = 77 \), orders.

For the U–PCA–MM–AR method, it is evident (Figure 3.12) that increasing the number of baseline beams to \( \nu = 15 \) leads to improved performance. Yet, in this case the best performance is achieved for model order \( n = 57 \); as already mentioned this is probably due to the fact that the parameter vector covariance matrix should be of dimensionality significantly lower than the number of signals used for its estimation (\( p >> n \); see subsection 3.5.2 and Table 3.4)\(^{21}\). With regard to \( \gamma \), the best performance is obtained for \( \gamma = 99.4\% \), maintained good for somewhat reduced values, but not so good for increased values.

3.6.5 Discussion and Guidelines

The best overall performance is achieved by the U–PCA–MM–AR method, with the U–MM–AR method following. For the detection of barely visible damages the ’conventional’ U–PCA–AR method may be also adequate, but this is not the case for invisible damages.

When considering the two MM–based methods, the U–MM–AR method is simpler, necessitating the determination of two (\( \nu, n \)) design parameters, instead of the three (\( \nu, n, \gamma \)) associated with the U–PCA–MM–AR method. The latter also requires somewhat higher computing power and skill on part of the operator.

Like with most methods, a concern relating to achieving the best performance with the MM–based methods is the determination of design parameters. In general, the use of as many as possible sample (healthy) structures in the baseline phase is recommended. The model order requires some attention, so that it is selected as low as possible, but still adequate for representing the dynamics. The selection of \( \gamma \) in conjunction with the U–PCA–MM–AR method also requires some caution and trials; it seems that values in the range [99–99.4]\% should be considered.

\(^{21}\)For this reason certain combinations of the design parameters are not considered.
3.6 Experimental Assessment of the MM–based Methods

Fig. 3.12. Sensitivity analysis for the U–PCA–MM–AR method. AUC for varying AR order $n$, number of baseline beams $\nu$, and $\gamma$. [50 baseline sets; the ‘optimal’ performance point is indicated by a magenta circle and an arrow; points with AUC values close to the ‘optimal’ are indicated by black arrows.]
3.7 Concluding Remarks

The problem of vibration–response–only based damage detection for a population of nominally identical structures, specifically composite beams representing the topology of the main part of a commercial Unmanned Aerial Vehicle (UAV) boom, was investigated via unsupervised methods. The study was based on a population sample consisting of 31 beams with manufacturing variability and other uncertainty sources inducing significant effects on the dynamics that nearly ‘mask’ those of damage. For each beam, damage was introduced via impact at one of two distinct energy levels and one of various possible positions.

Two Multiple Model (MM) based methods were postulated, an Unsupervised Multiple Model AR (U–MM–AR) method and its PCA–enhanced version (U–PCA–MM–AR). Comparisons were also made with two ‘conventional’ statistical time series type methods, an unsupervised AR parameter based (U–AR) and a PCA–enhanced version (U–PCA–AR).

The study may be claimed to constitute a first systematic assessment with a considerable population sample, with significant sample–to–sample variability in the dynamics. It also involved challenging damage scenarios (damage caused at various locations with low or higher impact energy), and thousands of inspection test cases based on which reliable performance results could be obtained.

The main conclusions of the study, which attempt to provide answers to the broad questions posed in the introduction, may be summarized as follows:

Q1. ‘Conventional’ statistical time series type methods exhibit serious difficulties in achieving effective damage detection, especially when low impact energy damages are considered. In this case their performance is judged as inadequate.

Q2. The MM–based methods lead to significant damage detection performance improvement, especially for low impact energy damages. Their overall performance is judged as very good. The U–PCA–MM–AR method may have a lead over its U–MM–AR counterpart. These performance characteristics are summarized (per damage severity) in Figure 3.13, by means of correct damage detection (true positive) rates, for two false alarm rates of interest.

Q3. Overall (all damage scenarios included) performance levels achieved by the MM–based methods: The correct detection rate approaching 100% for false alarm rates above 5%. It is remarkable that this performance was achieved for this challenging problem, with the use of only a single vibration response sensor and a limited frequency bandwidth.

![Figure 3.13](image_url)

Fig. 3.13. Postulated methods correct damage detection performance (True Positive) rate, versus two false alarm rates (FPR) of interest, by means of bar charts for: (a) Invisible damage scenarios, (b) Barely visible damage scenarios. [Based on the experiments of the benchmark application study (see chapter 2).]
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This chapter tackles the challenging problem of random vibration–based damage detection for a population of nominally identical structures using two supervised Principal Component Analysis (PCA) enhanced multiple model based methods. These are supervised in the sense that vibration response signals from the healthy and damaged structural states are used in their training phase. The objective is to automatically determine the PCA feature subspace that is sensitive to damage related changes in the dynamics, thereby relieving the user from critical selections. Their assessment is based on a population of 31 nominally identical composite beams with significant beam–to–beam variability in their dynamics, as well as on impact–induced type of damages, at various positions, using two distinct impact energy levels. Their assessment includes comparisons with the supervised version of a ‘conventional’ PCA–based method and their unsupervised counterparts, as well as a sensitivity analysis with respect to the damage scenarios included in the training phase. The assessment results indicate that the supervised methods may improve damage detection performance compared to their unsupervised counterparts, they are robust to the type and number of damage scenarios included in the training phase, and that the PCA–enhanced multiple model based methods yield the best performance.

4.1 Introduction

Random vibration–based damage detection forms an important part of the broader Structural Health Monitoring (SHM) problem, as vibration signals are often naturally available during normal operating conditions and easily measured via a variety of sensors and data acquisition systems of reasonable cost [Fassois and Kopsaftopoulos 2013][Deraemaeker 2010, pp. 19–32]. Methods tackling this problem use structural dynamics associated features (characteristic quantities), extracted from proper models of the vibration response signals (solely the challenging vibration–response–only case is herein considered). The idea is then that a damage changes the structural dynamics and subsequently these features [Chondros et al. 1989]. Yet, such changes may additionally manifest due to a multitude of damage irrelevant factors, such as varying Environmental and Operational Conditions (EOCs), thereby leading to subsequent changes in the features that may be misinterpreted as damage effects [Peeters, Maeck, and Roeck 2001a; Sohn 2007; Zhou et al. 2014]. The aggregated effect of such changes on the dynamics is interpreted as uncertainty in the present study, since it yields a subspace of healthy state associated features (denoted as ‘healthy
The latter issue is further amplified in an asset management based damage detection context [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b], where a population of nominally identical structures (such as an aircraft fleet) is monitored for damage via methods that infer on its dynamics using only 'sample' structures for their 'training'. In this context, the population dynamics exhibit uncertainty associated not only with the varying EOCs but also a combination of material, manufacturing, assembly and boundary conditions variability, thereby causing a challenging ‘masking’ effect of the damage induced changes on the dynamics. Challenging as it may be, asset management based damage detection yields some obvious time and cost associated advantages over the conventional separate 'training' for each individual population member.

The main premise for tackling damage detection under uncertainty is that the features associated with the healthy state dynamics occupy the ‘healthy subspace’, while their damaged state counterparts the complement ‘damage subspace’. Damage detection is then pursued by means of properly representing either subspace and subsequently deciding whether the current (unknown) health state feature is a subspace member.

Toward this end, and by taking into account the aggregated knowledge from the chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]) and the studies cited therein, the present study focuses on a Principal Component Analysis (PCA) enhanced Multiple Model (MM) based method that uses the AutoRegressive (AR) parameters as features (U–PCA–MM–AR method) and may achieve remarkable damage detection performance [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. Yet, this performance may be significantly compromised, when the method’s design parameters are not properly selected for a given population of structures [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

The selection of these important parameters is typically performed in the methods’ training (baseline) phase using a number of vibration response signals from the healthy population, as well as experience and proper tools (unsupervised training [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]). Among the method’s design parameters the one that often is not properly selected via such procedures, yet significantly impacts the damage detection performance, is the number of principal components that define the healthy subspace and are used for damage detection. This is typically determined either based on a–priori knowledge of the number of uncertainty sources or on eigenvalues diagrams [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b],[Jolliffe 2002, pp. 112-133],[Bellino et al. 2010; Giraldo 2006]. Yet, the number of uncertainty sources affecting the dynamics is usually unknown, while the eigenvalues diagram (which presents the PCA eigenvalues in decreasing order) often does not provide an evident cut–off based on which the eigenvalues are clearly separated in a high and a low valued sets. A number of the first components may be empirically selected instead, so that they cumulatively contribute a certain percentage (γ, often set as 90% or 95%) of the total model parameters variability, which however may yield a poor representation of the ‘healthy subspace’.

Toward relieving the user from the burdensome selection of components via such procedures, an alternative may be formulated by additionally employing vibration response signals from damaged cases in the baseline phase and then using proper algorithms for their automated selection (supervised training). Despite its importance, supervised training of damage detection methods has received limited attention that is mainly focused on the general (not for a population of structures or for principal components selection) problem of automating the damage sensitive features selection via proper supervised algorithms [Hoell et al. 2016b; Stull et al. 2012].

The objective of such supervised algorithms is the maximization of damage detection performance via proper damage sensitive feature selection (such as the AR model order or a subset of model parameters). The damage detection performance is measured either directly [Stull et al. 2012] via Receiver Operating Characteristic (ROC) curves [Fawcett 2006] and the Area Under the Curve (AUC), or indirectly via a proper measure of the distance between the healthy and damaged state features [Hoell et al. 2016b].
4.1 Introduction

The performance measure is then maximized with respect to the damage sensitive feature, either by means of a user defined collection of candidate features and selection of the one yielding the highest AUC value [Stull et al. 2012], or via one of the following optimization algorithms: forward selection, backward elimination, or a genetic algorithm [Hoell et al. 2016b]. Of course such a collection is formulated using prior knowledge or user experience and intuition, thus resulting into possibly suboptimal features selection (with respect to damage detection performance). On the other hand, the above mentioned optimization algorithms automatically explore the feature space for candidate features, with the genetic algorithm yielding the best damage detection performance among the three algorithms [Hoell et al. 2016b]. This is the case, since the genetic algorithm explores all the feature space for candidate features, while the alternative algorithms explore a small subspace of the feature space, assuming mutually independent features, and thereby losing significant information.

The forward selection algorithm [Gonzalez et al. 2016; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015a] and the genetic algorithm [Hoell et al. 2016a] have additionally been used for the current problem of supervised principal components selection for damage detection. An alternative denoted as fast forward selection algorithm, has also been used for tackling the same problem, yielding similar results with the genetic algorithm [Hoell et al. 2016a], while being significantly simpler and faster. The fast forward and forward selection algorithms are based on a similar procedure, where various combinations of principal components are sequentially created, starting with a single component and adding a new one per iteration. The new component is then retained if and only if it improves the damage detection performance.

The difference between the fast forward and forward selection algorithms is that the former algorithm assumes some correlation among the features, whereas the latter algorithm mutually independent features. Practically, their difference is that the latter algorithm starts from the last component (with the lowest variance) and sequentially adds components that improve the performance until it reaches the first component (each component contributes to the performance independently of the others), while the former starts from the component yielding the best performance and then incorporates a new component depending on which two components combination yields the best performance (multiple combinations are explored to account for potential correlations among the components), a procedure repeated for increasing components number. Thus, the fast forward algorithm assesses more principal components combinations than the forward one, yet given the components mutual independence this may be an unnecessary additional computational cost.

Of course the ‘optimal’ components selected by the algorithms are determined using only sample structures from the healthy and damaged structural states in the baseline phase, without generalizing to unobserved structural states. Therefore, the components yield the best detection performance for the structures of the baseline phase. Yet, new damage types or healthy structures may significantly affect this performance in the inspection phase, thereby creating an over–fitting problem.

Toward alleviating this problem, the present author postulated in a recent preliminary study [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018a] a sequential algorithm for selecting the principal components. The algorithm’s objective is to generalize to unobserved structural states, by penalizing the rejection of a high number of components, hence retaining components that may contain information for such states. This is achieved via empirically defined weights on the objective function that maximizes (with respect to the rejected principal components number) the separation between the healthy and damaged state features of the baseline phase. Yet, these weights are empirically determined for the specific experimental set–up and the structures studied in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018a], therefore requiring proper user manipulation when damage detection for other structures is tackled.

Among the aforementioned supervised damage detection relevant studies, only [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015a; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018a]...
Chapter 4. Supervised PCA–enhanced Multiple Model Type Methods

[72x771] iou, et al. 2018a] tackle the problem of damage detection for a population of nominally identical structures. Based on the lessons learned from these two studies and the limitations of the other relevant studies, the present study aims at postulating three novel supervised versions of the unsupervised PCA–enhanced methods presented in chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]), for tackling the present problem, while alleviating these limitations. The first two methods are the non–centered and centered versions of the supervised PCA–enhanced MM based method that employs AR models (S–nPCA–MM–AR and S–PCA–MM–AR respectively –the letter S designates supervised, while U unsupervised method) [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018a; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. The third method is the supervised version of a ’conventional’ type PCA–enhanced method that is also based on AR models (S–PCA–AR) [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] and is employed in the study for setting a damage detection performance standard.

All three supervised methods aim at the automated selection of the damage subspace defining principal components. These methods use the fact that uncertainty in the dynamics of the healthy state is the only source of variability in the baseline phase, along with an assumption for orthogonality between the healthy and damaged state features, as a regulator for the components selection. Based on this idea, a number of the first components, featuring high variance, corresponds to the healthy state, while the remaining $m$ components to damage. Therefore, a simple and computationally fast optimization algorithm is postulated for determining the number $m$ of the last components that define the damage subspace. This is achieved by sequentially increasing the number $m$ and selecting the value that maximizes the separation between the healthy and damaged state features (presently the AR model parameters projection on the $m$ dimensional PCA space) of the baseline phase. Of course, specific damaged states may be represented using only a subset of the $m$ components, while others using another subset, yet the last $m$ components construct a generalized damage subspace that does not over–fit the known in the baseline structural states.

The main questions posed and answered in the present study are presented below:

Q1. What are the supervised methods benefits to the user?
Q2. How does the supervised methods damage detection performance compare to that of their unsupervised counterparts?
Q3. Which supervised method yields the best damage detection performance?
Q4. How sensitive are the supervised methods to the type and number of damage scenarios included in the baseline phase?
Q5. Should the supervised methods be preferred over their unsupervised counterparts?

The main contributions of the present study are outlined as follows:

1. The problem of random vibration–based damage detection for a population of nominally identical structures is systematically considered within a supervised framework using the 31 nominally identical composite beams that were also employed in chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]) for tackling the unsupervised counterpart of the problem.
2. A combination of delamination, small cracks, and broken fibres caused by impact at various positions and at two distinct energy levels comprises the considered damage scenarios. Yet, the overall uncertainty in the dynamics of the healthy population nearly ’masks’ that caused by damage.
3. The non–centered and centered versions of the PCA–enhanced MM based method (S–nPCA–MM–AR and S–PCA–MM–AR respectively) are for the first time directly compared via a comprehensive study$^2$.
4. All three supervised methods are compared to their unsupervised counterparts (see [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]).
5. The limited availability of damage scenarios in the baseline phase and the existence of new (unexpected) scenarios in the inspection phase is considered via a sensitivity analysis to reveal its

$^2$The use of non-centered PCA is a debatable subject [Honeine 2014] since its properties and usefulness are still under investigation. Presently its usefulness is assessed in the SHM context in terms of damage detection performance.
effect on detection performance.

The rest of this chapter is organized as follows: The precise problem statement is presented in section 4.2. The paradigm application problem treated, along with the population of composite beams, the damage scenarios, the experiments, and the variability induced on the dynamics, is presented in section 4.3. The supervised PCA–enhanced methods are presented in section 4.4, while their experimental assessment is presented in section 4.5. Conclusions are finally summarized in section 4.9.

4.2 Precise Problem Statement –the Supervised Detection Problem

The precise problem addressed in the present study is stated as follows:

Given:
1. **Baseline (Training) Phase:**
   - A set of $p_o$ (subscript ‘o’ designates a healthy structure available in the baseline phase) sampled scalar vibration–response signals obtained from $v_o \leq p_o$ members (‘sample’ structures) of the considered healthy state population, each represented as $y_o[t]$, with $t = 1, 2, \ldots, N$ designating normalized (by the sampling period) discrete time and $N$ the signal length.
   - A set of $p_d$ scalar vibration–response signals obtained from $v_d \leq p_d$ damaged structures available in the baseline phase, being represented as $y_d[t]$ (the subscript ‘d’ designates a damaged structure available in the baseline phase).

2. **Inspection (Operational) Phase:**
   A vibration response signal, $y_u[t]$ (the subscript ‘u’ is used to designate a structure in its current/unknown state), is obtained from any member of the population.

Determine: The current health state for any member of the population.

Remarks:
1. Damage detection is implemented in a batch mode, meaning that the decision is made after the entire vibration response signal $y_u[t]$ ($t = 1, 2, \ldots, N$) has been obtained.
2. It is implicitly assumed that the health state of the structure as well as the respective environmental/operating conditions do not change during the short signal acquisition periods.
3. The measurement position and direction is fixed for all structures and both (baseline and inspection) phases.

4.3 The Structures, the Damage Scenarios & the Experimental Set-up

The population of 31 nominally identical composite beams presented in chapter 2 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]) is used for the methods assessment in the present study as well. Note that the population of beams herein employed is different than the respective population in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2015a], while includes 4 more damage scenarios compared to [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018a]. The beams consist of several layers of woven and unidirectional fabric manufactured based on one shot Resin Transfer Molding and represent the topology of the main part of a commercial Unmanned Aerial Vehicle. As explained in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] they exhibit high variability in their dynamics due to a combination of uncertainty sources such as manufacturing, material, and assembly variability. The experimental set-up for each beam is shown in Figure 4.1, while their nominal dimensions are shown in Figure 4.2.

Eight damage scenarios are considered in this study and induced in eight distinct beams via a pendulum type impact hammer. Each beam is damaged at one of the points (A,B,C,D) using either Low
Fig. 4.1. Experimental set-up: Point X represents the force excitation position, Point Y1 the vibration acceleration measurement position [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

Fig. 4.2. Sketch (top view) of the experimental set-up with the beam, the tail mass, the clamping, the damage locations and the measurement Point Y1 (dimensions in mm) [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

(L; invisible damage) or Higher (H; barely visible damage) impact energy level. Each combination of damage location and impact energy corresponds to a distinct damage scenario that is designate using two letters, with the first indicating damage location and the second impact energy level (see Figure 4.2 and Table 4.1). The dynamics of all healthy and damaged structures are presented in terms of envelopes of Welch–based Power Spectral Density (PSD) estimates [Ljung 1999, pp. 173–187] in Figure 4.3. It is evident that the effects of uncertainty ‘mask’ the effects of damages in the dynamics, a phenomenon especially pronounced for the low and intermediate frequencies as well as for the low impact energy damages.

Details on the data acquisition, the signals processing and subsequent non–parametric/parametric modeling are available in Table 4.2. Further information about the structures, the experiments and the AR model selection is available in chapter 2 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]). Presently, the AR(57) model is employed as this is the model order suggested by the sensitivity
4.3 The Structures, the Damage Scenarios & the Experimental Set-up

**Fig. 4.3.** Effects of damage and uncertainty on the population of nominally identical beams dynamics: (a) Invisible damage, (b) Barely visible damage. [All experiments per healthy/damaged beam; each color zone represents an envelope containing the Welch-based PSD estimates for all the beams corresponding to a particular structural health state].

**Table 4.1:** The damage scenarios and experimental details [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Impact Energy (J)</th>
<th>Damage Visual Characterisation</th>
<th>Impact Position</th>
<th>Number of beams</th>
<th>Number of experiments per beam</th>
<th>Total number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td></td>
<td></td>
<td></td>
<td>23</td>
<td>7</td>
<td>161</td>
</tr>
<tr>
<td>Damage AL</td>
<td>5</td>
<td>Invisible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BL</td>
<td>5</td>
<td>Invisible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CL</td>
<td>5</td>
<td>Invisible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DL</td>
<td>5</td>
<td>Invisible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage AH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

4.4 The Supervised PCA–enhanced Damage Detection Methods

Three supervised PCA–enhanced damage detection methods motivated by the work of chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]) are presented in the sequel. Two of the methods are MM–based and are postulated for tackling the damage detection for a population of nominally identical structures problem, while the third one is a robust ‘conventional’ type method employed for reasons of comparison as a reference method. All methods use vibration response signals from healthy and damaged structures in the baseline phase, in order to automatically select the healthy subspace defining principal components. The selection is made by means of the components combination that maximizes the discrepancy of the baseline phase healthy state test pseudo–statistics from their damaged state counterparts.

The three methods employ the AR model parameters as their characteristic quantity (damage detection features), which convey structural dynamics relevant information [Fassois and J. S. Sakellariou 2009] since the AR model is identified using a vibration response signal. The structure of such a model is the following [Fassois 2001; Ljung 1999]:

\[
y[t] + \sum_{i=1}^{n} a_i \cdot y[t-i] = e[t], \quad e[t] \sim \text{NID}(0, \sigma_e^2)\]  

(4.1)

where \( t = 1, \ldots, N \) is the normalized (by the sampling period) discrete time, \( y[t] \) the vibration response signal, \( n \) the model order, \( a_i \) the \( i \)–th AR parameter, and \( e[t] \) the model residual assumed to be a white Gaussian zero–mean sequence with variance \( \sigma_e^2 \). \( \text{NID} \) denotes a Normally Independently Distributed with the indicated mean and variance variable. The model is estimated using Ordinary Least Squares with QR implementation [Ljung 1999, pp. 318–320], [Fassois 2001], while its order is selected based on the Bayesian Information Criterion (BIC) and the Residual Sum of Squares / Signal Sum of Squares (RSS/SSS).

The AR model parameters are organized in a vector\(^3\) as shown below\(^4,5\):

\[
\alpha := [a_1 a_2 \ldots a_n]^T
\]

(4.2)

This parameter vector estimate is asymptotically (as the signal length \( N \to \infty \)) normally distributed with mean equal to each point estimate \( \alpha \) and estimated covariance \( \Sigma \) [Ljung 1999, p. 215].

\(^3\)Bold–face capital/lower letters designate matrices/vectors, respectively  
\(^4\)For notation simplicity no distinction is made between the true parameter vector and its estimate.  
\(^5\)\(T\) designates matrix transposition  
\(^6\)Covariance matrices expressing estimation uncertainty for a model identified from a single signal are designated by the symbol \( \Sigma \). On the other hand, sample (central or non–central) second moment matrices estimated using model parameter vectors from multiple signals are designated as \( P \).
4.4 The Supervised PCA–enhanced Damage Detection Methods

4.4.1 The Non–centred PCA–enhanced Multiple Model AR (S–nPCA–MM–AR) Method

This is the supervised version of the U–PCA–MM–AR method postulated in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] that is based on non–centered PCA [Jackson 1991, pp. 72–75][Cadima et al. 2009]. It employs a subspace defined by a number of automatically selected principal components, to model the major part of the affected by uncertainty healthy subspace. The remaining healthy subspace (also affected by uncertainty) is modeled via a MM representation (that is a crude RC model). The resulting healthy subspace representation is pictorially depicted for an indicative 3–dimensional parameters space in Figure 4.4. The healthy subspace is therein depicted as a deformed cylindrical tube, with uniform cross–section across its length, where the cylinder is defined by the first principal component and its deformation by the MM representation. This model provides modeling of complex healthy subspace geometries given a limited number of healthy state vibration response signals (experiments) in the baseline phase. This is the case, since PCA may provide a healthy subspace representation given limited experiments and MM is applied on a lower dimensional feature space (due to PCA dimensionality reduction), hence requiring a low number of experiments as well in the baseline phase. Discrepancies from this combined PCA–MM model indicate the existence of damage. The method’s operational phases are provided below.

Baseline phase. In this phase \( \nu_o \) healthy and \( \nu_d \) damaged nominally identical structures are used for vibration experiments, yielding \( p_o \) (>> \( n \)) and \( p_d \) (presently \( p_d = \nu_d \)) vibration response signals respectively. The method is then trained as follows.

Step 1: Identify an AR(\( n \)) model for each healthy and damaged state signal, with parameter vector estimates designated as \( \alpha_{oj} \) (\( j = 1, \ldots, p_o \)) and \( \alpha_{dj} \) (\( j = 1, \ldots, p_d \)) respectively.

Step 2: Determine the sample non–central second moment matrix of the AR parameter vectors \( \alpha_{oj} \):

\[
P := \frac{1}{p_o} \sum_{j=1}^{p_o} \alpha_{oj} \alpha_{oj}^T \quad (n \times n)
\]

Fig. 4.4. A schematic of the healthy subspace representation constructed by the PCA–MM–AR based methods. [Indicative 3–dimensional example]
and decompose it via Singular Value Decomposition (SVD) [Jolliffe 2002, pp. 44-46, 412-413] as:

\[ P = U S^2 U^T \]  

(4.4)

where:

\[ S^2 := \text{diag}(s_1^2, s_2^2, \ldots, s_n^2) \quad (n \times n), \quad U := [u_1 \ u_2 \ldots u_n] \quad (n \times n) \]  

(4.5)

\( U \) designates a real unitary matrix whose columns are the set of all available principal components and \( S^2 \) a diagonal matrix containing the eigenvalues of \( P \) arranged in decreasing order. The matrix \( S^2 \) is in fact the covariance matrix of the model parameter vectors projection on the space defined by the column vectors of \( U \) that is the PCA space.

**Note 1:** The objective in the following steps is the implicit determination of the subspace that models the major part of the healthy subspace and the explicit determination of its complement subspace that conveys damage related information. In the centred PCA context [Yan et al. 2005], the former subspace is defined by a set of the first principal components that exhibit the highest variance and hence are associated with the uncertainty imposed (healthy state) model parameters variability. Yet, in the current non–centred PCA–based formulation of the method the model parameters are not sample mean corrected, thus yielding a first principal component that is mostly associated with the sample mean [Jackson 1991, p. 73], instead of the variance (healthy state variability). Such a component conveys limited healthy state information and should be excluded from the subspace that models the major part of the healthy subspace.

**Note 2:** Of–course, when the sample mean tends to zero the centered and non–centered PCA versions coincide. In such a case, the first principal component is highly associated with the (healthy state) parameters variance, hence the healthy subspace as well, and should be included in the set of principal components that define the latter subspace. An indication of this special case is available via the percentage of model parameters variability contributed by the first principal component, which demonstrates values below 99% [Jackson 1991, p. 73].

**Note 3:** The complement subspace containing the damage subspace and the remaining part of the healthy subspace is defined by the remaining \( m \) components that are not included into the set that defines the major part of the healthy subspace. The projection of a model parameter vector onto this complement subspace provides a discrepancy measure (employed for damage detection) between this vector and the subspace modelling the major part of the healthy subspace. This projection is achieved by means of the so called loading matrix (say \( L \)) that is constructed as:

\[ L = [u_1 \ | \ u_{n-m+2} \ldots u_n] \quad (n \times m) \]  

(4.6)

The projection of any model parameter vector \( \alpha \) onto the complement subspace and the respective transformation of its covariance matrix \( \Sigma \) are then determined via the following expressions:

\[ \bar{\alpha} := L^T \alpha \quad (m \times 1), \quad \bar{\Sigma} := L^T \Sigma L \quad (m \times m) \]  

(4.7)

with the over–bar designating projection (or transformation) onto the truncated \( m \)–dimensional PCA space.

**Note 4:** The loading matrix \( L \) is constructed automatically by taking advantage of the supervised formulation of the method to determine the number, \( m \), of principal components. Toward this end, a useful ‘tool’ is the Kullback–Leibler (KL) divergence [Press et al. 2007, pp. 756-758], which is used as a pseudo–distance metric between two PCA transformed parameter vectors. Given two
such vectors $\bar{\alpha}_{oi}$, $\bar{\alpha}_{oj}$ with respective covariance matrices $\bar{\Sigma}_{oi}$, $\bar{\Sigma}_{oj}$, the KL divergence is expressed as follows (making use of the model parameters estimator Gaussian distribution):

$$d(\bar{\alpha}_{oj}, \bar{\alpha}_{oi}) := \frac{1}{2} \left[ tr\left( \bar{\Sigma}_{oj}^{-1}\bar{\Sigma}_{oi} \right) + (\bar{\alpha}_{oj} - \bar{\alpha}_{oi})^T \bar{\Sigma}_{oj}^{-1}(\bar{\alpha}_{oj} - \bar{\alpha}_{oi}) - 1 - \ln \left( \frac{det \bar{\Sigma}_{oi}}{det \bar{\Sigma}_{oj}} \right) \right]$$ (4.8)

with $tr(\cdot)$ designating trace and $det(\cdot)$ determinant of the indicated quantity. Since the KL–divergence is a non–symmetric pseudo–distance metric, it should be used as presented above (without interchanging the two parameter vectors and their covariances).

**Step 3:** Set $m = 1$ and obtain the corresponding PCA transformed parameter vectors using Equation 4.7.

**Step 4:** Determine the following (MM based) test pseudo–statistic, per transformed healthy state parameter vector:

$$D_{oi} := \frac{1}{p_o} \sum_{j=1}^{p_o} d(\bar{\alpha}_{oj}, \bar{\alpha}_{oi}), \quad \forall i = 1, 2, \ldots, p_o, \quad \text{with} \quad i \neq j$$ (4.9)

where $D_{oi}$ designates the $i$–th transformed healthy state parameter vector’s test pseudo–statistic. This is defined as the average of the $p_o$ KL–divergences between the transformed parameter vector $\bar{\alpha}_{oi}$ and each one of the remaining transformed healthy state parameter vectors $\bar{\alpha}_{oj}$ (see also Figure 4.5). Then, find the maximum among the $D_{oi}$ test pseudo–statistics (see also Figure 4.5):

$$D_o(m) := \max_i (D_{oi})$$ (4.10)

where $D_o(m)$ designates the maximum test pseudo–statistic for the transformed healthy state parameter vectors at the current $m$ value.

**Step 5:** Determine the following (MM based) test pseudo–statistic per transformed damaged state parameter vector:

$$D_{di} := \frac{1}{p_d} \sum_{j=1}^{p_d} d(\bar{\alpha}_{oij}, \bar{\alpha}_{di}), \quad \forall i = 1, 2, \ldots, p_d$$ (4.11)

where $D_{di}$ designates the $i$–th transformed damaged state parameter vector’s test pseudo–statistic (see also Figure 4.5). Then, find the minimum among the $D_{di}$ test pseudo–statistics (see also Figure 4.5):

$$D_d(m) := \min_i (D_{di})$$ (4.12)

where $D_d(m)$ designates the minimum test pseudo–statistic for the transformed damaged state parameter vectors at the current $m$ value.

**Step 6:** Repeat steps 3 to 7 for $m$ sequentially taking values $2, \ldots, n - 1$.

**Note 5:** When $m = n - 1$, the S–nPCA–MM–AR method uses the projection of the model parameters on all the principal components. In such a case, only the MM representation is used to model the healthy subspace and the method becomes similar to the U–MM–AR method that is presented in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

**Step 7:** Maximize the difference between minimum damaged and maximum healthy test pseudo–statistics, with respect to $m$ as (see also Figure 4.5):

$$m^* := \arg \max_m [(D_d(m) - D_o(m))]$$ (4.13)

with $m^*$ designating the desired $m$ value.
Step 8: Determine the loading matrix $L$ by setting $m = m^*$ in Equation 4.6. The selected $m$ value maximizes the separation of the healthy and damaged state test pseudo–statistics of the baseline phase, hence the separation of the respective parameter vectors as well, so as to maximize damage detection performance.

**Baseline phase**

![Truncated PCA space](image1)

- $D_o(m) = \max_i (D_{oi}) = \frac{1}{p_o} \sum_{j=1}^{p_o} d(\bar{\alpha}_{oj}, \bar{\alpha}_u)$
- $D_d(m) = \min_i (D_{di}) = \frac{1}{p_o} \sum_{j=1}^{p_o} d(\bar{\alpha}_{oj}, \bar{\alpha}_{d1})$
- $m^* = \arg \max_m [D_d(m) - D_o(m)]$

**Inspection phase**

![Truncated PCA space](image2)

- $D = \sum_{j=1}^{p_o} d(\bar{\alpha}_{oj}, \bar{\alpha}_u)$
- $D \leq l_{lim} \rightarrow$ Healthy structure
- otherwise $\rightarrow$ Damaged structure

\[ l_{lim} \text{ a user defined threshold; } p_o = 3 \]

Fig. 4.5. A schematic representation of the PCA–MM–AR based methods operational phases.

**Inspection phase.** A fresh vibration response signal is obtained in this phase from a ’sample’ structure of unknown health state and modelled using the model structure $AR(n)$ of the baseline phase. The respective parameter vector $\alpha_u$ and its covariance matrix $\Sigma_u$ are then transformed into the truncated $m$–dimensional PCA space via the following expressions:

\[ \bar{\alpha}_u = L^T \alpha_u \quad (m \times 1), \quad \bar{\Sigma}_u = L^T \Sigma_u L \quad (m \times m) \]  

(4.14)

The method’s test pseudo–statistic, $D$, is obtained as:

\[ D = \sum_{j=1}^{p_o} d(\bar{\alpha}_{oj}, \bar{\alpha}_u) \]  

(4.15)

Damage detection is then achieved by means of the following decision making mechanism:

\[ D \leq l_{lim} \rightarrow \text{Healthy structure} \]
\[ \text{otherwise } \rightarrow \text{Damaged structure} \]  

(4.16)
with \( l_{lim} \) designating a user-specified decision making threshold selected using the signals of the baseline phase.

### 4.4.2 The Centred PCA–enhanced Multiple Model AR (S–PCA–MM–AR) Method

This is the centered PCA [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] version of the S–nPCA–MM–AR method presented above and its operational phases follow:

**Baseline phase.** The model parameter vectors \( \alpha_{oj} \) and \( \alpha_{di} \) are obtained as in the S–nPCA–MM–AR method and centered by subtracting the *healthy parameter vectors sample mean*. The sample mean vector is a Gaussian distributed random variable obtained by the superposition of \( p_o \) independent Gaussian distributed random variables. Subtraction between two Gaussian distributed random variables (a parameter vector and the sample mean) yields a new Gaussian distributed random variable that takes into account the distributions of both variables. The respective covariance matrices, \( \Sigma_{oj} \) and \( \Sigma_{di} \), are thus adjusted via the following expressions, so as to take into account the sample mean estimation uncertainty (see proof in B.1):

\[
\Sigma_{oj} = \Sigma_{oj} + \frac{1}{p_o} \sum_{j=1}^{p_o} \Sigma_{oj} - \frac{2}{p_o} \Sigma_{oj}
\]

\[
\Sigma_{di} = \Sigma_{di} + \frac{1}{p_o} \sum_{j=1}^{p_o} \Sigma_{oj}
\]

For the rest of this section all the parameter vectors are assumed centred and the respective covariance matrices adjusted via Equation 4.17.

The rest of this phase is similar to the previous method including the sample covariance matrix \( P \) estimation (this is obtained via Equation 4.3 using centred parameter vectors), its analysis into its principal components yielding the matrices \( U, S^2 \), as well as the automated determination of the loading matrix \( L \) via the steps 3 to 8 and the following definition:

\[
L := [u_{n−m+1} \ldots u_n] \quad (n \times m)
\]

Note that the sample mean of the centred parameter vectors is zero, hence the first principal component is not sample mean affected, being instead strongly associated with uncertainty. The first component is therefore included into the set of components defining the major part of the healthy subspace.

**Inspection phase.** A fresh vibration response signal is obtained in this phase from a current ‘sample’ structure in unknown state. This is subsequently modeled using an AR(\( n \)) model yielding a parameter vector \( \alpha_u \) and the respective covariance matrix \( \Sigma_u \). The former is centred by subtracting the sample mean of the model parameter vectors estimated in the baseline phase, while the latter is adjusted using the following expression:

\[
\Sigma_u = \Sigma_u + \frac{1}{p_o} \sum_{j=1}^{p_o} \Sigma_{oj}
\]

Then, they are both transformed into the truncated \( m \)-dimensional PCA space via Equation 4.14 and damage detection is tackled by substituting the test pseudo-statistic \( D \) of Equation 4.15 into the decision making mechanism of Equation 4.16.

**Remark:** All the references to equations of the previous method assume that the latter use centred parameter vectors and their associated adjusted covariance matrices.
4.4.3 The ’conventional’ PCA–enhanced AR (S–PCA–AR) Method

This is the supervised version of the robust ’conventional’ type U–PCA–AR method postulated in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] that employs the centred PCA and may be interpreted as a simplification of the S–PCA–MM–AR. The idea is to automatically define two complement linear subspaces with the one representing the healthy subspace and the other–one the damage subspace. The latter is assumed to be affected solely by damage and as such is directly (without using MM or other RC representations) employed for damage detection. The method’s operational phases are provided below:

**Baseline phase.** The first part of this phase is identical to the corresponding part of the S–PCA–MM–AR method that is presented above. Following the analysis of the sample covariance matrix ($\mathbf{P}$) into its principal components and the computation of the matrix $\mathbf{U}$, the number $m$ of principal components is determined using steps 3 to 8 of the S–PCA–MM–AR method and by replacing the Equation 4.9 and Equation 4.11 with the respective equations below:

\[
D_{oi} := \| \bar{\alpha}_{oi} \|_{l_2}, \quad \forall i = 1, 2, \ldots, p_o
\]  

(4.20a)

\[
D_{di} := \| \bar{\alpha}_{di} \|_{l_2}, \quad \forall i = 1, 2, \ldots, p_d
\]  

(4.20b)

where $D_{oi}$ and $D_{di}$ respectively designate the $i$–th transformed healthy and damaged state parameter vectors test pseudo–statistic. These are defined as the Euclidean norm of the transformed parameter vectors $\bar{\alpha}_{oi}$ and $\bar{\alpha}_{di}$ respectively and indicate the vectors discrepancy from the truncated PCA space origin, which represents the healthy subspace defined by the principal components excluded from the $m$–dimensional set.

**Inspection phase.** As in the previous method the parameter vector $\alpha_u$ is estimated and centred (via sample mean subtraction). Damage detection is then based on the following decision making mechanism:

\[
D = \| \bar{\alpha}_u \|_{l_2} \leq l_{lim} \quad \rightarrow \quad \text{Healthy structure}
\]

\[
\text{otherwise} \quad \rightarrow \quad \text{Damaged structure}
\]

(4.21)

with $l_{lim}$ designating a user–defined threshold and $D$ the method’s test pseudo–statistic.

4.5 Performance Assessment

The three supervised methods damage detection performance is assessed in this section, via a systematic procedure that eliminates the damage detection performance dependence on the sample healthy structures used in the baseline phase (comparisons with an alternative principal components selection algorithm are available in B.2).

4.5.1 The Assessment Procedure and the Operating Conditions (OCs)

The methods performance is assessed through a systematic procedure employing $v_o = 15$ healthy beams in the baseline phase and the remaining 8 healthy beams in the inspection phase. Yet, the set of healthy beams in the baseline phase is not unique, since numerous sets are obtained by creating numerous combinations of 15 healthy beams from the available 23 beams. Each set is then used for damage detection tests, thereby yielding a high number of test cases. This procedure may be interpreted as a rotation between the healthy baseline and inspection beams, aiming at eliminating dependencies of the damage detection performance on a distinct set of beams employed for the methods training, thereby yielding robust performance assessment results (see details in [K. J. Vamvoudakis–Stefanou, J. S. ...]}
4.5 Performance Assessment

Table 4.3: Operating Conditions employed for the supervised methods performance assessment and sensitivity analysis.

<table>
<thead>
<tr>
<th>Study objective</th>
<th>Operating Condition (Acronym)</th>
<th>Baseline damage scenarios</th>
<th>Inspection damage scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance Assessment</td>
<td>PA&lt;sub&gt;L&lt;/sub&gt;</td>
<td>all&lt;sup&gt;*&lt;/sup&gt; L</td>
<td>all L</td>
</tr>
<tr>
<td></td>
<td>PA&lt;sub&gt;H&lt;/sub&gt;</td>
<td>all H</td>
<td>all H</td>
</tr>
<tr>
<td></td>
<td>PA&lt;sub&gt;all&lt;/sub&gt;</td>
<td>all</td>
<td>all</td>
</tr>
<tr>
<td>Sensitivity Analysis</td>
<td>SA&lt;sub&gt;L&lt;/sub&gt;</td>
<td>all L</td>
<td>all</td>
</tr>
<tr>
<td></td>
<td>SA&lt;sub&gt;H&lt;/sub&gt;</td>
<td>all H</td>
<td>all</td>
</tr>
<tr>
<td></td>
<td>SA&lt;sub&gt;LH&lt;/sub&gt;</td>
<td>{AL, BL, AH, BH}</td>
<td>all</td>
</tr>
<tr>
<td></td>
<td>SA&lt;sub&gt;LH&lt;/sub&gt;</td>
<td>{AL, BL, CH, DH}</td>
<td>all</td>
</tr>
</tbody>
</table>

<sup>*</sup> 'all' refers to all the damage scenarios of Table 4.1, while the acronyms 'L' and 'H' indicate the low and higher impact energy damage scenarios respectively.

<sup>**</sup> See damage scenarios in Table 4.1.

Sakellariou, et al. 2018b]). In addition, the assessment procedure is enhanced with a proper normalization of the methods test pseudo–statistic, D, (see description in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]), so as to eliminate scaling discrepancies among D values obtained from different sets of healthy baseline beams.

The three distinct Operating Conditions (OCs) presented in Table 4.3 are employed to assess the supervised methods performance using the above–mentioned assessment procedure. The respective number of healthy and damaged inspection beams, inspection experiments, employed sets of healthy baseline beams and aggregate inspection experiments per OC is provided in Table 4.4 (see definitions in the table).

The methods performance is assessed by means of the ROC curves and their AUC values (for detailed presentation of the ROC curves the reader is referred to [Fawcett 2006; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]). Presently, the True Positive Rate (TPR) and the False Positive Rate (FPR) in the ROC curve diagrams correspond to the correct damage detection and false alarm rate respectively. These are constructed by using the aggregate inspection experiments of Table 4.4, for damage detection tests with different decision thresholds.

4.5.2 Methods Design Parameters Selection and PCA Insights

The methods design parameters are presently selected by taking into account the sensitivity analysis results of chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]). In detail, the set of design parameters yielding the ‘best’ damage detection performance in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] for the PCA–enhanced methods (ν<sub>o</sub> = 15; n = 57), is used in the present study. On the other hand, the number m of principal components, defining the damage subspace, is automatically selected by the supervised methods.

Insight into the determination of the design parameter m is gained via Figure 4.6. This depicts the contribution of each eigenvalue of P to the parameter vectors total variability, defined as:

\[
\frac{s_j^2}{\sum_{i=1}^{n} s_i^2} \times 100\% 
\]

Figure 4.6(a) shows that the first principal component in the non–centered PCA case explains 98.24% of the total variability, thus depicts a great discrepancy from the other components. Based on this observation the first component is expected to be included by the S–nPCA–MM–AR method into the m principal components set (see also the notes of subsection 4.4.1). On the other hand, Figure 4.6(b) shows the respective values for the centered PCA case where all the components depict significantly
Chapter 4. Supervised PCA–enhanced Multiple Model Type Methods

Table 4.4: Number of baseline and inspection beams, baseline and inspection experiments, and aggregate inspection experiments per operating condition.

<table>
<thead>
<tr>
<th>Oper. conds. (OCs)</th>
<th>Baseline beams(1) ((v_0/v_d^*))</th>
<th>Inspection beams(1) ((i_0/i_d^*))</th>
<th>Baseline experiments(2)</th>
<th>Inspection experiments(2) ((z_0/z_d^*))</th>
<th>Employed sets of healthy baseline beams(3) ((h))</th>
<th>Aggregate inspection experiments(4) ((h(z_0 i_0 + z_d i_d)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA (_{all})</td>
<td>15/8*</td>
<td>8/8*</td>
<td>7/1*</td>
<td>7/6*</td>
<td>50</td>
<td>5 200</td>
</tr>
<tr>
<td>{PA(_L), PA(_H)}</td>
<td>15/4*</td>
<td>8/4*</td>
<td>7/1*</td>
<td>7/6*</td>
<td>50</td>
<td>4 000</td>
</tr>
<tr>
<td>SA</td>
<td>15/8*</td>
<td>8/8*</td>
<td>7/1*</td>
<td>7/6 or 7(5)</td>
<td>50</td>
<td>5 400</td>
</tr>
</tbody>
</table>

* # / #: Number of respective quantity per Healthy / Damaged state.

(1) Baseline and Inspection beams: all the beams used in the baseline and inspection phases respectively.
(2) Baseline and Inspection exp.: all the experiments used (per beam) in the baseline and inspection phases respectively.
(3) Employed sets of healthy baseline beams: Number of sets, consisting of \(v_0\) healthy baseline beams, employed for damage detection tests.
(4) Aggregate inspection experiments: all the inspection experiments from all the employed sets of healthy baseline beams.
(5) 6 experiments for the damaged beams used in both the baseline and inspection phases and 7 for those used solely in the inspection phase.

smaller discrepancies among each other, while the first component explains only the 29.46% of the total variability.

In order to further enhance the reader’s insight into the \(m\) determination, the finally selected \(m\) principal components are shown per supervised method in Figure 4.7 through bar diagrams. Each bar depicts how many times a specific component is included in the set of \(m\) components (frequency of use), for all 50 employed sets of baseline beams (the interested reader may compare with the figure’s unsupervised counterpart in [K. J. Vanvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]; see additional information in B.3). Figure 4.7(a) shows that the first component is always selected in the S–nPCA–MM–AR method as expected from the observations of Figure 4.6(a), while most of the remaining components are used in most of the 50 employed sets of baseline beams. Similar results are depicted for the S–PCA–MM–AR method in Figure 4.7(b), with the first component being the basic difference since this is almost never used in the \(m\) components set. On the other hand, the S–PCA–AR method frequently uses the last components \((i.e. \text{from 35 to 57})\) as depicted in Figure 4.7(c).

4.5.3 Damage Detection Results

\textit{Preliminaries}

Damage detection results obtained via the assessment procedure of subsection 4.5.1 are presented per supervised method and in terms of their test pseudo–statistic, \(D\), in Figure 4.8, using scatterplot (Figure 4.8(a),(c),(e)) and respective box plot (Figure 4.8(b),(d),(f)) diagrams. It is evident that the MM–based methods achieve better separation between the pseudo–statistics of the healthy and damaged states, compared to the S–PCA–AR method. The latter depicts overlap between the healthy and invisible damage states pseudo–statistics, while it yields significantly improved results for the barely visible damage scenarios. Among the three methods the S–nPCA–MM–AR method evidently achieves the best separation between the pseudo–statistics of the healthy and damaged states.

\textit{Performance assessment for invisible damage scenarios}

Damage detection results for the invisible damage scenarios are presented via ROC curves in Figure 4.9(a). The latter depicts detection performance results for all the healthy and the four low impact energy damage scenarios (OC PA\(_L\) in Table 4.3), demonstrating that the S–nPCA–MM–AR method yields the best damage detection performance among the supervised methods, with 92.5% correct damage
4.6 Comparisons with Unsupervised Methods

Fig. 4.6. Principal component analysis: Fraction of the total parameter vector variability explained by each principal component for the: (a) nPCA–MM–AR and (b) PCA–AR and PCA–MM–AR methods. [The values are provided for all 50 sets of employed baseline beams in the form of error bars; the central point of each error bar designates sample mean and the whiskers are a sample standard deviation above/below the sample mean.]

Detection rate for 3% false alarm rate. On the other hand, the other two methods exhibit significantly reduced performance with S–PCA–MM–AR yielding 88.79% correct damage detection rate for 5% false alarm rate and S–PCA–AR 69.1% correct damage detection rate for 5% false alarm rate.

**Performance assessment for barely visible damage scenarios**

Figure 4.9(b) presents damage detection results for the barely visible damage scenarios (OC PA\textsubscript{H} in Table 4.3), by means of ROC curves. It is evident that all three supervised methods yield similar performance, with small deviations being observed for false alarm rates below 1%. The best performance is depicted for the S–PCA–AR method, with 100% correct damage detection rate for 0.75% false alarm rate. The second best performance is depicted for the S–nPCA–MM–AR method with 100% correct damage detection rate for 1.18% false alarm rate, while the S–PCA–MM–AR method exhibits the worst performance.

**Performance assessment for both invisible & barely visible damage scenarios**

Overall performance results for all the damage scenarios (OC PA\textsubscript{all} in Table 4.3) are presented per supervised method in Figure 4.10. This depicts via ROC curves the evident performance improvement achieved by the S–nPCA–MM–AR method over the other two supervised methods, with the former yielding 96.93% correct damage detection rate for 5% false alarm rate. On the other hand, the S–PCA–MM–AR and S–PCA–AR methods exhibit 88.46% and 76.07% correct detection rate, respectively, for 5% false alarm rate.

4.6 Comparisons with Unsupervised Methods

The three supervised methods are compared with their unsupervised versions from chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]) in this section. In the unsupervised methods case, the design parameter \( m \) is herein selected based on predetermined \( \gamma \) values (see \( \gamma \) definition and selection details in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]), which are common in the literature for centered PCA–enhanced methods. On the other hand, the \( \gamma \) values yielding the most
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Fig. 4.7. Principal components frequency of use, in the set of $m$ components, by the methods for the 50 different sets of baseline beams: (a) S–nPCA–MM–AR, (b) S–PCA–MM–AR and (c) S–PCA–AR [$n_u = 57$, $\nu_o = 15$, $\nu_d = 8$].

prominent detection performance in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] are selected for the non–centered PCA–enhanced method, as this method is not used in other studies.

The supervised and unsupervised methods comparison via ROC curves is shown for all the damage scenarios (OC PA all of Table 4.3) in Figure 4.11. This depicts that the S–nPCA–MM–AR method yields results approaching and for a short false alarm range (of interest for damage detection) exceeding the best performance achieved by the U–nPCA–MM–AR method. Furthermore, it is evident that the S–PCA–MM–AR method outperforms its unsupervised counterpart, while the S–PCA–AR method yields slightly reduced performance compared with its unsupervised counterpart.

The supervised and unsupervised methods are additionally compared, under the PA L operating condition of Table 4.3, in Figure 4.12 via ROC curves. The results indicate that the supervised methods do not always outperform the unsupervised methods. Yet, in the worst case scenario they may achieve a performance that is only slightly reduced compared to the best performance exhibited by the unsupervised methods.

Remark: The three ROC curves depicted per unsupervised method are obtained by selecting the design parameter $m$ according to three commonly used values of $\gamma$. Of course, in the methods unsupervised setting the a priori determination of the $\gamma$ value that yields the best detection performance is impossible.
4.7 Sensitivity Analysis: Performance Assessment Under Incomplete Training

The test pseudo–statistics for the three supervised methods under OC PA_{all}: (a),(c),(e) D versus inspection experiment (experiments included in the inspection phase) and (b),(d),(f) the respective box plots (see Table 4.4; the top and bottom of each box are the 25th and 75th percentiles; the red line in the middle of each box is sample median; the lines extending above and below each box are called whiskers and are drawn from the ends of the interquartile ranges, with length 1.5 times the interquartile range; the red crosses represent observations out of the whiskers range; the black horizontal line is defined by the top whisker of the healthy state and separates the healthy and damaged states).

Therefore, supposing that the user has to decide among the (common) three γ settings utilized in Figure 4.12 per method, it is highly possible to use a setting that yields significantly reduced performance compared with the methods supervised version.

The respective comparative assessment between the supervised and unsupervised methods for the operating condition PA_{H} is presented in Figure 4.13. In this case, the methods supervised version always outperforms the unsupervised counterpart, with the performance difference being especially pronounced for the MM–based methods.

4.7 Sensitivity Analysis: Performance Assessment Under Incomplete Training

The supervised methods sensitivity to missing damage scenarios from the baseline phase is examined in terms of ROC curves in Figure 4.14. This shows with solid line the ROC curve corresponding to the OC PA_{all} of Table 4.3 and with dashed line the ROC curve representing the average correct detection rate per false alarm rate (ROC) curve of the OCs SA_L, SA_H, SA_{2LH}, and SA_{4LH} (see Table 4.3; sensitivity analysis
under additional operating conditions in B.4). It is evident that when four of the eight damage scenarios included in the inspection phase, are excluded from the baseline phase, the detection performance does not deviate significantly from the OC that uses all eight damage scenarios in both phases. What is more, the former OC may even exhibit improved performance when the S–PCA–MM–AR and S–PCA–AR methods are considered.

This improvement is exhibited due to the significantly different effects of the low and higher impact energy damage scenarios on the automated selection of the design parameter $m$ and therefore the detection performance as well. As depicted in Table 4.5, when the invisible damage scenarios are used in the baseline phase (SA$_L$), the S–PCA–MM–AR and S–PCA–AR methods yield significantly worse performance, compared to the barely visible damage scenarios (SA$_H$) case. Thus, the design parameter $m$
4.7 Sensitivity Analysis: Performance Assessment Under Incomplete Training

Fig. 4.11. Comparative assessment of the three supervised methods with their unsupervised counterparts, in terms of detection performance, using ROC curves under OC PA\_{all} (all damage scenarios included). \([n_a = 57, \nu_o = 15, \nu_d = 8]\).

Fig. 4.12. Comparative assessment of the three supervised methods with their unsupervised counterparts, in terms of detection performance, using ROC curves under OC PA\_L (all invisible damage scenarios included). \([n_a = 57, \nu_o = 15, \nu_d = 4]\).

selected based on the invisible damages, yields a poor performance for the barely visible damages (see SA\_L versus SA\_H in Table 4.5). In addition, the AUC values presented in Table 4.5 under the OCs PA\_{all} and SA\_L are similar, indicating that the invisible damage scenarios determine the design parameter \(m\)
Chapter 4. Supervised PCA–enhanced Multiple Model Type Methods

Fig. 4.13. Comparative assessment of the three supervised methods with their unsupervised counterparts, in terms of detection performance, using ROC curves under OC PA \(_H\) (all barely visible damage scenarios included). \([n_a = 57, \nu_o = 15, \nu_d = 4]\).

Table 4.5: AUC based detection performance per supervised method and operating condition of Table 4.3.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>PA(_{all})</td>
<td>0.9939(^1)</td>
<td>0.9768</td>
<td>0.9602</td>
</tr>
<tr>
<td>PA(_L)</td>
<td>0.9898</td>
<td>0.9773</td>
<td>0.9515</td>
</tr>
<tr>
<td>PA(_H)</td>
<td>0.9985</td>
<td>0.9981</td>
<td>0.9993</td>
</tr>
<tr>
<td>SA(_L)</td>
<td>0.9939</td>
<td>0.9768</td>
<td>0.9603</td>
</tr>
<tr>
<td>SA(_H)</td>
<td>0.9917</td>
<td>0.9807</td>
<td>0.9803</td>
</tr>
<tr>
<td>SA(_{2LH})</td>
<td>0.9859</td>
<td>0.9835</td>
<td>0.968</td>
</tr>
<tr>
<td>SA(_{4LH})</td>
<td>0.9875</td>
<td>0.9777</td>
<td>0.9699</td>
</tr>
</tbody>
</table>

\(^1\) The best performance for each damage scenario is shown in bold face.

selection even when barely visible damage scenarios are included into the baseline phase as well (PA\(_{all}\)). This is expected since the automated \(m\) selection algorithm is designed to prioritize the detection of the smallest and harder to detect damages. Hence, the average correct detection rate per false alarm rate ROC curves, which include also the OC SA\(_H\) that yields increased detection performance compared with its other SA counterparts, are better than the PA\(_{all}\) ROC curves. On the other hand, the S–nPCA–MM–AR method depicts performance decrease when the OCs SA (of Table 4.3) are considered, thereby indicating the significant effect of information loss (damage scenarios missing in the baseline phase) and the small discrepancy between the effects of the invisible and barely visible damage scenarios on the supervised selection of the design parameter \(m\).

4.8 Discussion and User Guidelines

The comparison between the unsupervised and supervised versions of the PCA–enhanced methods shows that the latter are not always better than the former, hence the automated design parameter \(m\)
4.9 Conclusions

The random vibration–based damage detection for a population of nominally identical structures problem was tackled via three supervised PCA–enhanced methods. These are novel supervised versions of the unsupervised PCA–enhanced AR model parameter based methods postulated in chapter 3 (also [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]) and include the S–nPCA–MM–AR, S–PCA–MM–AR and S–PCA–AR methods. Compared to their unsupervised counterparts the postulated supervised methods provide automated selection of the principal components that define the damage subspace, thereby improving performance and training difficulty. In this context the present study continuous the systematic assessment of the previous one, using the same considerable population of sample composite beams and the same challenging damage scenarios.

The main conclusions drawn from the present study are presented below in the form of answers to the introduction’s questions.

Q1. The supervised methods relieve the user from critical training selections.
Q2. The supervised methods yield improved performance compared to all or some of their unsupervised counterparts.

Q3. The S–nPCA–MM–AR method achieves the best damage detection performance among the supervised methods, with 96.93% correct damage detection for 5% false alarm rate. On the other hand, the S–PCA–MM–AR and S–PCA–AR methods yield 88.46% and 76.07% correct damage detection rate, for 5% false alarm rate, respectively.

Q4. The supervised methods exhibit robustness to the type and number of damage scenarios included in the baseline phase.

Q5. The supervised methods are preferred over their unsupervised counterparts, since they often achieve improved performance compared to the latter, while they also relieve the user from critical selections during the training phase. The postulated S–nPCA–MM–AR method in particular, achieves very good performance compared to the methods of the previous chapter and remarkable performance when the barely visible damage scenarios are considered, as illustrated in Figure 4.15.

**Fig. 4.15.** Comparison of the current chapter methods with their previous chapter counterparts, in terms of correct damage detection performance (True Positive) rate, versus two false alarm rates (FPR) of interest, by means of bar charts for: (a) Invisible damage scenarios, (b) Barely visible damage scenarios. [Based on the experiments of the benchmark application study (see chapter 2).]
Chapter 5

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An Automated Hyper–Sphere Based Healthy Subspace Method for Robust and Unsupervised Damage Detection via Random Vibration Response Signals

A novel unsupervised Hyper–Sphere based healthy subspace method for robust damage detection under uncertainty via random vibration response signals is postulated in the present chapter. The method is based on the approximate construction, within a proper feature space, of a “healthy subspace” representing the healthy structural dynamics under uncertainty as the union of properly selected hyper–spheres. This is achieved via a fully automated algorithm eliminating user intervention and subjective selections. The main asset of the proposed method lies in combining simplicity and full automation with high performance. Its performance is assessed via two experimental case studies featuring various uncertainty sources and distinct “healthy subspace” geometries, while comparisons with three well known robust damage detection methods are also performed. The results indicate excellent detection performance that is also superior to that of alternative methods.

5.1 Introduction

Robust vibration–based damage detection in structures operating under uncertainty is a crucial technological problem on which significant efforts have been devoted over the past several years [Peeters, Maecck, and Roeck 2001a; Sohn 2007; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b; Zhou et al. 2014]. The problem originates from the fact that varying Environmental and Operating Conditions (EOCs) may significantly affect the structural dynamics, oftentimes to the extent of “masking” changes attributed to damage, thus rendering effective damage detection highly challenging. This problem is further aggravated when considering damage detection for a population of nominally identical structures, as the effects of material, geometrical, and manufacturing uncertainties are compounded with those due to varying EOCs [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

A number of robust machine–learning–type, vibration data based, and unsupervised methods have been developed by either directly (explicitly) or indirectly (implicitly) modeling the healthy structural dynamics under uncertainty [Figueiredo, Park, et al. 2011; Figueiredo and A. Santos 2018; A. Santos, Figueiredo, Silva, Sales, et al. 2016; Worden and Manson 2007]. Although not necessarily explicitly stated, inherent within them is the construction of an appropriate “healthy subspace” for representing, within a properly selected feature space [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b], the healthy structural dynamics under all possible operating and environmental conditions. In the context
of direct (explicit) methods the “healthy subspace” is explicitly constructed, while in the context of indirect (implicit) methods the construction is implicit with the “healthy subspace” diluted into a lower dimensionality feature space.

The construction of the healthy subspace is, in all methods, achieved in an initial, baseline or learning phase, using signals either from the healthy and damaged structural states (supervised methods) or solely from the healthy state (unsupervised methods). In the inspection phase damage detection is then accomplished by examining whether a newly obtained (from the current structure, being in unknown health state) feature vector belongs or not to the healthy subspace. As already mentioned, indirect (implicit) methods employ a reduced dimensionality healthy subspace, based on features that are, presumably, sensitive only to damage, while being insensitive to changes in the EOCs. Feature selection is based on proper decomposition techniques such as Principal Component Analysis (PCA), Auto–Associative Neural Networks (AANN) [Figueiredo and A. Santos 2018], Factor Analysis (FA) [Bellino et al. 2010; Comanducci et al. 2016; Deraemaeker et al. 2008; Figueiredo, Park, et al. 2011; Figueiredo and A. Santos 2018; Giraldo 2006; Hu et al. 2016; Kojidi et al. 2014; Kullaa 2010; Manson 2002; Ram Mohan Rao et al. 2015; A. Santos, Silva, Sales, et al. 2015; A. Santos, Figueiredo, Silva, Sales, et al. 2016; Sen et al. 2019; Silva et al. 2019; Vanlanduit et al. 2005; Worden and Manson 2007; Yan et al. 2005], and cointegration [E. J. Cross, Manson, et al. 2012; E. J. Cross, Worden, et al. 2011; Tome et al. 2020; Worden, Baldacchino, et al. 2016].

On the other hand, the construction of the healthy subspace in the direct (explicit) methods is achieved through deterministic or probabilistic modeling of the effects of the varying EOCs on the dynamics. These methods may be classified in two categories according to whether the uncertainty sources are measurable or not. In the first case the healthy subspace may be constructed using cause–and–effect type modeling, with the “causes” being the measurable uncertainty sources, as for instance temperature, and the “effects” being the changes on the corresponding features [Comanducci et al. 2016; Hios et al. 2014; Hu et al. 2016; Ko et al. 2003; Lorenzoni et al. 2016; Peeters, Maeck, and Roeck 2001b; Worden and E. Cross 2018; Worden, Sohn, et al. 2002]. Evidently, the application of these methods is limited to cases where the uncertainty source is measurable, requiring also extra equipment for such measurements.


The aforementioned robust methods have been shown to achieve very good performance under unsupervised operation, yet they are characterized by their own limitations, such as the requirements for: (i) relatively high numbers of signal records for training [Avendano–Valencia et al. 2017b; Figueiredo and A. Santos 2018; Michaelides, Apostolellis, et al. 2011; Yan et al. 2005], (ii) the selection, often subjective, by the user of a number of hyper–parameters [Chakraborty et al. 2015; E. J. Cross, Worden, et al. 2011; Figueiredo, Radu, et al. 2014b; Figueiredo and A. Santos 2018; Kullaa 2014; Yan et al. 2005], (iii) optimization procedures of significant complexity within high–dimensional spaces and/or non–convex problems [Chakraborty et al. 2015; E. J. Cross, Worden, et al. 2011; Figueiredo, Park, et al. 2011; Figueiredo, Radu, et al. 2014b; Figueiredo and A. Santos 2018; Kullaa 2014; A. Santos, Figueiredo, Silva, Sales, et al. 2016; Worden and Manson 2007], (iv) assumptions regarding the healthy subspace.
5.1 Introduction

distribution or geometry [Kullaa 2010; Michaelides, Apostolellis, et al. 2011; Yan et al. 2005], (v) user expertise for the construction of the healthy subspace, which is not based on a simple and automated procedure [Chakraborty et al. 2015; E. J. Cross, Worden, et al. 2011; Figueiredo, Radu, et al. 2014b; Figueiredo and A. Santos 2018; Kullaa 2014; Yan et al. 2005].

The goal of the present study is the postulation of an Unsupervised machine–learning–type method in which the construction of the Healthy Subspace is simplified and automated, relieving the user from potentially critical selections, while at the same time achieving high detection performance without requiring high numbers of data records in its baseline (learning) phase. The method is presented for the response–only measurement case, while extensions to the excitation–response measurement case is straightforward. The method’s feature space is spanned by the parameters of an AutoRegressive (AR) representation of the dynamics, while the key idea lies with the approximation of the healthy subspace as the union of hyper–spheres, with properly determined centers and radii, thus offering conceptual and computational simplicity. The resulting Unsupervised Hyper–Sphere AR based method is hereafter abbreviated as U–HS–AR.

The underlying concept for the method is motivated by broader one–class classification [Stibor et al. 2007] and clustering [Silva et al. 2017] techniques. Yet, the postulated method employs multiple hyper–spheres of various radii, properly fitted on the required healthy subspace geometry for each distinct application, so that hyper–spheres near the subspace core are characterized by greater radius than their counterparts closer to the subspace bounds. Still, determining the hypersphere centers and radii is a challenging task. This is presently achieved via the injection of artificial points within an initially constructed healthy subspace, and the subsequent determination of the number of required hyper–spheres, their centers and radii via a novel and automated algorithm. The artificial points lead to dense covering of the original healthy subspace, yielding an initially crude hyper–surface approximation representing the healthy subspace bounds. The automated algorithm then assigns “healthy” or “damaged” pseudo–labels to each of the artificial points, and these are subsequently used for automatically selecting, via a proper criterion, the best hyper–sphere–based approximation of the healthy subspace. It is noted that a preliminary study on the method has been presented in our recent conference chapter [K. Vamvoudakis–Stefanou et al. 2019].

The postulated method is assessed via two experimental case studies: The first pertains to damage detection for a population of nominally identical composite aerostructures, each corresponding to half tail of an Unmanned Aerial Vehicle (UAV) and subject to impact induced damage, and in the presence of uncertainty sources related to variability in the materials, manufacturing, and assembly. The second pertains to damage detection on a single composite aerostructure subject to damage scenarios consisting of small, local, stiffness reduction (simulated by the attachment of small masses) under uncertainty related to varying temperature and tightening assembly torque. The postulated method’s performance is examined via thousands of test cases (inspection experiments) using Receiver Operating Characteristic (ROC) curves. Informative comparisons with three state–of–the–art robust methods, that is the U–MM–AR method, the Random Coefficient Gaussian Mixture (U–RC–GM–AR) based method, and a PCA (U–PCA–AR) based method, are also performed.

The rest of the chapter is organized as follows: The precise problem statement is presented in section 5.2. The postulated automated and unsupervised healthy subspace approximating method is presented in section 5.3. The experimental set–up and the respective experimental assessment for each of the case studies are presented in section 5.4. Discussion and conclusions are finally provided in section 5.5 and section 5.6, respectively.
Chapter 5. An Automated Hyper–Sphere Based Method

5.2 Precise Problem Statement

Let a structure operating under varying EOCs, considered constant over each measurement interval. Given:

1. **Baseline (Training) Phase:**
   A set of \( p \) random vibration response signals \( y_i[t] \) \((i = 1, 2, \ldots, p; t = 1, 2, \ldots, N; t: \) normalized by the sampling period discrete time, \( N: \) signal length) under healthy structural state and corresponding to a representative sample of the considered EOCs.

2. **Inspection (Operational) Phase:**
   A current (fresh) vibration response signal \( y_u[t] \) \((t = 1, 2, \ldots, N)\), with the subscript \( u \) designating unknown health state.

Determine: The health state (healthy or damaged) of the structure.

\( ^a \)It is noted that \( p \) should be sufficiently high in order to adequately represent the considered range of EOCs.

5.3 The Unsupervised Hyper–Sphere AR (U–HS–AR) Based Method for Robust Damage Detection

5.3.1 The Concept

As mentioned previously, this is an unsupervised vibration–based method for robust damage detection that is founded on a feature space spanned by the \( n \)–th order AR model parameters [Fassois and J. S. Sakellariou 2009], within which a “healthy subspace” is constructed in an initial, learning (baseline), phase. The AR model is identified using a random vibration response from the investigated structure under normal operation and sample EOCs. The training of the method requires a limited number, say \( p \), of vibration response signals, with the respective vectors of AR model parameters defining \( p \) points within the healthy subspace. It is worth noting that the exclusive use of healthy structures in the baseline phase renders the method as unsupervised, while the \( p \) vibration response signals should correspond to a sample of EOCs that cover the entire range of interest.

In the method’s diagnostic (inspection) phase, a single, fresh, random vibration signal is acquired from the structure in unknown health state, and its corresponding \( n \)–th order AR (current) model is obtained. The parameter vector of this model defines a current point in the feature space and damage detection is based on determining whether or not the current point resides within the healthy subspace.

Within this framework, the main issue is the construction, in the baseline phase, of the “healthy subspace” using the available \( p \) points. This is a generally difficult task typically requiring some form of a–priori knowledge [Yan et al. 2005], complicated probabilistic procedures [Chakraborty et al. 2015; Figueiredo and A. Santos 2018] and critical selections on part of the user.

The key idea behind the postulated method, aiming at simplifying and automating the procedure, is the deterministic approximation of the healthy subspace as the union of a proper number of hyper–spheres with distinct radii. For a proper approximation, each hyper–sphere is characterized by its own radius which depends on its relative position within the healthy subspace; getting “larger” if its center is close to the subspace core, and “smaller” if it is close to the boundaries. Proximity to the core is judged via a population density concept, defined by the percentage of healthy points lying within a distinct density–measuring “reference” hyper–sphere of fixed radius \( r^\star \), sharing the same center with the currently judged for its proximity to the subspace core hyper–sphere. The hyper–sphere centers are the centroids of (all possible) combinations of \( k^\star \) neighboring healthy points (out of the total \( p \) in the baseline phase) in the feature space obtained via a \( k \)–Nearest Neighbor type procedure [Stibor et al. 2007], while \( k^\star \) is automatically determined via a procedure maximizing the detection performance using Receiver Operating Characteristics.
The “reference” hyper–sphere radius $r^*$, the value $k^*$ and a decision making threshold needed for damage detection, constitute the method’s hyper–parameters. These are determined in the baseline phase of the postulated method via an automated algorithm without user intervention. This is achieved using the $p$ healthy points to artificially create additional points via probabilistic sampling, and proper manipulation so as they are densely distributed inside the healthy subspace and scarcely outside of it. Thus, a density–based discrimination is realized, which is subsequently used to pseudo–label the densely distributed points as “healthy” and the rest as “damaged” via the “reference” hyper–sphere radius $r^*$. Based on this procedure, a number of healthy subspaces (characterized by certain hyper–parameters) is available and the one containing the maximum number of “healthy” and a minimum number of “damaged” points is automatically selected as optimal by the method. A detailed presentation of the postulated method through distinct steps of its two operational phases follows.

5.3.2 The Baseline (Learning) Phase

In this phase, $p$ vibration response signals are obtained from the structure (or from a set of structures if a population is considered), each one corresponding to a distinct set of EOCs, with the employed sample of EOCs covering the range of interest within which the structure operates. Each of these signals is subsequently modeled using a conventional AR($n$) model, with $\alpha_i$ and $\Sigma_i$ the $i$–th model’s estimated parameter vector (bold-face capital/lower-case letters designate matrices/vectors, respectively) and its covariance matrix, respectively; no distinction is made between the true parameter vector and its estimate for reasons of notation simplicity. Each parameter vector is asymptotically (that is for $N \rightarrow \infty$) associated to a Gaussian distribution with mean the true parameter vector and covariance $\Sigma_i$ [Ljung 1999].

The obtained $p$ parameter vectors are points of the healthy subspace and thus they are used to construct the previously mentioned hyper–spheres and determine the method’s hyper–parameters via the following two–part algorithm.

**PART 1: Artificial point generation and pseudo–labeling.**

Two sets of artificial parameter vectors (feature space points) are constructed and pseudo–labeled as “healthy” or “damaged” in this part of the algorithm, which is pictorially presented in Figure 5.1.

Step 1: A number of, say $3n$, artificial points, say $\theta_{i,j}$ with $j = 1,\ldots,3n$, are generated by sampling from each $N(\alpha_i, \Sigma_i)$ distribution ($\forall i = 1,\ldots,p$). Then, the Euclidean distances between each artificial $\theta_{i,j}$ and each $\alpha_i$ are obtained, with the minimum and maximum non–zero values among these designated as:

$$d_{i}^{\min} := \min_{j} |\theta_{i,j} - \alpha_i|_{l_2}$$ \hspace{1cm} (5.1)

and

$$d_{i}^{\max} := \max_{j} |\theta_{i,j} - \alpha_i|_{l_2}$$ \hspace{1cm} (5.2)

Step 2: The artificial points $\theta_{i,j}$ are expanded using the following equation that scales and re–centers them to $\alpha_i$:

$$\tilde{\theta}_{i,j} = s_i \theta_{i,j} - s_i \frac{3n}{3n} \sum_{j=1}^{3n} \theta_{i,j} + \alpha_i$$ \hspace{1cm} (5.3a)

where

$$s_i := \frac{d_i^{\max} + d_i^{\min}}{d_i^{\min}}$$ \hspace{1cm} (5.3b)
with $\bar{\theta}_{i,j}$ designating the $j$–th scaled artificial point corresponding to the $N(\alpha_i, \Sigma_i)$ distribution and $s_i$ an empirical scaling factor yielding scaled artificial points that surround the hyper–sphere containing their original counterparts (that is the $\theta_{i,j}$ points), with center $\alpha_i$ and radius $\max_j |\theta_{i,j} - \alpha_i|_{l_2}$. This is achieved based on the minimum Euclidean distance between $\bar{\theta}_{i,j}$ and $\alpha_i$ which should be greater than the maximum Euclidean distance between $\theta_{i,j}$ and $\alpha_i$, that is:

$$\min_j |\bar{\theta}_{i,j} - \alpha_i|_{l_2} > \max_j |\theta_{i,j} - \alpha_i|_{l_2}$$

(5.4)

Thus, the scaled artificial points are sparsely distributed compared with the rest of the points in the feature space, especially in areas outside of the healthy subspace, providing a measure for assigning a “healthy” or “damaged” pseudo–label to each point (also see Figure 5.1).

**Artificial points generation**

![Diagram of artificial points generation](image)

**Fig. 5.1.** Schematic representation of the U–HS–AR method’s PART 1: generation of artificial points (steps 1 - 6). [For colored interpretation of the schematic, the reader is referred to the web version of this article].

**Step 3:** All available parameter vectors are organized in a set $\{b\} = \{\alpha_i, \theta_{i,j}, \bar{\theta}_{i,j}\}$, $\forall i, j$, while the set’s elements are hereafter denoted as $b_i$, with $i = 1, \ldots, p + 6np$, for convenience. These are subsequently standardized as $\Sigma^{-1}b_i$, where $\Sigma = \frac{1}{p} \sum_{i=1}^{p} \Sigma_i$ is the covariance matrix of the $\alpha_i$ parameter vectors’ sample mean. It is noted that in the sequel all vectors $b_i, \alpha_i, \theta_{i,j}, \bar{\theta}_{i,j}$ are assumed standardized, even if this is not explicitly stated. This standardization is motivated by similarity measures, such as the Mahalanobis distance [Rencher et al. 2012], taking crudely into account the estimation uncertainty of all AR model parameter vectors $\alpha_i$, while retaining their relative position in the feature space.

The feature space points determined by the vectors in $\{b\}$ are subsequently labeled as “healthy” or “damaged” with respect to their population density as well as the constraints provided below:
5.3 The Unsupervised Hyper–Sphere AR Based Method

(A) All $\alpha_i$'s and at least 95% of the $\theta_{i,j}$ vectors are labeled as “healthy”.

(B) At most a fraction $\delta$ (say 30%) of the $\theta_{i,j}$ vectors are labeled as “healthy”.

These constraints are used to determine a critical population density value based on which a point may be labeled as “healthy” or “damaged”. This is selected so that the $\alpha_i$ points, which are known to be healthy, along with most of their respective samples $\theta_{i,j}$ are labeled as “healthy”, while a maximum fraction $\delta$ of the $\theta_{i,j}$ points gets the same label and residing thus inside the healthy subspace. The fraction $\delta$, which is not available, is empirically initialized ($\bar{\delta} = 30\%$), and subsequently updated in the following steps until both constraints are satisfied.

Step 4: The Euclidean distances, for any pair of standardized vectors $\alpha_i$ are obtained:

$$d_o(i,j) := |\alpha_i - \alpha_j|_2, \quad i, j = 1, \ldots, p, \quad i \neq j \quad (5.5)$$

Step 5: The obtained distances are organized into a $(p - 1) \times p$ matrix $C$ so that its $i$–th column contains all the distances $d_o(i,j)$ between $\alpha_i$ and each $\alpha_j$ for $j = 1, \ldots, p$ (with $i \neq j$), organized in increasing order, that is $c_{1,i} < c_{2,i} < \cdots < c_{p-1,i}$, with $c_{1,i} = \min_j d_o(i,j)$ and $c_{p-1,i} = \max_j d_o(i,j)$, while $c_{j,i}$ designates the $(j,i)$–th element of matrix $C$:

$$C = \begin{pmatrix} c_{1,1} & \cdots & c_{1,p} \\ \vdots & \ddots & \vdots \\ c_{p-1,1} & \cdots & c_{p-1,p} \end{pmatrix} = \begin{pmatrix} \min_j d_o(1,j) & \cdots & \min_j d_o(p,j) \\ \vdots & \ddots & \vdots \\ \max_j d_o(1,j) & \cdots & \max_j d_o(p,j) \end{pmatrix} \quad (5.6)$$

Step 6: The Euclidean distances of all possible pairs of standardized vectors in $\{b\}$ are calculated ($i \neq j$):

$$d(i,j) := |b_i - b_j|_2, \quad i, j = 1, \ldots, p + 6np \quad (5.7)$$

Then, the density $\rho_i$ of each point $b_i$ is obtained through a “reference” hyper–sphere of radius:

$$r = \min_i c_{p-1,i} \quad (5.8)$$

and center $b_i$ (see Figure 5.1), by determining the number $\rho_i$ of points $b_j$ ($j = 1, \ldots, p + 6np$) residing within it via the test:

$$d(i,j) \leq r \quad (5.9)$$

The “reference” hyper–sphere radius is defined as the smallest one among the $c_{p-1,i}$ values. Thus, the resulting “reference” hyper–spheres are large enough to avoid characterizing a point of the healthy subspace core as one near the subspace bounds, due to local proximity fluctuations among the points. Yet, the hyper–spheres are not so large as to include all or most of the healthy subspace while their center moves from the subspace core to its bounds. Therefore, these “reference” hyper–spheres may yield distinguishable density values between points of the subspace core and their counterparts near its bounds.

Step 7: The points in $\{b\}$ are then labeled as “healthy” or “damaged” via the following decision making mechanism:

$$\text{If } \rho_i \leq l_p \rightarrow \text{“Damaged” label}$$
$$\text{otherwise } \rightarrow \text{“Healthy” label} \quad (5.10)$$

with $l_p$ designating a decision making threshold that is determined by sequentially assigning a $\rho_i$ density, from $\max_i \{\rho_i\}$ to $\min_i \{\rho_i\}$ in decreasing order until constraints A and B are satisfied. Based on this procedure, the algorithm minimizes the number of “healthy” $\theta_{i,j}$ points, by prioritizing labeling of points in denser neighborhoods as “healthy”.

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Chapter 5. An Automated Hyper–Sphere Based Method

Step 8: If either constraint A or B is violated, repeat steps 6 and 7, with the updated radius:

\[ r = \min_i c_{p-1-h,i} \]  

(5.11)

where \( h \) sequentially takes the values 1, \ldots, \( p - 2 \) until all the constraints are satisfied. The algorithm automatically examines different values of radius \( r \), starting from the largest one with the respective “reference” hyper–spheres tending to contain a fairly large number of \( b_i \) points, thereby yielding density values that are not sensitive to local proximity discrepancies among the points (“noise”), and continues with decreasing values until the maximum \( r \) that satisfies the constraints is reached.

Step 9: When a constraint is violated after all radii \( r \) are examined in step 8, repeat steps 6 to 8 with the redefined fraction \( \delta = \delta + 10\% \) (constraint B). Note that lower values of this step may increase the computational complexity.

Step 10: The finally selected “reference” hyper–sphere radius (say \( r^* \)) takes the \( r \) value used in the algorithm’s last iteration. The radius \( r^* \) along with the standardized artificial points and their respective pseudo–labels are subsequently provided to the second part of the algorithm.

**PART 2: Hyper–parameters determination.**

The second part of the algorithm aims at determining the remaining hyper–parameters, searching for the healthy subspace that includes the maximum possible number of “healthy” points for the minimum number of “damaged” ones. Toward this end, damage detection is performed for each point in \{\( b_i \)\} using varying hyper–parameter values and the assigned “healthy” / “damaged” pseudo–labels, for the construction of respective ROC curves. The set of hyper–parameters that leads to the best ROC curve, indicating the method’s best damage detection performance, is employed for use in the inspection phase. This part of the algorithm is pictorially presented in Figure 5.2, while the corresponding distinct steps are described in the sequel.

Step 11: The Euclidean distances, \( d_k(i, j) \), between all possible pairs of \( \alpha_i \) and \( b_j \) points are calculated (Figure 5.2):

\[ d_k(i, j) := |a_i - b_j|_{l_2}, \quad i = 1, \ldots, p, \quad j = 1, \ldots, p + 6np \]  

(5.12)

The \( k \) nearest to \( b_j, \alpha_i \) points are obtained, and their arithmetic mean \( \bar{\alpha}_{k,j} \) representing their centroid, is computed.

Step 12: A “reference” hyper–sphere with center the centroid \( \bar{\alpha}_{k,j} \) and radius \( r^* \) is constructed (Figure 5.2), and the number \( \rho \) of \( \alpha_i \) points residing inside it is determined using the following test:

\[ d_k(i, j) \leq r^* \]  

(5.13)

The population density \( \rho \) is subsequently standardized as \( \rho_{k,j} = \rho / p \) so as \( \rho_{k,j} \in [0, 1] \), and the population density being independent from the number of \( \alpha_i \) points.

Step 13: The method’s damage detection test pseudo–statistic \( D_{k,j} \) based on point \( b_j \) is then determined:

\[ D_{k,j} := \frac{|b_j - \bar{\alpha}_{k,j}|_{l_2}^2}{e^{\rho_{k,j}}} \]  

(5.14)

The numerator corresponds to the distance of point \( b_j \) from its nearest hyper–sphere with center \( \bar{\alpha}_{k,j} \), the denominator adjusts the hyper–sphere’s radius with respect to the sensitive to the population density value \( e^{\rho_{k,j}} \) (also see inspection phase), while the exponential \( e \) is used to amplify the radius difference among the hyper–spheres of the core and the bounds. This is necessary as the population density \( \rho_{k,j} \) often takes values in a limited range of \([0, 1]\) and the density difference among the core and the bounds
5.3 The Unsupervised Hyper–Sphere AR Based Method

Pseudo-labeling & hyper-parameters determination

Fig. 5.2. Schematic representation of the method’s PART 2: assignment of “healthy” / “damaged” pseudo–labels and hyper–parameters determination (steps 7 - 12). [For better interpretation of the schematic, the reader is referred to the web version of this article].

may be insufficient to significantly affect the respective hyper–spheres. Therefore, the method’s capability to properly approximate any healthy subspace would be compromised.

Step 14: Damage detection for the $j$-th point in $\{b\}$ is thus performed via the decision making mechanism:

\[
\text{If } D_{k,j} \leq l_{\text{lim}} \quad \rightarrow \quad \text{Healthy structure}
\]
\[
\text{otherwise} \quad \rightarrow \quad \text{Damaged structure} \quad (5.15)
\]

where $l_{\text{lim}}$ designates a decision making threshold. A ROC curve is constructed through this mechanism using the obtained $D_{k,j}$ values, the respective labels from PART 1 and varying $l_{\text{lim}}$ values. The “shortest” distance $\xi_k$ between the curve and the (0,1) point of the curve (Figure 5.3) determines the best performance achieved for the current $k$ value.

Steps 11 to 14 are repeated for increasing $k$ values ($k = 1, 2, \ldots, p$) and the optimal $k$ is determined as (also see Figure 5.3):

\[
k^* = \arg \min_k \xi_k \quad (5.16)
\]

while the optimal threshold $l_{\text{lim}}$ as that corresponding to $\xi_{k^*}$. It is noted that $k^*$ decreases as the healthy subspace geometry complexity increases.
Chapter 5. An Automated Hyper–Sphere Based Method

5.3.3 The Inspection (Diagnostic) Phase

In this phase a new vibration response signal from the structure under normal operation is obtained and the respective AR\( (n) \) model is identified. Its parameter vector, say \( \mathbf{a}_u \), is then standardized as \( \Sigma^{-\frac{1}{2}} \mathbf{a}_u \) (\( \mathbf{a}_u \) is assumed standardized hereafter) and the squared Euclidean distances between \( \mathbf{a}_u \) and each \( \mathbf{a}_i \) point are obtained. The \( k^* \) nearest to \( \mathbf{a}_u \) points \( \mathbf{a}_i \) are then determined according to these distances and their centroid \( \hat{\mathbf{a}}_{k^*} \) is calculated. The standardized population density \( \rho_{k^*} \) is then calculated as in step 12 of the baseline phase.

Damage detection is then based on the following decision making mechanism:

\[
D := \frac{|\mathbf{a}_u - \hat{\mathbf{a}}_{k^*}|_2^2}{l_{lim}} \leq l_{lim} \rightarrow \text{Healthy structure}
\]

\[
\text{otherwise} \rightarrow \text{Damaged structure}
\]

where \( D \) designates the method’s test pseudo–statistic. The above expression may be written as:

\[
|\hat{\mathbf{a}}_{k^*} - \mathbf{a}_u|_2^2 \leq e^{\rho_{k^*} l_{lim}}
\]

indicating that the healthy subspace is locally represented by a hyper–sphere of radius \( \sqrt{e^{\rho_{k^*} l_{lim}}} \) (population density dependent) and center \( \hat{\mathbf{a}}_{k^*} \), and that the current structure is declared as healthy if \( \mathbf{a}_u \) resides in this hyper–sphere.

5.4 Experimental Assessment and Comparisons

5.4.1 Preliminaries

The postulated method’s damage detection performance assessment is presented via two experimental case studies. Informative comparisons with state–of–the–art robust methods are also performed. The first case study concerns damage detection for a population of nominally identical composites structures under material, manufacturing, and assembly uncertainty, while the second, damage detection for one of these composites structures operating under varying EOCs such as temperature, tightening torque on the mounting, and other potential uncertainties. Further information on the experiments of the first case study
5.4 Experimental Assessment and Comparisons

Fig. 5.4. Case Study A: Experimental set-up with the force excitation at Point X, and the measurement of the vibration acceleration at Point Y1 [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].


The employed structure represents half of the tail of a commercial twin tail Unmanned Aerial Vehicle (UAV) that consists of a composite beam, where the considered damage scenarios are induced, and an additional aluminum part that simulates the half of the horizontal stabilizer. The structure is free from the one end as it is on the actual UAV, while it is tightly clamped on the other side simulating its attachment to the fuselage as shown in Figure 5.4. The composite beam has a square hollow cross section, uniform along its length, and it is made of several layers of woven and unidirectional fabric based on one shot transfer molding, while its basic dimensions are shown in Figure 5.5.

The U–HS–AR method’s damage detection performance is presented via scatter and box plots of its $D$ test pseudo–statistic, as well as Receiver Operating Characteristic (ROC) curves indicating the True Positive Rate (TPR: percentage of correct detection) versus the False Positive Rate (FPR: percentage of false alarms) for a varying decision making threshold $l_{lim}$. The best performance is achieved when the ROC curve includes the (0,1) point, that is on the upper left corner in such graphs.

Three state–of–the–art powerful and unsupervised methods for robust damage detection are also employed in this section for a comparative assessment of the U–HS–AR method’s damage detection performance. The first method is the U–MM–AR [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] that is based on a Multiple Model representation for a probabilistic approximation of the healthy subspace. The second method is a Random Coefficient Gaussian Mixture (U–RC–GM–AR) based method [K. J. Vamvoudakis–Stefanou and Fassois 2017] through which the healthy subspace is constructed using a Gaussian Mixture model, while the third method [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] is based on the well–known PCA where a subset of the principal components with the greatest variability is used to formulate the healthy subspace.
Fig. 5.5. Case Study A: Sketch of the half tail with the basic dimensions (in mm) of the composite beam, the mass (horizontal stabilizer) and the clamping, along with the damage locations (Points A, B, C, D), the measurement Point Y1, and the excitation Point X: (a) top view, (b) side view.

Table 5.1: Case Study A. Main characteristics.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of structures</td>
<td>23 healthy (15 in baseline phase), 8 damaged (inspection phase)</td>
</tr>
<tr>
<td>Damage scenarios</td>
<td>Impact induced of 15 J or 5 J energy at Points A, B, C, D</td>
</tr>
<tr>
<td>Uncertainty sources (inter–beam)</td>
<td>manufacturing, material, assembly</td>
</tr>
</tbody>
</table>

5.4.2 Case Study A: Damage Detection for a Population of Nominally Identical Structures

Damage detection is attempted in this case study for a population of nominally identical structures as the one described previously. In particular, the study is based on a population sample consisting of 23 healthy and 8 damaged structures characterized by significant uncertainty in their dynamics (see Table 5.1) due to: (a) variability in the manufacturing and the materials of the composite beams, (b) variability due to the assembly of the aluminum mass (horizontal stabilizer) on the different composite beams, (c) variability in the clamping of each distinct beam to the side of the fuselage.

Each considered damage (Table 5.2) in this case study is impact induced at certain points on each composite beam with different impact energy, and it is designated by two letters with the first indicating the damage position (Figure 5.5) and the second low (L; invisible damage) or higher (H; barely visible damage) impact energy. For instance Damage AL indicates a damage at Point A via impact of low energy.

As shown in Figures 5.4, 5.5 each structure is excited at its free end with a random white Gaussian force applied vertically at Point X using an electromechanical shaker, while the vibration acceleration at
5.4 Experimental Assessment and Comparisons

Table 5.2: Case Study A. Experimental details [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Impact Energy (J)</th>
<th>Damage Visual Characterisation</th>
<th>Impact Position</th>
<th>Number of beams</th>
<th>Number of experiments per beam</th>
<th>Total number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>—</td>
<td>—</td>
<td>23</td>
<td>7</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>Damage AL</td>
<td>5</td>
<td>Invisible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BL</td>
<td>5</td>
<td>Invisible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CL</td>
<td>5</td>
<td>Invisible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DL</td>
<td>5</td>
<td>Invisible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage AH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Signal details: Sampling freq. $f_s = 4654.5$ Hz, Signal bandwidth $[5 - 2327.25]$ Hz, damage detection $N = 10,000$ samples (2.15s), Signal normalized via sample mean removal and division by its sample standard deviation.

Point Y1 is measured via a lightweight accelerometer. Sample mean correction and normalization by the sample standard deviation is performed for each measured vibration signal.

The dynamics of the healthy population, as well as under the considered damages on the composite beams are presented via envelopes of Welch [Ljung 1999] based Power Spectral Density (PSD) estimates in Figure 5.6. These are constructed using all experiments of Table 5.2, Hamming windowing, segment length of 8192 samples and 90% overlap via the Matlab function, pwelch.m. It is obvious that the envelope corresponding to the healthy dynamics under the mentioned uncertainty sources is significantly overlapped with the low impact energy damage envelope, indicating a challenging damage detection problem, while the problem seems easier for the impact damages with the higher energy basically due to the discrepancies between the envelopes in the range $[1800 - 2200]$ Hz.

It is important to note in this case study for which there is no measure to quantify the effects of the combined uncertainty sources (manufacturing, materials and assembly variability), as it could be possible for varying conditions such as the temperature (see Case Study B in the next section), that the selection of a certain set of structures representing the healthy population dynamics is not possible. Due to this fact, the selection of a random set of healthy structures for the U–HS–AR training in the baseline phase, may not be representative of the healthy dynamics, thereby significantly affecting the method’s damage detection performance. Taking this into account, a special assessment procedure that is also used in Vamvoudakis et al. [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] is presently adopted. According to this, a set of 15 healthy structures (“baseline structures”) is initially reserved for the baseline phase, while the remaining 8 healthy and all damaged ones that compose the set of “inspection structures”, are exclusively used for the method’s assessment in the inspection phase. Once the damage detection results are obtained for these two sets, a number of structures is mutually exchanged between them creating thus two new sets which are used again in the method for new damage detection results. This procedure is repeated a sufficient number of times so as: (a) multiple combinations of “baseline structures” are used, and (b) each distinct healthy structure is included at least in one set with “baseline structures”. Thus, all experiments (baseline/inspection experiments) with the tail structures are rotated between the two sets. Presently, 50 different sets of “baseline structures” (with 50 respective sets of “inspection structures”) are considered, yielding a high number of test cases (aggregate inspection experiments) as presented in Table 5.3. Based on this special procedure, the method’s detection performance is judged...
Chapter 5. An Automated Hyper–Sphere Based Method

Fig. 5.6. Case Study A: Envelopes of Welch-based PSD estimates for the healthy and damaged population of structures under (a) invisible and (b) barely visible damage scenarios [Healthy population: 161 experiments, Invisible / Barely visible damage at all considered points: 28 / 28 experiments].

Table 5.3: Case Study A. Details of the assessment procedure [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]

<table>
<thead>
<tr>
<th>No. of baseline structures (ν)</th>
<th>No. of inspection structures</th>
<th>No. of baseline experiments (p)</th>
<th>Inspection experiments (test cases) per set of baseline structures</th>
<th>Employed sets of baseline structures</th>
<th>Aggregate inspection experiments (test cases)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>16</td>
<td>105</td>
<td>112</td>
<td>50</td>
<td>5 600</td>
</tr>
</tbody>
</table>

based on results from 50 different sets of “baseline structures” warranting statistical reliability.

Baseline phase

\[ p = 105 \] vibration response \( N = 10\,000 \) sample long signals (2.15s), obtained from \( ν = 15 \) healthy sample structures from the population (also see Table 5.2), are used for the estimation of corresponding AR(57) models through a typical identification procedure for the minimization of the Bayesian Information Criterion (BIC) using Ordinary Least Squares (OLS) estimation (Matlab func.: \texttt{arx.m}) with QR implementation (see details in Vamvoudakis et al. [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]). Then, the U–HS–AR method’s hyper–parameters are automatically determined via the algorithm presented in section 5.3. In particular, the algorithm is repeated per each set of baseline structures starting with the sampling of \( 3 \times 57 \) artificial points per AR(57) model, yielding a total number of \( 17\,955 \) artificial points per set. The obtained hyper–parameters per set of baseline structures are shown in Figure 5.7, indicating through their significant discrepancies per set, the method’s capability to adapt on the different healthy subspace geometries. It is noted that a constant \( δ = 30\% \) was found adequate in this case study implying that step 9 is skipped.

In the following, the AR models used previously for the U–HS–AR method are also employed for the construction of the healthy subspace based on the three methods which are used for the comparative assessment. Furthermore, the hyper–parameters per method are selected as follows: (a) All the above AR(57) models are used in the U–MM–AR method as the MM representation of the healthy subspace; (b)
5.4 Experimental Assessment and Comparisons

![Graphs showing the *k* value, δ value, and r value per baseline parameter vectors set.](image)

Fig. 5.7. Case study A: Selected U–HS–AR method’s hyper–parameters per employed set of baseline structures.

The number of the Gaussian Mixture’s Gaussian components for the U–RC–GM–AR method is selected based on the Bayesian Information Criterion (BIC); 8 to 10 components are selected among the sets of baseline structures (see details in [K. J. Vamvoudakis–Stefanou and Fassois 2017]); (c) the critical (user selected) fraction $\gamma$ of the U–PCA–AR method, which determines the subset of the first principal components with the greatest variability, is selected among three commonly used values, 90%, 95% and 99%, as the one yielding the best damage detection performance.

**Inspection phase**

The U–HS–AR method’s damage detection performance is initially judged through the scatter plots of its test pseudo-statistic $D$ in Figure 5.8(a), and the respective box plots in Figure 5.8(b). These depict that the method’s damage detection performance is perfect for damages BL, BH, CH, while it may be judged as very good for damages AL, CL, AH, DH due to a slight overlap, and as good for damage DL for which the overlap is greater. Respective results are presented in the rest subplots of Figure 5.8 for the U–MM–AR, U–RC–GM–AR and U–PCA–AR methods indicating the superiority of the U–HS–AR method. However, the performance of the U–MM–AR method is very close to that of the U–HS–AR.
Fig. 5.8. Case study A: (a),(c),(e),(g) Scatter plots of the methods’ test pseudo-statistics $D$ and, (b),(d),(f),(h) respective box plots [the top and bottom of each box are the 25th and 75th percentiles; the red line in the middle of each box is sample median; the lines extending above and below each box are called whiskers and are drawn from the ends of the interquartile ranges, with length 1.5 times the interquartile range; the red crosses represent observations out of the whiskers range; the black horizontal line is defined by the top whisker of the healthy state and separates the healthy and damaged states].

The above results are also confirmed by the ROC curves illustrated in Figure 5.9 for the invisible and barely visible damages, where again the very good performance of the U–HS–AR method is evident, achieving 96.2% TPR for 4% FPR for the invisible damage and 100% TPR for 1.7% FPR for the barely visible damage. Furthermore, the comparisons via the ROC curves with the alternative methods demonstrate the overall superiority of the U–HS–AR method. All methods’ performance in terms of TPR and FPR are summarized in Table 5.4.

Additional comparisons between the U–HS–AR and the U–PCA–AR methods for different, frequently used, values of the latter’s user selected hyper–parameter $\gamma$ are presented in Figure 5.10 based on ROC curves. These demonstrate that the U–HS–AR outperforms the U–PCA–AR, as well as that the U–PCA–AR method’s performance is significantly affected by the selection of $\gamma$ hyper–parameter.

5.4.3 Case Study B: Damage Detection for a Composite Structure Under Varying EOCs

The problem of damage detection for a single composite tail structure under varying EOCs is investigated in this case study (see Table 5.5) using a different experimental set–up with the structure into...
5.4 Experimental Assessment and Comparisons

Table 5.4: Damage detection performance in terms of TPR and FPR per method and case study.

<table>
<thead>
<tr>
<th>Method</th>
<th>TPR (%)</th>
<th>FPR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case Study A</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invisible damages (5 J impact)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U–HS–AR</td>
<td>96.2</td>
<td>4</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>95.14</td>
<td>4</td>
</tr>
<tr>
<td>U–RC–GM–AR</td>
<td>88.5</td>
<td>4</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>67.79</td>
<td>4</td>
</tr>
<tr>
<td>Barely visible damages (15 J impact)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U–HS–AR</td>
<td>100</td>
<td>1.7</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>96.79</td>
<td>1.7</td>
</tr>
<tr>
<td>U–RC–GM–AR</td>
<td>98.64</td>
<td>1.7</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>98.71</td>
<td>1.7</td>
</tr>
<tr>
<td><strong>Case Study B</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damage 1: Low mass damage (0.41% mass increase)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U–HS–AR</td>
<td>93.33</td>
<td>4.92</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>7.69</td>
<td>5</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>100</td>
<td>2.88</td>
</tr>
<tr>
<td>Damage 2: High mass damage (1.15% mass increase)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U–HS–AR</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>99.49</td>
<td>1.7</td>
</tr>
<tr>
<td>U–RC–GM–AR</td>
<td>100</td>
<td>1.7</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

a freezer as shown in Figure 5.11. Based on this, the structure operates: (a) under varying temperature in the range [0 –28]°C, (b) under varying tightening torque at Bolt A (Figures 5.11(a),(b)) simulating assembly uncertainty and, (c) under material / manufacturing variability (resin thickness, fiber orientation and so on) via the use of reinforced with plastic mesh adhesive tape of two different sizes at Point T (Figure 5.11(b)).

Moreover, two damage scenarios are simulated in this case study via mass addition at Point D on the surface of the composite beam (Figure 5.11(b)). The first damage scenario (Damage 1) corresponds to an added mass of 4.5g (0.41% mass increase), while the second (Damage 2) to a mass of 12.6g (1.15% mass increase). All considered varying EOCs and damage scenarios are summarized in Table 5.6.

The structure is excited at its free end with a random white Gaussian force applied vertically at Point X (Figure 5.11(b)) via an electromechanical shaker, and the resulting vibration response signal is measured at Point Y1 (Figures 5.4 and 5.5) via a lightweight accelerometer. All signals are normalized via sample mean removal and division by its sample standard deviation. The experimental details for both operational phases are shown in Table 5.6.

The dynamics of the healthy and damaged structure, under the considered uncertainty sources are presented through envelopes of Welch based Power Spectral Density (PSD) estimates in Figure 5.12. These are constructed using all the experiments of Table 5.6, Hamming windowing, segment length of 8192 samples and 90% overlap via the Matlab function, pwelch.m. It is evident that the envelope corresponding to the healthy dynamics is significantly overlapped with that of the small damage scenario.
Fig. 5.9. Case Study A: Comparative assessment via ROC curves for the U–HS–AR, the U–MM–AR, the U–RC–GM–AR and the U–PCA–AR ($\gamma = 90\%$) methods. (a) Invisible and (b) barely visible damages.

Table 5.5: Case Study B. Main characteristics.

<table>
<thead>
<tr>
<th>Number of structures:</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage scenarios:</td>
<td>4.5 g or 12.6 g added mass at Point D</td>
</tr>
<tr>
<td>Uncertainty sources:</td>
<td>Temperature (0 . . . 28°C), Tightening torque (Bolt A; 1,2,3,4 Nm), Adhesive tape (small / large)</td>
</tr>
</tbody>
</table>

(4.5g), indicating a challenging damage detection problem, while the discrepancies from the envelope of the bigger damage (12.6g mass) are more prominent.

**Baseline phase**

$p = 160$ vibration response signals are obtained from the structure under a sample of EOCs as shown in Table 5.6. As in the previous case study, an AR(41) model is identified via BIC minimization based on each acquired signal ($N = 10 \, 000$ sample long or 2.15s). The final AR model order is selected as the one that leads to the minimum BIC sample mean.

Based on the parameter vectors of the above models, the hyper–parameters of U–HS–AR method are determined based on its automated algorithm, starting with the sampling of $3 \times 41$ artificial points per AR(41) model, yielding thus a total number of 19,680 artificial points. The obtained hyper–parameters are $k^* = 3$, $r^* = 245$, and $\delta = 30\%$ indicating a healthy subspace of complex geometry as a small $k^*$ is required, while Step 9 of the algorithm is again skipped.

Furthermore, the hyper–parameters of the three methods which are used for the comparative assessment of the U–HS–AR method are selected as follows: (a) All the above AR(41) models are used in
5.4 Experimental Assessment and Comparisons

**Fig. 5.10.** Case Study A: Comparative damage detection results via ROC curves between the U–HS–AR method and the U–PCA–AR method for different values of the hyper-parameter $\gamma$. (a) Invisible and (b) barely visible damages.

**Fig. 5.11.** Case study B: Experimental set-up: Point X represents the force excitation position, Point Y1 the vibration acceleration measurement position, Point D the position where a mass is added as damage, Point T the position where an adhesive tape is attached, and Bolt A the bolt where tightening uncertainty is introduced at the clamping [Aravanis, Kolovos, et al. 2018].

the U–MM–AR method for the representation of the healthy subspace; (b) the number of the Gaussian components for the U–RC–GM–AR method is set to 33 based on the BIC [Bishop 2006] minimization; (c) the critical fraction $\gamma$ of the U–PCA–AR method, is selected as in the previous case study among the values 90%, 95%, 99% based on a supervised procedure.
Table 5.6: Case study B. Experimental details.

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Uncertainty 1: Temperature (°C)</th>
<th>Uncertainty 2: Torque (Nm)</th>
<th>Number of Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>[0–28] with a step of 4°C</td>
<td>1,2,3,4</td>
<td>160°</td>
</tr>
<tr>
<td><strong>Inspection (damage detection) phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>A° &amp; {3, 21}</td>
<td>1</td>
<td>45°</td>
</tr>
<tr>
<td>Healthy</td>
<td>A° &amp; {9, 19, 25}</td>
<td>2</td>
<td>50°</td>
</tr>
<tr>
<td>Healthy</td>
<td>A° &amp; {15, 25}</td>
<td>3</td>
<td>45°</td>
</tr>
<tr>
<td>Healthy</td>
<td>A° &amp; {3, 9, 15, 19, 21}</td>
<td>4</td>
<td>60°</td>
</tr>
<tr>
<td>Healthy 1 (small tape)</td>
<td>B°</td>
<td>1,3,4</td>
<td>195°</td>
</tr>
<tr>
<td>Healthy 2 (large tape)</td>
<td>B°</td>
<td>1,3,4</td>
<td>195°</td>
</tr>
<tr>
<td>Damage 1 (4.5 g)</td>
<td>B°</td>
<td>1,3,4</td>
<td>195°</td>
</tr>
<tr>
<td>Damage 2 (12.6 g)</td>
<td>B°</td>
<td>1,3,4</td>
<td>195°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>980 (total)</td>
</tr>
</tbody>
</table>

A°: Set of experiments including [2–26]°C with a step of 4°C
B°: Set of experiments including {0, 1, 3, 7, 9, 10, 14, 15, 17, 21, 23, 25, 27}°C
° 5 experiments per temperature and torque values
Sampling frequency: $f_s = 4654.5$ Hz. Signal bandwidth: [5 – 2327.25] Hz.
Signal length (damage detection): $N = 10,000$ samples (2.15s).
Signal normalized via sample mean removal and division by its sample standard deviation.

**Fig. 5.12.** Case Study B: Envelopes of Welch–based PSD estimates for the healthy and damaged structure under simulated damage of: (a) 4.5g (Damage 1) and (b) 12.6g (Damage 2) added mass [Healthy state: 360 experiments, Healthy 1 state: 195 experiments, Healthy 2 state: 195 experiments, Damage 1 / Damage 2: 195 / 195 experiments].

**Inspection phase**

The values of the U–HS–AR test pseudo–statistic $D$, as obtained from all inspection experiments (Table 5.6), are presented through scatter and box plots in Figures 5.13(a),(b), respectively. These depict
5.4 Experimental Assessment and Comparisons

Fig. 5.13. Case study B: (a),(c),(e),(g) Scatter plots of the methods’ test pseudo-statistics $D$ and, (b),(d),(f),(h) respective box plots [the top and bottom of each box are the 25th and 75th percentiles; the red line in the middle of each box is sample median; the lines extending above and below each box are called whiskers and are drawn from the ends of the interquartile ranges, with length 1.5 times the interquartile range; the red crosses represent observations out of the whiskers range; the black horizontal line is defined by the top whisker of the healthy state and separates the healthy and damaged states].

The above results are confirmed through the ROC curves of Figure 5.14 with the U–HS–AR method achieving TPR of 93.33% with FPR 4.92% for Damage 1 (left subplot), and TPR of 100% with FPR equal to 0% for Damage 2 (right subplot). All methods’ performance in terms of TPR and FPR values is summarized in Table 5.4.

Further comparisons between the U–HS–AR and the U–PCA–AR methods using the other two common values of $\gamma$ to the latter are presented in Figure 5.15 through ROC curves. It is obvious that the detection of Damage 1 based on the U–PCA–AR method for $\gamma = 90\%, 95\%$ is poor and significantly worse than that of the U–HS–AR method. This pinpoints the U–PCA–AR method’s high sensitivity to its user selected parameter $\gamma$ that significantly affects its performance, as well as the need for a supervised training procedure that will potentially lead to the best selection of $\gamma$. 

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Fig. 5.14. Case Study B: Comparative assessment via ROC curves for the U–HS–AR, the U–MM–AR, the U–RC–GM–AR and the U–PCA–AR ($\gamma = 99\%$) methods. (a) Damage 1 and (b) Damage 2.

Fig. 5.15. Case Study B: Comparative damage detection results via ROC curves between the U–HS–AR and U–PCA–AR methods for different values of the hyper–parameter $\gamma$. (a) Damage 1 and (b) Damage 2.
The damage detection performance of the postulated U–HS–AR method is assessed in this study based on two distinct experimental case studies. Each of these is intentionally characterized by uncertainty sources of different nature that significantly affect the healthy dynamics and mask early stage damages, aiming at testing the method’s robustness to different healthy subspace geometries. Indicative projections of the considered geometries on the first three parameters of the employed AR models, are shown per each case study in Figure 5.16. As it is shown the healthy subspace geometry corresponding to Case Study B (Figure 5.16(b)) is significantly more complex compared to that of Case Study A (Figure 5.16(a)).

However, based on the very good to excellent damage detection results of the U–HS–AR method in both case studies as presented in the previous section, the method’s capability to adapt to any healthy subspace is evident, warranting thus robust damage detection. On the other hand, based on the comparisons with the alternative methods it is clear that none of the employed U–MM–AR, U–RC–GM–AR and U–PCA–AR methods can cope with both of the examined healthy subspace geometries. The U–PCA–AR method yields poor performance in the first case study and the U–MM–AR and U–RC–GM–AR in the second, while all of them, and especially the U–PCA–AR, require user intervention and subjective selections that may significantly affect their performance in contrast to the U–HS–AR method which is fully automated.

The above mentioned damage detection robustness of the U–HS–AR method is attributed to the union of hyper–spheres based representation of the healthy subspace that may approximate any geometry. This procedure yields a compact, non–fragmented, healthy subspace approximation that may expand to unknown healthy state dynamics provided that vibration signals from a representative sample of the considered uncertainty sources are available. This is achieved using artificial “healthy” and “damaged” points to construct the healthy subspace, thus performing an artificially supervised learning. Of course, the constructed healthy subspace may deviate from its true counterpart which is unknown, but this issue may be alleviated using fresh vibration response signals from healthy or damaged structural states to automatically update the healthy subspace.

In addition to all the above, it is important to note that the algorithm, for the method’s hyper–parameters determination, does not pose any requirement for a high number of vibration response signals. Instead, other methods require a high signals number for a good estimation of a sample covariance matrix or other statistical quantities and model parameters.

**Healthy subspace geometry**

![Fig. 5.16. Indicative parts of the healthy subspace geometry per case study presented through the first three parameters of the employed AR model in each case: (a) Case Study A; (b) Case Study B.](image-url)
5.6 Concluding Remarks

A novel, machine–learning–type, Unsupervised Hyper–Sphere based healthy subspace approximation method using AutoRegressive representations of the dynamics (U–HS–AR method) for robust vibration–based damage detection under uncertainty has been postulated. The method is based on the concept of constructing a healthy subspace representing the healthy dynamics under varying Environmental and Operating Conditions (EOCs) and other uncertainty. This has been achieved via a conceptually simple deterministic approximation using a union of properly selected hyper–spheres with distinct centers and radii for approximating arbitrary healthy subspace geometries. The method offers full automation, relieving the user from critical selections, while also avoiding complex optimization procedures.

The method has been assessed via two experimental case studies and high numbers of experiments (test cases), while comparisons with three state–of–the–art robust methods have been also performed. The main conclusions of the study may be summarized as follows:

1. The postulated U–HS–AR method is capable of constructing approximations of healthy subspaces of various geometries using relatively low numbers (105 in case study A and 160 in case study B) of vibration response signals and through a fully automated procedure. This renders the method suitable for damage detection under any type of uncertainty.

2. The method’s experimental assessment has indicated high detection performance even with a single accelerometer and under limited and low–frequency bandwidth.

3. In addition, the comparisons with three state–of–the–art robust methods has demonstrated superiority in performance, further underlying its robustness to different sources of uncertainty.

4. Compared to the methods of the previous chapters, the postulated one yields very good performance when invisible damages are considered and the best performance in the case of the barely visible damages, as depicted in Figure 5.17.

![Fig. 5.17.](image-url) Comparison of the current chapter methods with their previous chapters counterparts, in terms of correct damage detection performance (True Positive) rate, versus two false alarm rates (FPR) of interest, by means of bar charts for: (a) Invisible damage scenarios, (b) Barely visible damage scenarios. [Based on the experiments of the benchmark application study (see chapter 2).]
A Crude Gaussian Mixture Model Based Healthy Subspace Method for Automated and Unsupervised Damage Detection via Random Vibration Response Signals

A novel unsupervised and automated crude Gaussian mixture model based healthy subspace method, for robust damage detection under uncertainty via random vibration response signals, is postulated in the present chapter. The method is based on the approximate construction, within a proper, AutoRegressive parameters based, feature space, of a “healthy subspace” representing the healthy structural dynamics under uncertainty as the superposition of a proper collection of isotropic Gaussian distributions. This is achieved via a simple and automated procedure that eliminates user intervention and subjective selections. The method’s detection performance is assessed through two experimental case studies featuring various uncertainty sources and distinct “healthy subspace” geometries. In addition, comparisons with five powerful state–of–the–art robust damage detection methods are considered. The assessment results demonstrate the excellent detection performance of the postulated method and its superiority against the alternative methods in terms of detection performance and robustness to uncertainty.

6.1 Introduction

The problem of robust vibration–based damage detection for structures operating under uncertainty due to varying environmental, operational, or other conditions, has received significant attention over the past two decades [Peeters, Maeck, and Roeck 2001a; Sohn 2007; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b; Zhou et al. 2014]. The reason is that vibration signals are easy to measure, the measurement and acquisition equipment is of reasonable cost, and the signals are naturally available without interrupting the structure’s normal operation or requiring specific excitation equipment. Yet, the presence of varying Environmental and Operating Conditions (EOCs) during the structure’s operation, may introduce significant uncertainty in its structural dynamics that often “masks” the respective damage effects and jeopardizes the damage detection effectiveness. This phenomenon may be significantly amplified, when damage detection for a population of nominally identical structures is considered, since variability in the structures materials, geometry, and manufacturing, further aggravates the varying EOCs masking of the damages effects in the dynamics [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

The problem has been investigated in numerous studies, using several methods, with the most prominent ones being of the robust data–based machine–learning–type. These are typically unsupervised
methods, hence requiring only signals from the healthy structure(s) for their training, that provide either direct (explicit) or indirect (implicit) representations of the healthy structural dynamics under uncertainty [Figueiredo, Park, et al. 2011; Figueiredo and A. Santos 2018; A. Santos, Figueiredo, Silva, Sales, et al. 2016; Worden and Manson 2007]. These representations provide an approximation of the subspace spanned by the healthy structural dynamics under all possible EOCs, which is defined within a properly selected feature space and may be denoted as “healthy subspace” [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

The healthy subspace representations are typically obtained in an initial training or baseline phase, based on a proper set of vibration signals and the construction of a proper feature space (proper damage sensitive features are extracted from the signals). Subsequently, damage detection is pursued in the inspection or diagnostic phase, by investigating whether a new damage sensitive feature, obtained from the current structure of unknown health state, belongs or not to the healthy subspace.

The healthy subspace may be approximated by means of two main approaches. In the indirect (implicit) methods approach, this is achieved through a reduced dimensionality feature space, wherein a damage causes significant changes in the respective features, which however are insensitive to changes in the EOCs. Such features are obtained via proper decomposition techniques, as for instance the Principal Component Analysis (PCA), the Auto–Associative Neural Networks (AANN) [Figueiredo and A. Santos 2018], the Factor Analysis (FA), [Bellino et al. 2010; Comanducci et al. 2016; Deremaeker et al. 2008; Figueiredo, Park, et al. 2011; Figueiredo and A. Santos 2018; Giraldo 2006; Hu et al. 2016; Kojidi et al. 2014; Kullaa 2010; Manson 2002; Rama Mohan Rao et al. 2015; A. Santos, Silva, Sales, et al. 2015; A. Santos, Figueiredo, Silva, Sales, et al. 2016; Sen et al. 2019; Silva et al. 2019; Vanlanduit et al. 2005; Worden and Manson 2007; Yan et al. 2005], and the cointegration [E. J. Cross, Manson, et al. 2012; E. J. Cross, Worden, et al. 2011; Tome et al. 2020; Worden, Baldacchino, et al. 2016].

On the other hand, the direct (explicit) methods approach constructs deterministic or probabilistic models of the effects of the varying EOCs on the dynamics. Such methods may be further categorized based on whether they require measurable uncertainty sources or not. In the first category the healthy subspace may be constructed using cause–and–effect type modeling, with the “causes” being the measurable uncertainty sources, as for instance temperature, and the “effects” being the changes on the corresponding features [Comanducci et al. 2016; Hios et al. 2014; Hu et al. 2016; Ko et al. 2003; Lorenzoni et al. 2016; Peeters, Maecck, and Roeck 2001b; Worden and E. Cross 2018; Worden, Sohn, et al. 2002]. The pitfall in this case, is that the uncertainty sources may be non–measurable, while extra measurement equipment is required.


Each one of the above–mentioned robust damage detection methods, corresponding to either the indirect or the direct approaches, has achieved its own significant merit toward tackling the current problem. Yet, these methods also feature one or more limitations, which are associated with requirements for: (i) relatively high numbers of signal records for training (especially when high dimensional feature spaces are considered) [Avendaño–Valencia et al. 2017b; Figueiredo and A. Santos 2018; Michaelides, Apostolellis,

Among the damage detection under uncertainty robust methods, those based on Gaussian Mixture (GM) models are of the most prominent [Figueiredo and A. Santos 2018]. Such models, consist of linear superpositions of a number of Gaussian (component) distributions and may approximate almost any arbitrary distribution [Bishop 2006, p. 111]. As no closed form solution is available, their estimation is typically based on the Expectation Maximization (EM) algorithm [Bishop 2006, pp. 435–439][Figueiredo and E. Cross 2013]. Yet, this algorithm features a number of limitations, associated with: (a) singularity issues in the estimation of the GM components, (b) difficulty into estimating the optimal GM model, (c) a significantly high number of parameters to be estimated when a high dimensional space is considered, (d) the requirement for a GM components number selection procedure.

Several alternatives to the EM based GM model estimation have been proposed, mainly focusing into improving the robustness of the GM estimation. Toward this end, they provide methods that effectively explore the feature space for potential solutions of the GM estimation problem, avoiding local optima and determining the global one. Such methods are based on Genetic algorithms [A. Santos, Figueiredo, Silva, R. Santos, et al. 2016], Particle Swarm Optimization [A. Santos, Silva, R. Santos, et al. 2016], and clustering techniques [Qiu, Fang, and Yuan 2019; Qiu, Fang, Yuan, et al. 2018], yet they increase the conceptual and computational complexity, often without significantly improving the damage detection performance [Figueiredo and A. Santos 2018], while they do not alleviate the rest of the above–mentioned limitations. Another alternative based on Bayesian modelling is proposed [Figueiredo, Radu, et al. 2014b], so as to alleviate the singularity issues of the EM algorithm and automate the selection of the optimal number of GM components, through the introduction of proper prior distributions over the GM parameters and Gibbs sampling estimation. Nevertheless, this alternative does not improve the damage detection performance compared to its EM counterpart [Figueiredo, Radu, et al. 2014b]. The alternative method alleviating most of the above–mentioned limitations is based on a Bayesian non–parametric density estimation that introduces a non–parametric Dirichlet Process (DP) prior over the GM parameters. This achieves optimal GM model estimation, without requiring the GM components number determination, yet features a conceptually complex estimation procedure.

The goal of the present study is the postulation of a robust, unsupervised and automated GM type method, in which the approximation of the healthy subspace is achieved via a conceptually and computationally simplified procedure that: (i) relieves the user from hyper–parameters selections, (ii) achieves high detection performance, (iii) alleviates the requirement for high numbers of vibration signals in the baseline phase. The method is postulated for the response–only measurement case, while its extension to the excitation–response case is straightforward. The postulated method is inspired by the U–HS–AR method presented in chapter 5 and attempts a healthy subspace approximation through a collection of probabilistic hyper–spheres (isotropic Gaussian distributions). The corresponding centres and radii are determined via a significantly simpler and computationally more effective procedure compared with the U–HS–AR method, while each probabilistic hyper–sphere is estimated based on the local characteristics of the healthy subspace, hence enhancing the method’s robustness to the various healthy subspace geometries. The probabilistic hyper–spheres collection is herein interpreted as a crude GM model that is defined within a feature space spanned by the parameters of an AutoRegressive (AR) representation of the dynamics. The resulting Unsupervised crude Gaussian Mixture AR based method is hereafter abbreviated as U–cGM–AR.
In contrast with the other GM–based methods, the herein postulated crude Gaussian mixture model does not aim at an optimal estimation of the healthy dynamics distribution within the feature space (hence it is characterized as crude GM). Instead, its objective is the determination of a good approximation of the healthy subspace geometry, under limited numbers of vibration response signals and within a high–dimensional feature space. Toward this end, the method employs a significantly simpler estimation procedure that is relieved from the limitations of the EM algorithm and the other GM estimation methods.

The U–cGM–AR method is herein assessed via two experimental case studies: (a) The first study investigates the damage detection for a population of nominally identical composite aerostructures problem, through a population of half tails from an Unmanned Aerial Vehicle (UAV) that are subject to impact induced damage; (b) The second study considers damage detection for a single composite aerostructure, subject to small, local, stiffness reduction (simulated by the attachment of small masses) damage scenarios, and operating under varying temperature and tightening assembly torque conditions. The method’s performance is examined via thousands of test cases and comparisons with five powerful state–of–the–art robust methods using Receiver Operating Characteristic (ROC) curves.

The rest of the chapter is organized as follows: The precise problem statement is presented in section 6.2. The postulated U–cGM–AR method is presented in section 6.3. The experimental set–up and the respective experimental assessment per case study are presented in section 6.4, while conclusions are provided in section 6.5.

6.2 Precise Problem Statement - the Supervised Detection Problem

Consider a structure operating under varying EOCs, assumed constant during a measurement interval. 

Given:
1. **Baseline (Training) Phase:**
   * p random vibration response signals \( y_i[t] \) (\( i = 1, 2, \ldots, p; t = 1, 2, \ldots, N; t: \) normalized by the sampling period discrete time, \( N: \) signal length\(^a\) from the healthy structure operating under a representative sample of the considered EOCs.

2. **Inspection (Operational) Phase:**
   A new vibration response signal \( y_u[t] \) (\( t = 1, 2, \ldots, N \)) from the current state of the structure, with the subscript \( u \) designating unknown health state.

Determine: The health state (healthy or damaged) of the structure.

\(^a\)It is noted that \( p \) should be sufficiently high in order to adequately represent the considered range of EOCs.

6.3 The Unsupervised Crude Gaussian Mixture Model Based Method (U–cGM–AR) for Damage Detection

6.3.1 The Concept

This is an unsupervised robust vibration–based damage detection method that uses a feature space spanned by the parameters of an AR(\( n \)) model (AR stands for AutoRegressive) [Fassois and J. S. Sakellariou 2009], within which a healthy subspace is constructed during the learning (baseline) phase. The AR model identification is based on a random vibration response from the investigated structure under normal operation, while in the baseline phase only a limited number of vibration response signals from the healthy structural state is required for the healthy subspace construction. On the other hand, in the diagnostic (inspection) phase of the method, a fresh random vibration signal from the structure in unknown health state is obtained and the respective AR(\( n \)) model parameter vector is estimated. Then, the structure is deemed as healthy if and only if the “fresh” vector resides within the healthy subspace.
6.3 The Unsupervised Crude Gaussian Mixture Model Based Method

The postulated method’s main idea is the simple and automatic healthy subspace construction, via the hyper–position of a proper number of isotropic Gaussian components. The resulting healthy subspace approximation is a crude Gaussian Mixture (GM) model, estimated using the limited number, say \( p \), of AR\((n)\) model parameter vectors of the baseline phase. The GM model estimation is based on the separation of the \( p \) model parameter vectors into all possible unique (yet overlapped) sets of \( \kappa^* \) neighboring parameter vectors, through a \( k–\)Nearest Neighbor type procedure [Stibor et al. 2007]. Each set is subsequently used to estimate a respective isotropic Gaussian component of the GM. The \( \kappa^* \) is the method’s hyper–parameter that is automatically determined via a proper likelihood maximization criterion.

The postulated U–cGM–AR method is in detail presented below, by means of its two operational phases.

6.3.2 The Baseline (Training) Phase

**The signals and the feature space construction.** In the baseline (learning) phase of the method, \( p \) vibration response signals are obtained from the structure (or from a set of structures when a population of structures is considered) in its healthy state. Each signal is measured under a distinct set of EOCs, so that the \( p \) signals yield a collection of sample EOCs that span the range within which the structure is expected to operate. A conventional AR\((n)\) model is then identified per signal, with \( \alpha_i \) and \( \Sigma_i \) designating the \( i–th \) model’s estimated parameter vector (bold–face capital/lower–case letters designate matrices/vectors, respectively) and its covariance matrix, respectively. Presently, the true parameter vector is not distinguished from its estimate for reasons of notation simplicity.

**Accounting for the estimation uncertainty.** Each parameter vector \( \alpha_i \) is, due to estimation uncertainty, asymptotically \((N \to \infty)\) associated to a Gaussian distribution, with mean the true parameter vector and covariance \( \Sigma_i \) [Ljung 1999]. In order to take this uncertainty into account within the postulated method, each signal is standardized as \( \Sigma^{-1} b_i \), with \( \Sigma = \frac{1}{p} \sum_{i=1}^{p} \Sigma_i \) designating the covariance matrix of the \( p \) parameter vectors \( (\alpha_i) \) sample mean. In the sequel all the parameter vectors \( \alpha_i \) are assumed standardized, even if this is not explicitly expressed. The proposed standardization is motivated by similarity measures, such as the Mahalanobis distance [Rencher et al. 2012], to approximately take into account the estimation uncertainty of all AR model parameter vectors \( \alpha_i \), while maintaining their relative position within the feature space.

**Isotropic Gaussian components estimation for varying \( \kappa \).** The Euclidean distances, for any pair of parameter vectors \( \alpha_i \) are obtained:

\[
d_p(i, j) := ||\alpha_i - \alpha_j||_2, \quad i, j = 1, \ldots, p, \quad i \neq j
\]  

(6.1)

The \( \kappa – 1 \) nearest vectors to each parameter vector \( \alpha_i \) are then determined and included into a set of vectors, \( \{\alpha_j\} \), along with the vector \( \alpha_i \). Among the resulting \( p \) parameter vectors sets, only the \( \kappa \) unique sets are retained and used so as to estimate \( \mathcal{K} \) respective isotropic Gaussian distributions. Let \( \alpha_{cj} \) designate one of the \( \kappa \) vectors contained into the \( \{\alpha_j\} \) set, with \( c = 1, \ldots, \kappa \) and \( j = 1, \ldots, \mathcal{K} \). Then, the mean and covariance matrix of the respective isotropic Gaussian distribution, \( N(\alpha|\mu_{kj}, \Sigma_{kj}) \), are obtained as:

\[
\mu_{kj} := \frac{1}{\kappa} \sum_{c=1}^{\kappa} \alpha_{cj}, \quad \Sigma_{kj} := \frac{1}{n} \left[ \frac{1}{\kappa - 1} \sum_{c=1}^{\kappa} \left( \alpha_{cj} - \mu_{kj} \right) \left( \alpha_{cj} - \mu_{kj} \right)^T \right] I
\]  

(6.2)

with \( tr\left[ \frac{1}{\kappa - 1} \sum_{c=1}^{\kappa} \left( \alpha_{cj} - \mu_{kj} \right) \left( \alpha_{cj} - \mu_{kj} \right)^T \right] \) the trace of the sample covariance matrix \( \frac{1}{\kappa - 1} \sum_{c=1}^{\kappa} (\alpha_{cj} - \mu_{kj})(\alpha_{cj} - \mu_{kj})^T \) and \( I \) the \( n \times n \) identity matrix. The covariance matrix \( \Sigma_{kj} \) is isotropic, with off–diagonal elements equal to zero, and diagonal elements equal to the sample mean of the \( n \) parameters variance obtained from the parameter vectors \( \{\alpha_j\} \).
Chapter 6. A Crude Gaussian Mixture Model Based Method

Note 1: The isotropic Gaussian distributions are herein used as Gaussian mixture components, in order to significantly reduce the requirement for a high number of AR parameter vectors ($\kappa$ may take values starting from as low as 18 parameter vectors; that is 18 samples per parameter) that is especially pronounced in high dimensional feature spaces, as well as to avoid over–fitting the Gaussian mixture model to these vectors. The latter is achieved via a crude representation of the feature space through a collection of probabilistic hyper–spheres (isotropic Gaussian distributions). These are preferred over the ellipsoids of a Gaussian distribution, which may yield over–fitting issues when estimated with the limited number of parameter vectors available in this phase. This procedure of isotropic Gaussian distributions estimation is repeated for $\kappa = 18, 19, \ldots, p$. The $\kappa = 18$ case produces a large collection of isotropic Gaussian distributions, thereby offering a detailed representation of the healthy subspace, at the cost of potentially over–fitting the available parameter vectors. On the other hand, the $\kappa = p$ case yields the generic model of a single isotropic Gaussian distribution, which may generalize to unobserved parameter vectors, yet provides only a crude representation of the healthy subspace. The rest of the $\kappa$ values provide a trade off between the two cases.

The $\kappa$ selection criterion. The $\kappa$ selection is the objective of the baseline phase and is achieved by employing all the obtained Gaussian mixture models for the various $\kappa$ values and a proper selection criterion. Let $f_\kappa(\alpha)$ the probability density function of the Gaussian mixture for a distinct $\kappa$ value that is expressed as:

$$f_\kappa(\alpha) = \sum_{j=1}^{\mathcal{K}} \lambda_{\kappa j} N(\alpha | \mu_{\kappa j}, \Sigma_{\kappa j}), \quad \text{with} \quad \lambda_{\kappa j} = \frac{1}{\mathcal{K}}$$

(6.3)

Let also $A$ the $p \times n$ matrix containing all the available parameter vectors of the current phase. The following $\kappa$ selection criterion may then be defined as follows:

$$\kappa^* = \arg \min_{\kappa} \left[ -\ln f_\kappa(A) - (n + 1) \frac{\kappa}{\mathcal{K}} \right]$$

(6.4a)

where

$$\ln f_\kappa(A) = \sum_{i=1}^{p} \ln \left( \sum_{j=1}^{\mathcal{K}} \lambda_{\kappa j} N(\alpha_i | \mu_{\kappa j}, \Sigma_{\kappa j}) \right)$$

(6.4b)

with $\ln f_\kappa(A)$ designating the log likelihood function of the parameter vectors in $A$.

Note 2: The postulated $\kappa$ selection criterion is inspired by criteria such as the Akaike Information Criterion (AIC) and selects the $\kappa$ value that maximizes a penalized log likelihood function. The log likelihood function is estimated based on the probability density function $f_\kappa(\alpha)$, for all the available in the baseline phase AR parameter vectors. On the other hand, the penalty term $((n + 1) \frac{\kappa}{\mathcal{K}})$ is a trade–off between the penalization of the small number of Gaussian components (that yields a crude approximation of the healthy subspace) and the penalization of a small $\kappa$ value (that may produce over–fitting issues). This trade–off is expressed as the weighting of the number of Gaussian mixture parameters, by the percentage of the $p$ parameter vectors $\alpha$, employed for the estimation of a single isotropic Gaussian component.

With the completion of this phase only the parameter vectors $\alpha_i$, the covariance matrix $\Sigma$, and the hyper–parameter $\kappa^*$ are retained for use in the method’s inspection phase.
6.4 Experimental Assessment and Comparisons

6.3.3 The Inspection (Diagnostic) Phase

In this phase a fresh vibration response signal from the operating structure in “unknown” state is obtained and an AR(n) model with estimated parameter vector $\mathbf{\alpha}_u$ is identified. This is then standardized as $\Sigma^{-\frac{1}{2}} \mathbf{\alpha}_u$ ($\mathbf{\alpha}_u$ is assumed standardized in th sequel) and the Euclidean distances between $\mathbf{\alpha}_u$ and each $\mathbf{\alpha}_i$ point are determined. The $\kappa^*$ nearest to $\mathbf{\alpha}_u$, parameter vectors $\mathbf{\alpha}_i$ are obtained based on these distances and the respective isotropic Gaussian component is estimated as:

$$
\mathbf{\mu}_u := \frac{1}{\kappa^*} \sum_{c=1}^{\kappa^*} \mathbf{\alpha}_c, \quad \Sigma_u := \frac{1}{\kappa^*} \sum_{c=1}^{\kappa^*} \left( \mathbf{\alpha}_c - \mathbf{\mu}_u \right) \left( \mathbf{\alpha}_c - \mathbf{\mu}_u \right)^T \frac{1}{n} I
$$

(6.5)

where $\mathbf{\alpha}_c$ designates the $c$–th nearest to $\mathbf{\alpha}_u$, parameter vector $\mathbf{\alpha}_i$.

Note 3: In order to reduce the computational complexity, only the Gaussian component which is nearest to the parameter vector $\mathbf{\alpha}_u$ is used for damage detection in this phase. Thus, the structure is deemed as healthy, if and only if the vector $\mathbf{\alpha}_u$ is a sample from this Gaussian component.

The following test pseudo–statistic is then defined:

$$
D := -\ln N(\mathbf{\alpha}_u | \mathbf{\mu}_u, \Sigma_u)
$$

(6.6)

and damage detection is tackled via the decision making mechanism below:

$$
D \leq l_{\text{lim}} \quad \implies \quad \text{Healthy structure}
$$

$$
\text{otherwise} \quad \implies \quad \text{Damaged structure}
$$

(6.7)

where $l_{\text{lim}}$ is a decision making threshold initially defined based on a significance level of 3% and subsequently properly updated to account for changes in the healthy structure.

6.4 Experimental Assessment and Comparisons

6.4.1 Preliminaries

The damage detection performance of the postulated U–cGM–AR method is assessed using two experimental case studies and comparisons with state–of–the–art robust methods.

The experimental case studies

The first case study (case study A) is based on a population of nominally identical composites structures with significant material, manufacturing, and assembly uncertainty. On the other hand, the second case study (case study B), is based on one of these composites structures, which operates under varying temperature, tightening torque on the mounting, and other potential uncertainties. Details on the experiments and the structures of each case study are provided in [Aravanis, J. Sakellariou, et al. 2020; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] respectively.

The structure

The structure employed in both case studies represents one of the tail booms of a commercial twin tail Unmanned Aerial Vehicle (UAV). The structure consists of an aluminum mass attached at the one end of a composite beam featuring a square hollow cross section, uniform along its length, made of several layers of woven and unidirectional fabric based on one shot transfer molding (see dimensions in Figure 6.1). The structure is tightly clamped on the other end representing the attachment of the tail boom to the fuselage (see experimental set–up in Figure 6.2).
Fig. 6.1. Case Study A: Sketch of the half tail with the basic dimensions (in mm) of the composite beam, the mass (horizontal stabilizer) and the clamping, along with the damage locations (Points A, B, C, D), the measurement Point Y1, and the excitation Point X: (a) top view, (b) side view.

The assessment procedure

The damage detection performance of the U–cGM–AR and the other herein utilized methods is depicted by means of box plots of their test pseudo–statistics values (D) and Receiver Operating Characteristic (ROC) curves. The latter indicate the True Positive Rate (TPR: percentage of correct damage detection) versus the False Positive Rate (FPR: percentage of false alarms) for different values of the decision making threshold \( l_{\text{lim}} \). The optimal performance (with TPR = 100% and FPR = 0%) is achieved when the ROC curve includes the (0,1) point, which is located on the upper left corner in these diagrams.

The other methods

Five state–of–the–art methods for robust unsupervised damage detection under uncertainty are herein utilized for comparisons with the postulated method. These are properly adjusted to use the parameters of AR models as damage sensitive features, in order to achieve a proper comparative assessment of the postulated method. The main idea of each method is presented below, while proper references for detailed descriptions are also provided.

**U–HS–AR** This is an automated method that constructs a healthy subspace approximation using a collection of deterministic hyper–spheres in the feature space, with proper centres and radii. The method determines all its hyper–parameters automatically and its in detail presentation is available in chapter 5.

**U–MM–AR** This method is based on a Multiple Model representation for a probabilistic approxi-
6.4 Experimental Assessment and Comparisons

Fig. 6.2. Case Study A: Experimental set–up with the force excitation at Point X, and the measurement of the vibration acceleration at Point Y1 [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]

U–PCA–AR This method is based on the well–known PCA where a subset of the principal components with the greatest variability is used to approximate the healthy subspace. The number of principal components included in this subset is the method’s user–defined hyper–parameter. The method is formally presented in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

U–EM–GM–AR This is a typical Gaussian Mixture model based method that approximates the healthy subspace using a GM estimated via Expectation Maximization (EM). The number of GM components is selected based on the Bayesian Information Criterion (BIC) and constitutes the method’s hyper–parameter. Further details on the method are available in [Figueiredo and E. Cross 2013].

U–DP–GM–AR This method is an improved alternative of the U–EM–GM–AR that uses a Gaussian Mixture model with possibly infinite Gaussian components for approximating the healthy subspace. A non–parametric Dirichlet Process (DP) prior is introduced over the GM model parameters distribution that is estimated via Gibbs sampling. The Gaussian components number is automatically estimated via a Bayesian clustering framework that sequentially updates their number and parameters for every available feature vector. Compared to its EM counterpart this method avoids suboptimal solutions to the GM estimation problem. Details about the method may be found in [Rogers et al. 2019].
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Table 6.1: Case Study A. Main characteristics [K. Vamvoudakis–Stefanou et al. 2021].

<table>
<thead>
<tr>
<th>Number of structures:</th>
<th>23 healthy (15 in baseline phase), 8 damaged (inspection phase)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage scenarios:</td>
<td>Impact induced of high or low energy at Points A, B, C, D</td>
</tr>
<tr>
<td>Uncertainty sources (inter–beam):</td>
<td>manufacturing, material, assembly</td>
</tr>
</tbody>
</table>

Table 6.2: Case Study A. Experimental details [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Impact Energy (J)</th>
<th>Damage Visual Characterisation</th>
<th>Impact Position</th>
<th>Number of beams</th>
<th>Number of experiments per beam</th>
<th>Total number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>—</td>
<td>—</td>
<td>23</td>
<td>7</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>Damage AL</td>
<td>5</td>
<td>Invisible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BL</td>
<td>5</td>
<td>Invisible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CL</td>
<td>5</td>
<td>Invisible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DL</td>
<td>5</td>
<td>Invisible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage AH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point A</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage BH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point B</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage CH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point C</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Damage DH</td>
<td>15</td>
<td>Barely visible</td>
<td>Point D</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Signal details: Sampling freq. $f_s = 4,654.5$ Hz, Signal bandwidth $[5 – 2,327.25]$ Hz,
Signal length: non–parametric analysis $N = 112,000$ samples $(24.06s)$,
damage detection $N = 10,000$ samples $(2.15s)$.
Signal normalized via sample mean removal and division by its sample standard deviation.

6.4.2 Case Study A: Damage Detection for a Population of Nominally Identical Structures

In this case study the problem of damage detection for a population of nominally identical structures is tackled. Towards this end, a population sample consisting of 23 healthy and 8 damaged structures as the one described in section 6.4.1, characterized by significant uncertainty in their dynamics (see Table 6.1) due to a combination of: (a) manufacturing and materials variability, and (b) variability in the assembly of the different composite beams to the aluminum mass (horizontal stabilizer) and the clamping (attachment to the fuselage).

Each damaged beam (see Table 6.2) is impacted at certain points on its surface with a distinct impact energy, and the respective damage scenario is designated by two letters, with the first indicating the damage position (see Figure 6.1) and the second low (L; 5 J) or high (H; 15 J) impact energy (see also Table 6.2). Each healthy/damaged structure is excited at its free end with a random white Gaussian force applied vertically at Point X (see Figure 6.1 and Figure 6.2) using an electromechanical shaker. The resulting vibration acceleration is measured at Point Y1 using a lightweight accelerometer, while the respective vibration response signal is sample mean corrected and normalized by its sample standard deviation.

The significant effect of uncertainty on the population dynamics is shown in Figure 6.3 via envelopes of Welch [Ljung 1999] based Power Spectral Density (PSD) estimates for all the healthy and damaged structures. These are estimated using all experiments of Table 6.2, Hamming windowing, segment length of 8,192 samples and 90% overlap via the Matlab function, pwelch.m. This effect is so significant that the low impact energy damage envelope is highly overlapped with its healthy state counterpart, hence indicating a challenging damage detection problem. On the other hand, such overlap is only partially observed for the impact damages of higher energy, where discrepancies between the healthy and damaged
Fig. 6.3. Case Study A: Envelopes of Welch–based PSD estimates for the healthy and damaged population of structures under (a) invisible and (b) barely visible damage scenarios [Healthy population: 161 experiments, Invisible / Barely visible damage at all considered points: 28 / 28 experiments] [K. Vamvoudakis–Stefanou et al. 2021].

envelopes are evident in the range $[1800 – 2200]$ Hz.

At this point, it should be noted that the present case study represents a special type of damage detection under uncertainty problem, where the uncertainty sources (manufacturing, materials, and assembly variability) effects are not measurable\(^1\). Therefore, the selection of a distinct set of structures that properly represents the healthy population dynamics is not possible and a random set of healthy structures is selected in the baseline phase for the methods training. Yet, this set may not be representative of the healthy population dynamics, hence significantly affecting the methods damage detection performance. In order to take this phenomenon into account and produce reliable performance results, a special assessment procedure that is proposed in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b] is herein adopted and concisely explained below.

A set of 15 healthy structures, denoted as “baseline structures”, is used in the baseline phase, while the other 8 healthy and all the damaged structures, which formulate the “inspection structures” set, are exclusively used in the inspection phase. Then, once damage detection results are obtained using these two sets, a number of healthy structures is mutually exchanged between them, thereby yielding two new sets that are employed for a new round of damage detection results. This procedure is repeated a sufficient number of times until multiple combinations of “baseline structures” are used, and each distinct healthy structure is included in at least one “baseline structures” set. Presently, this procedure is repeated 50 times, thereby yielding 50 respective sets of “baseline structures” and a high total number of test cases (denoted as aggregate inspection experiments), which are presented in Table 6.3.

Baseline phase

The postulated method’s training is based on $p = 105$ vibration response $N = 10 000$ sample long signals (2.15s), obtained from $\nu = 15$ healthy sample structures of the population (also see Table 6.2). The respective AR(57) models, whose parameters span the feature space, are identified using a typical

\(^1\)This is possible for uncertainty sources such as the varying temperature (see Case Study B in the next section).
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Table 6.3: Case Study A. Details of the assessment procedure. [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]

<table>
<thead>
<tr>
<th>No. of baseline structures (V)</th>
<th>No. of inspection structures</th>
<th>No. of baseline experiments (p)</th>
<th>Inspection experiments (test cases) per set of baseline structures</th>
<th>Employed sets of baseline structures</th>
<th>Aggregate inspection experiments (test cases)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>16</td>
<td>105</td>
<td>112</td>
<td>50</td>
<td>5 600</td>
</tr>
</tbody>
</table>

**Fig. 6.4.** Case study A: Selected U–cGM–AR method’s hyper-parameter κ (i.e. κ⋆) per employed set of baseline structures.

identification procedure that minimizes the BIC using Ordinary Least Squares (OLS) estimation (Matlab func.: *arx.m*) with QR implementation (see details in Vamvoudakis et al.[K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]). Then, the procedure described in section 6.3 is applied per set of baseline structures, the respective hyper-parameter κ⋆ is determined and its values per baseline structures set are presented in Figure 6.4. The κ⋆ values exhibit significant discrepancies per baseline structures set, hence indicating the method’s capability to adapt on the dynamics of each set. In addition, most of the selected κ⋆ values are significantly large, thus indicating a compact geometry for the underlying healthy subspace.

The AR(57) models identified and used for the U–cGM–AR method are also utilized for the training of the other five methods that are used for comparisons with the postulated method. At the end of their training, the corresponding hyper-parameters per method are selected as follows: (a) The hyper-parameters of the U–HS–AR method are automatically determined and the finally selected values per baseline beams set may be found in chapter 5; (b) all the above AR(57) models are used in the U–MM–AR method as the MM representation of the healthy subspace; (c) the critical (user selected) fraction γ of the U–PCA–AR method, which determines the subset of the first principal components with the greatest variability, is selected among three commonly used values, 90%, 95% and 99%, as the one yielding the best damage detection performance (that is 90%); (d) the number of the Gaussian Mixture components for the U–EM–GM–AR method is selected based on the Bayesian Information Criterion (BIC), with 20 to 33 components being selected among the sets of baseline structures (see
also Figure C.1.1); (e) the strength parameter of the U–DP–GM–AR method that controls the clustering procedure determining the GM components, is equal to 2 and it is selected as the value yielding the best performance in the range of values [1,20] (see Figure C.1.2).

Remark: The U–EM–GM–AR method requires a significantly high number of parameter vectors, so as to achieve a statistically reliable GM estimation, in such a high dimensional feature space and avoid singularity issues (see [K. J. Vamvoudakis–Stefanou and Fassois 2017]). In order to overcome this requirement, a number of additional parameter vectors is sampled from the Gaussian distribution of each one of the \( p \) original AR parameter vectors, following the procedure described in [K. J. Vamvoudakis–Stefanou and Fassois 2017] and yielding a total number of 89,250 vectors, which are used for the method’s training.

**Inspection phase**

The postulated U–cGM–AR method’s damage detection performance is presented in Figure 6.5 by means of box plots of its test pseudo–statistic \( D \). It is evident that the method’s damage detection performance is excellent for damages BL, BH, CH, and very good for damages AL, CL, DL, AH, DH where a slight overlap between the healthy and damaged state test pseudo–statistics is observed. The damage detection performance of the other five methods is also provided in Figure 6.5, indicating the superiority of the U–cGM–AR method over the U–PCA–AR, U–EM–GP–AR, and U–DP–GM–AR methods. On the other hand, the U–HS–AR and U–MM–AR methods achieve performance very close to that of the U–cGM–AR method.

The U–cGM–AR method’s performance is further illustrated in Figure 6.6, by means of the informative ROC curves, for the invisible and barely visible damages. The postulated method’s very good performance is therein verified, by achieving 96.5% TPR at 4% FPR for the invisible damage and 100% TPR at the impressive 0.68% FPR for the barely visible damage. The same figure provides respective curves for the U–HS–AR, U–MM–AR, and U–PCA–AR methods, which have been previously used for tackling the problem of damage detection for a population of nominally identical structures. These curves demonstrate the superiority of the U–cGM–AR method for the invisible as well as the barely visible damages, an observation additionally confirmed via the summarized (“best”) TPR and FPR results of Table 6.4 for all the methods.

Furthermore, the U–cGM–AR method is compared with the powerful U–EM–GM–AR and U–DP–GM–AR robust damage detection methods, by means of the ROC curves of Figure 6.7, for the invisible and barely visible damages. These depict that the U–cGM–AR method evidently outperforms the other two GM–based methods, a fact additionally demonstrated via the respective methods (“best”) TPR and FPR results of Table 6.4.

**6.4.3 Case Study B: Damage Detection for a Single Structure Under Varying EOCs**

This case study considers the problem of damage detection for a single structure under varying EOCs (see Table 6.5). Toward this end, the UAV tail structure described in section 6.4.1 is set–up into a freezer as depicted in Figure 6.8, where it operates under: (a) varying temperature in the range [0 –28°C], (b) varying tightening torque at Bolt A (Figures 6.8(a),(b)) simulating assembly uncertainty, and (c) material / manufacturing variability (resin thickness, fiber orientation and so on) simulated via the attachment of (reinforced with plastic mesh) adhesive tape, of two different sizes, at Point T (see Figure 6.8(b)).

Two simulated damage scenarios are considered in this case study, by attaching a proper mass at Point D on the surface of the composite beam (see Figure 6.8(b)). The first damage scenario, which is denoted as Damage 1, uses an added mass of 4.5g (0.41% mass increase), whereas the second damage scenario, denoted as Damage 2, uses a higher mass of 12.6g (1.15% mass increase). Both damage scenarios, along with all considered varying EOCs are presented in Table 6.6.

As in the previous case study, the structure is excited at its free end with a random white Gaussian force applied vertically at Point X (Figure 6.8(b)) via an electromechanical shaker, while the respective vibration response signal is measured at Point Y1 (see Figure 6.2 and Figure 6.1) using a lightweight

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Fig. 6.5. Case study A: Box plots of the methods test pseudo–statistics $D$ [the top and bottom of each box are the 25th and 75th percentiles; the red line in the middle of each box is sample median; the lines extending above and below each box are called whiskers and are drawn from the ends of the interquartile ranges, with length 1.5 times the interquartile range; the red crosses represent observations out of the whiskers range; the black horizontal line is defined by the top whisker of the healthy state and separates the healthy and damaged states].

accelerometer. Each signal is subsequently normalized via sample mean removal and division by its sample standard deviation. Details about the experiments used in the baseline and inspection phases of the methods, are available in Table 6.6.

The effects of uncertainty in the dynamics of the three healthy and the two damaged structural states presented in Table 6.6, are demonstrated via envelopes of Welch based Power Spectral Density (PSD) estimates in Figure 6.9. The envelopes construction is based on all the experiments of Table 6.6, Hamming windowing, segment length of 8 192 samples and 90% overlap (Matlab function: pwelch.m). Figure 6.9 shows that the envelopes of the healthy states are significantly overlapped with their small damage scenario (4.5g) counterpart, thus indicating a challenging damage detection problem. On the other hand, the bigger damage, of 12.6g mass, envelope exhibits discrepancies from its healthy counterparts, thereby indicating a less challenging problem.
6.4 Experimental Assessment and Comparisons

Fig. 6.6. Case Study A: Comparative assessment via ROC curves for the U–cGM–AR, the U–HS–AR, the U–MM–AR, and the U–PCA–AR ($\gamma = 90\%$) methods. (a) Invisible and (b) barely visible damages.

Fig. 6.7. Case Study A: Comparative assessment via ROC curves for the U–cGM–AR, the U–EM–GM–AR, and the U–DP–GM–AR methods. (a) Invisible and (b) barely visible damages.

Baseline phase

Table 6.6 provides the number of experiments employed for the postulated method’s training in the baseline phase. These are $p = 160$ vibration response signals, obtained from the structure under the sample EOCs provided in Table 6.6. An AR(41) model is identified by following the procedure of section 6.4.2 and using each acquired signal (with $N = 10,000$ samples or $2.15s$). The final AR model order is selected as the one that leads to the minimum BIC sample mean, obtained from all the AR models of the baseline phase. The parameter vectors extracted from these models are then employed for the training of the U–cGM–AR method based on the procedure of section 6.3. The postulated method’s baseline phase concludes with the automated estimation of its hyper–parameter $\kappa^*$ that takes the lowest possible value of 18, thereby indicating a complex healthy subspace geometry, requiring a model of high
### Chapter 6. A Crude Gaussian Mixture Model Based Method

#### Table 6.4: Damage detection performance in terms of the “best” TPR and FPR per method and case study.

<table>
<thead>
<tr>
<th>Case study</th>
<th>Method</th>
<th>TPR (%)</th>
<th>FPR (%)</th>
<th>TPR (%)</th>
<th>FPR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Invisible damages (5J impact)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U–cGM–AR</td>
<td><strong>96.5</strong></td>
<td>4</td>
<td>100</td>
<td><strong>0.68</strong></td>
</tr>
<tr>
<td></td>
<td>U–HS–AR</td>
<td>96.2</td>
<td>4</td>
<td>100</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>U–MM–AR</td>
<td>95.14</td>
<td>4</td>
<td>100</td>
<td>2.61</td>
</tr>
<tr>
<td></td>
<td>U–PCA–AR</td>
<td>71.36</td>
<td>5</td>
<td>99.86</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>U–EM–GM–AR</td>
<td>70.43</td>
<td>5</td>
<td>100</td>
<td>2.89</td>
</tr>
<tr>
<td></td>
<td>U–DP–GM–AR</td>
<td>69.86</td>
<td>5</td>
<td>100</td>
<td>2.75</td>
</tr>
<tr>
<td>B</td>
<td>Damage 1: Low mass damage (7% mass increase)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U–cGM–AR</td>
<td><strong>100</strong></td>
<td>1.2</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–HS–AR</td>
<td>93.33</td>
<td>4.92</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–MM–AR</td>
<td>7.69</td>
<td>5</td>
<td>99.49</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>U–PCA–AR</td>
<td>100</td>
<td>2.88</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–EM–GM–AR</td>
<td>100</td>
<td>2.2</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–DP–GM–AR</td>
<td>98.97</td>
<td>1.2</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>Damage 2: High mass damage (19.9% mass increase)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U–cGM–AR</td>
<td><strong>100</strong></td>
<td>1.2</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–HS–AR</td>
<td>93.33</td>
<td>4.92</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–MM–AR</td>
<td>7.69</td>
<td>5</td>
<td>99.49</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>U–PCA–AR</td>
<td>100</td>
<td>2.88</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–EM–GM–AR</td>
<td>100</td>
<td>2.2</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U–DP–GM–AR</td>
<td>98.97</td>
<td>1.2</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
</tbody>
</table>

#### Fig. 6.8. Case study B: Experimental set-up: Point X represents the force excitation position, Point Y1 the vibration acceleration measurement position, Point D the position where a mass is added as damage, Point T the position where an adhesive tape is attached, and Bolt A the bolt where tightening uncertainty is introduced at the clamping. [Aravanis, Kolovos, et al. 2018]

#### Table 6.5: Case Study B. Main characteristics [K. Vamvoudakis–Stefanou et al. 2021].

<table>
<thead>
<tr>
<th>Number of structures:</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage scenarios:</td>
<td>4.5 g or 12.6 g added mass at Point D</td>
</tr>
<tr>
<td>Uncertainty sources:</td>
<td>Temperature (0...28°C), Tightening torque (Bolt A; 1,2,3,4 Nm), Adhesive tape (small / large)</td>
</tr>
</tbody>
</table>
6.4 Experimental Assessment and Comparisons

Table 6.6: Case study B. Experimental details [K. Vamvoudakis–Stefanou et al. 2021].

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Uncertainty 1: Temperature (°C)</th>
<th>Uncertainty 2: Torque (Nm)</th>
<th>Number of Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>[0–28] with a step of 4°C</td>
<td>1,2,3,4</td>
<td>160♦</td>
</tr>
<tr>
<td><strong>Inspection (damage detection) phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>A* &amp; [3, 21]</td>
<td>1</td>
<td>45♦</td>
</tr>
<tr>
<td></td>
<td>A* &amp; [9, 19, 25]</td>
<td>2</td>
<td>50♦</td>
</tr>
<tr>
<td></td>
<td>A* &amp; {15, 25}</td>
<td>3</td>
<td>45♦</td>
</tr>
<tr>
<td></td>
<td>A* &amp; {3, 9, 15, 19, 21}</td>
<td>4</td>
<td>60♦</td>
</tr>
<tr>
<td>Healthy 1 (small tape)</td>
<td>B*</td>
<td>1,3,4</td>
<td>195♦</td>
</tr>
<tr>
<td>Healthy 2 (large tape)</td>
<td>B*</td>
<td>1,3,4</td>
<td>195♦</td>
</tr>
<tr>
<td>Damage 1 (4.5 g)</td>
<td>B*</td>
<td>1,3,4</td>
<td>195♦</td>
</tr>
<tr>
<td>Damage 2 (12.6 g)</td>
<td>B*</td>
<td>1,3,4</td>
<td>195♦</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td>980 (total)</td>
</tr>
</tbody>
</table>

A*: Set of experiments including [2–26]°C with a step of 4°C
B*: Set of experiments including {0, 1, 3, 7, 9, 10, 14, 15, 17, 21, 23, 25, 27}°C
♦ 5 experiments per temperature and torque values
Sampling frequency: \( f_s = 4654.5 \) Hz, Signal bandwidth: \([5 – 2327.25]\) Hz.
Signal length (damage detection): \( N = 10\,000\) samples (2.15 s).
Signal normalized via sample mean removal and division by its sample standard deviation.

and takes the value 99%; (d) as in the previous case study a significantly high number of parameter vectors is sampled yielding a total of 38400 vectors that are used for the U–EM–GM–AR method’s training, while 33 GM components are selected by mean of the BIC; (e) the strength parameter of the U–DP–GM–AR method that controls the clustering procedure determining the GM components, is equal to 5 (see Figure C.1.3).

**Inspection phase**

The postulated method’s performance is initially judged by means of its test pseudo–statistic \( D \), which is presented in terms of box plots for all the inspection experiments in Figure 6.10. These illustrate the U–cGM–AR method’s perfect detection performance when Damage 2 is considered, and its excellent performance when Damage 1 is considered instead. Such diagrams are additionally presented in Figure 6.10 for the other five methods, indicating the postulated method’s superiority for both damage scenarios and over all the other methods.

These observations become more evident through the ROC curves of Figure 6.11 and Figure 6.12, where the U–cGM–AR method achieves the impressive TPR of 100% at FPR 1.2% for Damage 1 (left subplot), and TPR of 100% with 0% FPR for Damage 2 (right subplot), thereby outperforming all the other methods. The methods “best” performance, in terms of TPR and FPR values, is additionally provided in Table 6.4, so as to further confirm the aforementioned observations.

6.4.4 Discussion and User Guidelines

The U–cGM–AR method is herein assessed based on two experimental case studies. Each case study considers different uncertainty sources that significantly affect the healthy dynamics, thereby yielding a challenging damage detection problem. These two case studies, feature two respective healthy subspaces of distinct geometries, with the one corresponding to Case Study B being more complex compared to that of Case Study A. The postulated method’s damage detection assessment through these two case studies, demonstrates its very good to excellent performance in both of them, thus indicating its robustness under
different uncertainty sources. Such robustness is additionally observed for the U–HS–AR method as well, yet its performance is significantly reduced compared to that of the U–cGM–AR method. On the other hand, the other methods show significant difficulties to effectively address both of the experimental case studies.

The postulated method’s robustness is attributed to the hyper–position of a collection of probabilistic hyper–spheres that may approximate any healthy subspace geometry. This collection may be seen as a crude Gaussian mixture model, featuring isotropic Gaussian components and a very simple estimation procedure that alleviates singularity issues, complex estimation procedures, and the requirement for a significantly high number of vibration response signals. In addition, the method’s hyper–parameters are automatically selected, thereby relieving the user from their burdensome selection.

A potential weakness of the U–cGM–AR method is the significant memory required, when a very high number (thousands) of vibration response signals is used in the baseline phase. In such a case, a data compression or dimensionality reduction strategy may be employed before the baseline phase, so as to reduce these requirements.

Fig. 6.9. Case Study B: Envelopes of Welch–based PSD estimates for the healthy and damaged structure under simulated damage of: (a) 4.5g (Damage 1) and (b) 12.6g (Damage 2) added mass [Healthy state: 360 experiments, Healthy 1 state: 195 experiments, Healthy 2 state: 195 experiments, Damage 1 / Damage 2: 195 / 195 experiments] [K. Vamvoudakis–Stefanou et al. 2021].
6.4 Experimental Assessment and Comparisons

![Box plots of the methods’ test pseudo–statistics $D$](image)

**Fig. 6.10.** Case study B: Box plots of the methods’ test pseudo–statistics $D$ [the top and bottom of each box are the 25th and 75th percentiles; the red line in the middle of each box is sample median; the lines extending above and below each box are called whiskers and are drawn from the ends of the interquartile ranges, with length 1.5 times the interquartile range; the red crosses represent observations out of the whiskers range; the black horizontal line is defined by the top whisker of the healthy state and separates the healthy and damaged states].
Fig. 6.11. Case Study B: Comparative assessment via ROC curves for the U–cGM–AR, the U–HS–AR, the U–MM–AR, and the U–PCA–AR (γ = 99%) methods. (a) Damage 1 and (b) Damage 2.

Fig. 6.12. Case Study B: Comparative assessment via ROC curves for the U–cGM–AR, the U–EM–GM–AR, and the U–DP–GM–AR methods. (a) Damage 1 and (b) Damage 2.
6.5 Conclusions

A novel, machine-learning-type, Unsupervised crude Gaussian Mixture based healthy subspace approximation method, using the AutoRegressive model parameters as representative features of the structural dynamics (U–cGM–AR method), is herein postulated for robust vibration-based damage detection under uncertainty. The method employs a crude Gaussian mixture model, with isotropic Gaussian distributions as its components, so as to represent the healthy state dynamics under varying Environmental and Operating Conditions (EOCs), as well as other potential uncertainty. This model is estimated via a conceptually and computationally simple procedure that relieves the user from critical selections, avoids complex optimization procedures, and requires a relatively low number of vibration response signals for its training.

The method has been assessed with respect to two experimental case studies, through comparisons with five powerful state-of-the-art robust methods and using numerous test cases. The main conclusions of the study are provided below:

1. The postulated U–cGM–AR method provides very good approximations of the two healthy subspace geometries herein considered, using relatively (with respect to the respective high dimensional feature spaces) low numbers (105 in case study A and 160 in case study B) of vibration response signals and without requiring any user judgement or selection for their training and implementation. Thus the method is judged as suitable for automated damage detection under various types of uncertainty.

2. The method’s experimental assessment that is based on vibration response signals obtained from a single accelerometer, under limited and low-frequency bandwidth, illustrates its very good damage detection performance and robustness.

3. The method’s comparison with a deterministic Hyper-Spheres-based, a Multiple Model, a PCA-based, and two Gaussian mixture model based methods, indicates its superiority over all the alternatives, in terms of damage detection performance and robustness to different sources of uncertainty.

4. Compared to the methods of the previous chapters, the postulated U–cGM–AR method yields remarkable overall performance, and the best performance when the barely visible damages are considered, as depicted in Figure 5.17. This performance is achieved while alleviating the limitations of the previous chapters methods, via low number of vibration response signals, automated training, and computational simplicity.

![Fig. 6.13. Comparison of the current chapter methods with their previous chapters counterparts, in terms of correct damage detection (True Positive) rate, versus two false alarm rates (FPR) of interest, for: (a) Invisible, and (b) Barely visible damage scenarios. [Based on the experiments of the benchmark application study (see chapter 2).]]
Chapter 7

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Conclusions

In this chapter the main conclusions for each one of the thesis’ four main chapters are provided. The chapter is then concluded by presenting further, relevant to the current thesis, issues that may be addressed in future studies.

7.1 Thesis Summary

The present thesis addresses the problem of vibration data–based damage detection for a population of nominally identical structures. The problem is herein properly defined and its importance explained. The challenges pertaining this problem, which are associated with uncertainty in the population structural dynamics, due to a combination of variability in the materials/manufacturing and environmental/operational conditions (EOCs), are investigated through a benchmark experimental application study.

This application study represents the first systematic attempt of experimentally addressing the problem and the respective vibration response signals constitute a valuable data–base that may be used as benchmark for future studies. The study employs a population sample of 31 nominally identical composite aerostructures, featuring significant material, manufacturing, and assembly variability that affects their dynamics. Each population member represents one of the tail booms of a commercial twin tail Unmanned Aerial Vehicle (UAV), while the considered damage scenarios are characterized by a combination of delamination, small cracks, and broken fibres, caused by impact at two distinct energy levels.

The benchmark application study is formally presented in chapter 2, along with the corresponding uncertainty and damage effects in the structural dynamics. Then, the damage detection challenges, stemming from the uncertainty effects in the dynamics, are illustrated in chapter 3, by means of conventional statistical time series based methods, as well as a well–known PCA based method. The methods limitations are therein discussed and proper Multiple Model (MM) alternatives are proposed. The most prominent among these methods is a PCA enhanced MM based method that achieves very good damage detection performance, under the significant uncertainty effects of the considered application study.

Yet, this performance may be achieved, provided that proper hyper–parameters selections, requiring user judgement and experience, are made in the method’s training phase. In order to address this limitation, a supervised version of the PCA–enhanced MM based method is postulated in chapter 4. This method
Chapter 7. Conclusions

employs vibration response signals from healthy as well damaged structures in its training phase, so as to automatically determine its hyper–parameters. The respective damage detection results, for the herein considered population of nominally identical structures, indicate that the supervised method yields improved performance compared to all or some of its unsupervised counterparts, while it also relieves the user from critical selections during the training phase.

Of–course, damaged structures are typically not available during the methods training phase, therefore a novel unsupervised method is postulated instead in chapter 5. This method tackles the problem, under fully automated training that uses vibration response signals solely from structures in their healthy state. Its idea is based on representing the healthy state dynamics, within a properly defined feature space, through the union of a number of deterministic hyper–spheres with distinct centres and radii. The method’s damage detection performance is assessed via two experimental case studies featuring various uncertainty sources and distinct “healthy subspace” geometries. The first case study pertains to damage detection for a population of nominally identical composite aerostructures, and is the one presented in the previous chapters. The second case study pertains to damage detection on a single composite aerostructure subject to damage scenarios consisting of small, local, stiffness reduction (simulated by the attachment of small masses) under uncertainty related to varying temperature and tightening assembly torque. The method is additionally compared with three well known robust damage detection methods, with the assessment results indicating the postulated method’s excellent detection performance that is superior to that of the alternative methods and robustness to the various uncertainty sources.

The method postulated in chapter 5, although tackles the limitations of the previous methods and most of the state–of–the–art limitations, features increased computational complexity. This issue is tackled in chapter 6, where a novel crude Gaussian mixture model based method is postulated for tackling the herein posed damage detection problem. The method’s probabilistic framework, along with a simplified estimation procedure, significantly reduce the computational and conceptual complexity compared to the deterministic hyper–spheres based method of the previous chapter. The method’s main idea pertains to the approximation of the previously mentioned healthy subspace, through the superposition of a number of isotropic Gaussian distributions, with distinct, properly defined, means and covariances. The method’s detection performance is assessed by means of the two experimental case studies of the previous chapter and through comparisons with five powerful state–of–the–art methods. The respective results demonstrate the postulated method’s excellent performance, outperforming its hyper–sphere based predecessor from the previous chapter, as well as all the other methods, while exhibiting robustness to the various uncertainty sources.

7.2 Thesis Concluding Remarks

The current thesis may be considered the first systematic experimental exploration of the damage detection for a population of nominally identical structures problem. The underlying challenges and limitations with respect to the state of the art are experimentally illustrated, while viable solutions are provided. The main conclusion of the thesis, may be summarised as follows:

• The problem of response–only random vibration–based damage detection for a population of nominally identical structures is for the first time systematically explored and addressed, by means of a benchmark experimental application study and herein postulated novel methods.

• The benchmark experimental application study is based on a population sample consisting of 31 nominally identical composite aerostructures with significant uncertainty in their dynamics and 8 impact induced damage scenarios, with either invisible or barely visible type of damage.

• The herein postulated methods effectively tackle the problem and overcame the state of the art barriers by featuring:
  – Low number of training vibration response signals (105 used in the present thesis)
  – Automated training that does not require user involvement.
  – Conceptual and computational simplicity, through convex optimization procedures.
7.3 Future Work Suggestions

– Robustness under any Healthy Subspace geometry.

• High damage detection performance (see summarised performance results in Figure 7.1) is achieved using only a single sensor and limited frequency bandwidth, without measuring the uncertainty sources. Particularly, in the case of the barely visible damages, which is of higher interest (compared to the invisible damages), since it may require further maintenance actions, the herein postulated methods achieve remarkable performance that may reach up to 99.93% correct damage detection rate, for only 0.5% false alarm rate.

• The herein postulated methods, significantly outperform three powerful state of the art methods as illustrated in Figure 7.1.

![Figure 7.1. Comparison of the postulated in the current thesis methods (methods inside the dotted box) to one another, as well as with three state of the art methods (methods outside the dotted box), in terms of correct damage detection (True Positive) rate, versus two false alarm rates (FPR) of interest, for the: (a) Invisible, and (b) Barely visible damage scenarios. [Based on the experiments of the benchmark application study (see chapter 2).]](image)

7.3 Future Work Suggestions

The present thesis effectively addresses the vibration-based damage detection for a population of nominally identical structures problem and sets the groundwork for addressing the population-based Structural Health Monitoring (SHM) problem. Toward this end, a number of important issues need to be overcome, thus giving rise to respective research topics, which are described below:

• The damage detection for a population of nominally identical structures problem is herein addressed using a single experimental application study, which although exhaustive (involving numerous vibration experiments), it is not sufficient to completely investigate the problem. Therefore, a number of alternative experimental case studies, preferably using real life structures and higher number of population samples, is required.

• Toward the commercial utilization of the herein postulated methods, the proper automated selection of the postulated methods decision making thresholds needs to be addressed. Although, some user guidelines are herein provided for the thresholds selection, these may not feature automation or optimal damage detection performance for each method. Therefore, this would be a very interesting and important future research topic.

• In addition, the postulated methods are formulated to address the problem of damage detection under non-measurable uncertainty sources. Of course, these methods may be also employed when measurable uncertainty sources are acting on the structure. Yet, in the latter case, the additional information stemming from the uncertainty sources measurement may be also incorporated within the
methods, via a cause and effect type of model, so as to improve the damage detection performance. Thus, another research topic is the development of methods that separate the uncertainty effects on the dynamics into those stemming from measurable and those from non–measurable sources. The former effects are then represented via a regression model or a polynomial function and the rest via the herein postulated methods. The resulting method represents the Healthy subspace via a combination of deterministic and probabilistic functions, with the former’s points corresponding to a distinct measurable EOC, and the latter function yielding a non–Gaussian probability defined around each such point.

- As indicated in various chapters of the current thesis, the methods damage detection effectiveness depends on their training through a set of sample nominally identical structures that properly represent the respective population dynamics. Thus, a procedure that guarantees the selection of such a proper set for the methods training phase, under non–measurable uncertainty sources, is of crucial importance.

- The PCA–enhanced method postulated in chapter 3, achieves very high damage detection performance, yet an unsupervised and automated procedure for the determination of the PCA features dimensionality that yields such a performance is not presently available. A research toward this end would be of high importance not only for the herein postulated methods, but for a multitude of scientific topics where PCA may be used.

- In chapter 4 two versions of the PCA technique are investigated, one standardized (centred) and another non–standardized (non–centred). In the latter case, the postulated PCA–enhanced MM based method achieves significantly improved performance, thus raising the questions: (a) Why this performance difference is observed? (b) How does the standardization affects the resulting Healthy Subspace approximation? (c) When should each PCA version be selected for optimal damage detection performance?

- The methods postulated in chapter 5 and chapter 6, incorporate a standardization that takes into account the AR model parameters estimation uncertainty. This standardization, although only crudely accounts for the aforementioned uncertainty, leads to improved damage detection performance. The properties of this standardization, its limitations, and underlying assumptions may be further investigated.

- In order to further improve the damage detection performance, the AR parameters estimation uncertainty may be properly incorporated into the probabilistic model of chapter 6, possibly via a Bayesian framework.

- The methods postulated in chapter 5 and chapter 6 are based on instance based representations and may be continuously updated as more data records from the healthy state of the structures become available, thereby improving the resulting Healthy Subspace approximation. Of course, the representation size may become significantly large, thus increasing the methods computational complexity. In order to avoid such a scenario, a subset of the training data records that best represent the population structural dynamics should be used instead. Toward this end, an investigation of clustering techniques or similarity measures is of high interest, in order to eliminate the redundant records.
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Damage detection methods:

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>S–nPCA–MM–AR</td>
<td>Supervised non-centred Principal Component Analysis (PCA) enhanced Multiple Model (MM) AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>S–PCA–AR</td>
<td>Supervised Principal Component Analysis (PCA) enhanced AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>S–PCA–MM–AR</td>
<td>Supervised centred Principal Component Analysis (PCA) enhanced Multiple Model (MM) AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>U–AR</td>
<td>Unsupervised AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>U–cGM–AR</td>
<td>Unsupervised crude Gaussian Mixture (cGM) AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>U–HS–AR</td>
<td>The Unsupervised Hyper–Sphere AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>U–MM–AR</td>
<td>Unsupervised Multiple Model (MM) AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>U–nPCA–MM–AR</td>
<td>Unsupervised non-centred Principal Component Analysis (PCA) enhanced Multiple Model (MM) AutoRegressive (AR) parameter based method</td>
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<tr>
<td>U–PCA–AR</td>
<td>Unsupervised Principal Component Analysis (PCA) enhanced AutoRegressive (AR) parameter based method</td>
</tr>
<tr>
<td>U–PCA–MM–AR</td>
<td>Unsupervised centred Principal Component Analysis (PCA) enhanced Multiple Model (MM) AutoRegressive (AR) parameter based method</td>
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</table>
## ACRONYMS

### Additional acronyms:

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>Akaike Information Criterion</td>
</tr>
<tr>
<td>AR</td>
<td>AutoRegressive model</td>
</tr>
<tr>
<td>ARX</td>
<td>AutoRegressive with eXogenous excitation model</td>
</tr>
<tr>
<td>AUC</td>
<td>Area Under the ROC Curve</td>
</tr>
<tr>
<td>BIC</td>
<td>Bayesian Information Criterion</td>
</tr>
<tr>
<td>EOCs</td>
<td>Environmental and Operational Conditions</td>
</tr>
<tr>
<td>FPR</td>
<td>False Positive Rate (in ROC curves)</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>GM</td>
<td>Gaussian Mixture</td>
</tr>
<tr>
<td>HS</td>
<td>Hyper–Sphere</td>
</tr>
<tr>
<td>KL</td>
<td>Kullback–Leibler (divergence)</td>
</tr>
<tr>
<td>MM</td>
<td>Multiple Model</td>
</tr>
<tr>
<td>OC</td>
<td>Operating Condition</td>
</tr>
<tr>
<td>PA</td>
<td>Performance Assessment</td>
</tr>
<tr>
<td>PCA</td>
<td>Principal Component Analysis</td>
</tr>
<tr>
<td>RC</td>
<td>Random Coefficient</td>
</tr>
<tr>
<td>ROC</td>
<td>Receiver Operating Characteristic curve</td>
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<tr>
<td>RSS</td>
<td>Residual Sum of Squares</td>
</tr>
<tr>
<td>SA</td>
<td>Sensitivity Analysis</td>
</tr>
<tr>
<td>SHM</td>
<td>Structural Health Monitoring</td>
</tr>
<tr>
<td>SPP</td>
<td>Samples Per Parameter</td>
</tr>
<tr>
<td>SSS</td>
<td>Signal Sum of Squares</td>
</tr>
<tr>
<td>TPR</td>
<td>True Positive Rate (in ROC curves)</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
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## Important Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(a_i)</td>
<td>(i)-th AR model (scalar) parameter</td>
</tr>
<tr>
<td>({\mathbf{b}})</td>
<td>set of all the (\mathbf{a}<em>i, \mathbf{\theta}</em>{i,j},) and (\mathbf{\tilde{\theta}}_{i,j}) vectors; (\mathbf{b}_j) designates the set’s (j)-th element</td>
</tr>
<tr>
<td>(\mathbf{C})</td>
<td>matrix containing the Euclidean distances between all possible pairs of (\mathbf{a}<em>i)’s; (c</em>{i,j}) its elements</td>
</tr>
<tr>
<td>(d)</td>
<td>Kullback–Leibler divergence (distance metric)</td>
</tr>
<tr>
<td>(d(m_u, m_{o,k}))</td>
<td>distance between the models (m_u) and (m_{o,k})</td>
</tr>
<tr>
<td>(D)</td>
<td>Distance between a model and a MM representation</td>
</tr>
<tr>
<td>(e[t])</td>
<td>model residual</td>
</tr>
<tr>
<td>(k)</td>
<td>hyper–parameter of the U–HS–AR method determining the hyper–spheres’ center and affecting their radius</td>
</tr>
<tr>
<td>(\mathcal{K})</td>
<td>number of isotropic Gaussian component</td>
</tr>
<tr>
<td>(\mathbf{L})</td>
<td>loading matrix used in the non–centred PCA</td>
</tr>
<tr>
<td>(l_{lim})</td>
<td>damage detection threshold</td>
</tr>
<tr>
<td>(l_p)</td>
<td>threshold for assigning “healthy” or “damaged” pseudo–labels</td>
</tr>
<tr>
<td>(m)</td>
<td>number of the last eigenvectors of (\mathbf{P}) that are associated with damage</td>
</tr>
<tr>
<td>(m_o)</td>
<td>multiple model representation of the healthy structural dynamics</td>
</tr>
<tr>
<td>(m_{o,k})</td>
<td>(k)-th AR model representing the healthy structural state</td>
</tr>
<tr>
<td>(m_u)</td>
<td>AR model corresponding to an unknown structural state</td>
</tr>
<tr>
<td>(N)</td>
<td>signal length (in samples)</td>
</tr>
<tr>
<td>(n)</td>
<td>AR model order</td>
</tr>
<tr>
<td>(p)</td>
<td>number of signals obtained from (\nu) healthy structures in the baseline phase</td>
</tr>
<tr>
<td>(p_d)</td>
<td>number of signals obtained from (\nu_d) damaged structures in the baseline phase</td>
</tr>
<tr>
<td>(p_o)</td>
<td>number of signals obtained from (\nu_o) healthy structures in the baseline phase</td>
</tr>
<tr>
<td>(\mathbf{P})</td>
<td>sample non–central or central second moment matrix associated with ({\mathbf{a}_o}) (multiple model)</td>
</tr>
<tr>
<td>(q)</td>
<td>minimum number of the largest eigenvalues of (\mathbf{P}) that cumulatively explain (\gamma) (%) of the transformed parameter vector variability</td>
</tr>
</tbody>
</table>
**IMPORTANT SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>radius of the “reference” hyper–sphere</td>
</tr>
<tr>
<td>$s_i$</td>
<td>scaling factor used for the $\theta_{i,j}$’s determination</td>
</tr>
<tr>
<td>$y[t]$</td>
<td>vibration response signal</td>
</tr>
<tr>
<td>${\alpha_j}$</td>
<td>set of healthy state AR model parameter vectors from the baseline phase used for the estimation of a single isotropic Gaussian component</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>the $c$–th nearest to $\alpha_u$, AR model parameter vector</td>
</tr>
<tr>
<td>$\alpha_{c,j}$</td>
<td>the $c$–th AR model parameter vector included in the set ${\alpha_j}$ of healthy state vectors from the baseline phase</td>
</tr>
<tr>
<td>$\alpha_{d,j}$</td>
<td>AR model parameter vector corresponding to the $j$–th damaged structure of the baseline phase</td>
</tr>
<tr>
<td>${\alpha_o}$</td>
<td>set of AR model parameter vectors corresponding to the healthy Multiple Model $m_o$</td>
</tr>
<tr>
<td>$\alpha_o$</td>
<td>AR model parameter vector corresponding to a healthy structure</td>
</tr>
<tr>
<td>$\alpha_{o,j}$</td>
<td>AR model parameter vector corresponding to the $j$–th healthy structure of the baseline phase</td>
</tr>
<tr>
<td>$\alpha_u$</td>
<td>AR model parameter vector corresponding to an unknown state structure</td>
</tr>
<tr>
<td>$\bar{\alpha}_{k,j}$</td>
<td>the centroid (arithmetic mean) of the $k$, closest to $\bar{b}_{j}$, vectors $\alpha_i$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>pre–specified percentage of the parameter vector variability</td>
</tr>
<tr>
<td>$\delta$</td>
<td>maximum allowed fraction of “healthy” labeled $\theta_{i,j}$ vectors</td>
</tr>
<tr>
<td>$\theta_{i,j}$</td>
<td>artificial vectors generated via sampling from all $N(\alpha_i, \Sigma_i)$ distributions</td>
</tr>
<tr>
<td>$\bar{\theta}_{i,j}$</td>
<td>properly manipulated $\theta_{i,j}$ vectors</td>
</tr>
<tr>
<td>$\kappa$</td>
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</tr>
<tr>
<td>$\kappa^*$</td>
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</tr>
<tr>
<td>$\mu_{\alpha}$</td>
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<td>$\mu_{\kappa,j}$</td>
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<tr>
<td>$\nu$ or $\nu_o$</td>
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</tr>
<tr>
<td>$\nu_d$</td>
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</tr>
<tr>
<td>$\rho$</td>
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</tr>
<tr>
<td>$\rho_{k,j}$</td>
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<tr>
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<tr>
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</tr>
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A.1 The Signals and their Non-parametric Analysis

Figure A.1.1 presents an indicative pair of input/excitation (force) - output/response (acceleration) signals along with respective zooms in an indicative time range. It is evident that both signals have constant mean and variance over time.

The response signal’s Welch based PSD estimate is presented in Figure A.1.2(a) depicting 11 main peaks, hence indicating that the structure has got at least 11 resonance frequencies. This is additionally depicted by Figure A.1.2(c) in terms of the Welch based FRF magnitude estimate. The whiteness of the force excitation signal is shown in Figure A.1.2(b) via the respective Welch based PSD estimate, while the linear relationship between the excitation-response signals is shown in Figure A.1.2(d) in terms of the coherence function.
Fig. A.1.1. Indicative input/excitation (a) and output/response (b) signals, along with their zoomed counterparts (c),(d) respectively [single experiment, single beam].
Fig. A.1.2. Welch-based PSD estimate of the response (a) and excitation signals (b), along with their Welch-based FRF magnitude (c) and coherence estimates (d) [single experiment, single beam].
A.2 Parametric Analysis

Figure A.2.1(a) depicts the ’optimal’ AR model order per beam according to the minimum of the BIC, with the highest ’optimal’ value being 77. Figure A.2.1(b) shows the sample mean BIC value per AR model order and for all the healthy beams which achieves minimum for order 59. The respective values are shown in Figure A.2.1(c) for the 5 model orders employed for assessing the damage detection methods’ sensitivity to model order selection. It is evident that the minimum sample mean BIC is achieved for AR(57).

Figure A.2.2 depicts the residuals AutoCorrelation Function (ACF) for various AR model orders corresponding to a single indicative vibration response signal. It is evident that all the models exhibit some autocorrelation in the first lags due to absence of a Moving Average (MA) model part. This issue is not tackled by increasing the model order up to 97 and requires a significantly higher order. However, such high orders require a significantly higher number of experiments in the baseline phase of the PCA–enhanced methods that of course is usually impossible. Nevertheless, it is evident that model orders above 47 have only 5% of their ACF values lying outside the 95% confidence bounds.

Figure A.2.3 depicts the AR model residuals normality via normal probability plots (Matlab function: normplot.m). This depicts that the residuals are white for any model order.
Fig. A.2.1. AR model order selection based on BIC: (a) AR model order per beam corresponding to the minimum BIC value, (b) sample mean BIC and respective standard deviation of all the healthy beams per model order, (c) sample mean BIC of all the healthy beams for selected orders [in (b),(c) the curve (or points) in the middle correspond(s) to the sample mean BIC value, while the length of the vertical lines (errorbars) equals two times the standard deviation].
Fig. A.2.2. Indicative ACF of the residual sequence obtained using model: (a) AR(47), (b) AR(57), (c) AR(67),
(d) AR(77), (e) AR(87), (f) AR(97), [single experiment, single beam; the horizontal blue lines designate the 95%
confidence bounds].
Fig. A.2.3. Indicative normal probability plot of the residual sequence obtained using model: (a) AR(47), (b) AR(57), (c) AR(67), (d) AR(77), (e) AR(87), (f) AR(97), [single experiment, single beam].
Appendix A: Chapter 3 Appendix

A.3 Unsupervised Principal Components Selection

Table A.3.1 presents the numbers \( m \) and \( m^* \) of used principal components for the U–PCA–AR and U–PCA–MM–AR methods respectively. These numbers are presented per each combination of \( \gamma \) and \( \nu \) values, depicting that increasing \( \nu \) while retaining constant \( \gamma \) does not affect the number of used principal components as much as increasing \( \gamma \) while retaining constant \( \nu \). In addition, for any common combination of \( \gamma \) and \( \nu \) values between the two PCA–enhanced methods, the U–PCA–MM–AR method yields the highest number of used principal components. The latter is observed due to the fact that the U–PCA–MM–AR method employs the non–centred PCA, while its ‘conventional’ counterpart the centred one.

Table A.3.1: Number of principal components used for damage detection per method and set of design parameters.

<table>
<thead>
<tr>
<th>( \gamma(%) )</th>
<th>( \nu = 9 )</th>
<th>( \nu = 11 )</th>
<th>( \nu = 13 )</th>
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<tr>
<td>70</td>
<td>49 − 52</td>
<td>49 − 51</td>
<td>49 − 51</td>
<td>49 − 51</td>
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<tr>
<td>80</td>
<td>46 − 50</td>
<td>45 − 51</td>
<td>46 − 48</td>
<td>45 − 47</td>
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<td>99.6</td>
<td>17 − 21</td>
<td>14 − 17</td>
<td>13 − 15</td>
<td>12 − 14</td>
</tr>
<tr>
<td>99.9</td>
<td>10 − 13</td>
<td>8 − 10</td>
<td>6 − 8</td>
<td>5 − 7</td>
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<th>( \gamma(%) )</th>
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<th>( \nu = 11 )</th>
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<td>40 − 42</td>
<td>38 − 40</td>
<td>37 − 38</td>
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* (−) − (:): ‘minimum – maximum’ number of used principal components among the 50 sets of baseline beams

A.4 MM Representations Interpreted As Gaussian Mixture Models

Let \( f_{o1}(x) = N(x|\mu_{o1}, \Sigma_{o1}) \), \( f_{o2}(x) = N(x|\mu_{o2}, \Sigma_{o2}) \), ... , \( f_{oK}(x) = N(x|\mu_{oK}, \Sigma_{oK}) \) designate the Probability Density Functions (PDFs) of \( K \) multivariate normal distributions, with \( \mu, \Sigma \) designating their mean and covariance matrix respectively and \( x \) the feature vector (e.g. AR model parameter vector). Let also these distributions be the components of a GMM, approximating the healthy state damage features distribution under uncertain environmental/operating conditions. The healthy state GMM PDF is defined as [Bishop 2006, p. 111]:

\[
    f_o(x) = \sum_{k=1}^{K} w_k N(x|\mu_{ok}, \Sigma_{ok}), \quad \sum_{k=1}^{K} w_k = 1, \quad 0 \leq w_k \leq 1
\]  

(A.4.1)

with \( w_k \) designating the mixing coefficients (weights) of the respective Gaussian components (Gaussian distributions of the GMM).
A GMM with 2 components is then defined as follows:

\[ f_o(x) = w_1 f_{o1}(x) + w_2 f_{o2}(x) \]  
(A.4.2)

with \( w_1, w_2 \) the GMM weights (obviously \( w_1 + w_2 = 1 \)) and \( f_{o1}(x), f_{o2}(x) \) the respective GMM components.

Let also \( f_u(x) = N(x|\mu_u, \Sigma_u) \) a new PDF corresponding to an unknown structural state.

**Remark:** The PDFs \( f_{o1}(x), f_{o2}(x), f_o(x), f_u(x) \) are used in the sequel as \( f_{o1}, f_{o2}, f_o, f_u \) respectively, for reasons of simplicity.

In order to determine whether the current structure is healthy, the PDFs \( f_u \) and \( f_o \) are compared via the KL divergence. The KL divergence measures the average additional information (in nats) needed to specify a value of \( x \) that follows the distribution \( f_o \) when the distribution \( f_u \) is used instead [Bishop 2006, p. 55]. This may be additionally seen as a generalized asymmetric distance metric (pseudo-distance metric) between the two distributions that is defined as:

\[ KL(f_o||f_u) = \int f_o \ln \frac{f_o}{f_u} dx \]  
(A.4.3)

By substituting \( f_o \) of Equation A.4.2 in Equation A.4.3 the following is obtained:

\[
KL(f_o||f_u) = \int (w_1 f_{o1} + w_2 f_{o2}) \ln \frac{w_1 f_{o1} + w_2 f_{o2}}{f_u} dx \\
= \int (w_1 f_{o1} + w_2 f_{o2}) (\ln(w_1 f_{o1} + w_2 f_{o2}) - \ln f_u) dx \\
= \int w_1 f_{o1} \ln(w_1 f_{o1} + w_2 f_{o2}) + w_2 f_{o2} \ln(w_1 f_{o1} + w_2 f_{o2}) - w_1 f_{o1} \ln f_u - w_2 f_{o2} \ln f_u \\
(A.4.4)
\]

Another pseudo–test statistic for determining the current structural state is the weighted sum of KL divergences between each GMM component and the unknown state PDF, that is:

\[
W\text{SumKL}(f_o||f_u) = w_1 KL(f_{o1}||f_u) + w_2 KL(f_{o2}||f_u) \\
= w_1 \int f_{o1} \ln \frac{f_{o1}}{f_u} dx + w_2 \int f_{o2} \ln \frac{f_{o2}}{f_u} dx \\
= \int w_1 f_{o1} (\ln f_{o1} - \ln f_u) dx + \int w_2 f_{o2} (\ln f_{o2} - \ln f_u) dx \\
= \int w_1 f_{o1} \ln f_{o1} - w_1 f_{o1} \ln f_u + w_2 f_{o2} \ln f_{o2} - w_2 f_{o2} \ln f_u dx \\
(A.4.5)
\]

It is evident that the terms affected by the unknown state PDF \( (\int -w_1 f_{o1} \ln f_u - w_2 f_{o2} \ln f_u dx) \) are common between Equation A.4.4 and Equation A.4.5. Thus, the two pseudo–test statistic yield constant discrepancy when different \( f_u \) distributions are tested as shown below:

\[
KL(f_o||f_u) - W\text{SumKL}(f_o||f_u) = \int w_1 f_{o1} \ln f_{o1} + w_1 f_{o1} \ln(w_1 f_{o1} + w_2 f_{o2}) + w_2 f_{o2} \ln f_{o2} + w_2 f_{o2} \ln(w_1 f_{o1} + w_2 f_{o2}) dx \\
(A.4.6)
\]

Therefore, whichever pseudo–test statistic is employed for damage detection, the same performance results are obtained.

In the special case of \( w_1 = w_2 \), Equation A.4.5 becomes equal to the mean of the KL divergences, that is:

\[
\text{SumKL} = \frac{1}{2} (KL(f_{o1}||f_u) + KL(f_{o2}||f_u)) \\
(A.4.7)
\]
This is in fact proportional to the test pseudo–statistic defined in the U–MM–AR method for damage detection (Equation 3.14). Thus, based on the above–mentioned statements the U–MM–AR method’s test pseudo–statistic is equivalent (yielding the same performance results) to the KL divergence between a GMM with (uniformly weighted) components the baseline phase AR model parameter vectors’ PDFs, and the Gaussian PDF of the current unknown state model parameter vector.

Remark: It is evident that these statements can generalize to any GMM with $K$ components.

A.5 Geometric Interpretation of MM–based Methods

In this section Equation 3.14 of the U–MM–AR method takes the following form:

$$d(m_{o,k}, m_u) := (\alpha_{o,k} - \alpha_u)^T \Sigma_{o,k}^{-1} (\alpha_{o,k} - \alpha_u)$$  \hspace{1cm} (A.5.1)

Hence, the KL divergence is replaced by the Mahalanobis squared distance. This is an alternative version of the method that assumes common covariance matrix for both parameter vectors compared via the distance metric. Although, this assumption may sometimes be inaccurate it provides some useful geometric interpretations for the method.

Let two baseline healthy parameter vectors $\alpha_{o,1}$, $\alpha_{o,2}$ with covariance matrices $\Sigma_{o,1}$ and $\Sigma_{o,2}$ respectively, and a parameter vector $\alpha_u$ of unknown state with covariance matrix $\Sigma_u$ ($\Sigma_u$ is assumed equal to one of the covariances $\Sigma_{o,1}$, $\Sigma_{o,2}$). Then the Mahalanobis squared distance between the parameter vectors $\alpha_{o,1}$ and $\alpha_u$ is defined as follows [Rencher et al. 2012, p. 85]:

$$d(m_{o,1}, m_u) = (\alpha_{o,1} - \alpha_u)^T \Sigma_{o,1}^{-1} (\alpha_{o,1} - \alpha_u) = ||\Sigma_{o,1}^{-1/2} (\alpha_{o,1} - \alpha_u)||^2_{l_2}$$  \hspace{1cm} (A.5.2)

Then, according to Equation 3.14 the pseudo–statistic between the unknown and the two baseline healthy vectors is the following:

$$D = ||\Sigma_{o,1}^{1/2} (\alpha_{o,1} - \alpha_u)||^2_{l_2} + ||\Sigma_{o,2}^{1/2} (\alpha_{o,2} - \alpha_u)||^2_{l_2} =$$

$$||\Sigma_{o,1}^{1/2} \alpha_{o,1} - \Sigma_{o,1}^{1/2} \alpha_u||^2_{l_2} + ||\Sigma_{o,2}^{1/2} \alpha_{o,2} - \Sigma_{o,2}^{1/2} \alpha_u||^2_{l_2}$$  \hspace{1cm} (A.5.3)

Of course in the damage detection context any expression proportional to the right side of Equation A.5.3 yields identical damage detection performance results, since the $D$ values obtained from different vibration experiments are compared to each other in terms of inequalities. Obviously any inequality is not affected when both sides are multiplied by the same constant value. Therefore, Equation A.5.3 may be replaced by the respective sample mean of the squared Euclidean distances as:

$$D = \frac{1}{2} (||\Sigma_{o,1}^{-1/2} \alpha_{o,1} - \Sigma_{o,1}^{-1/2} \alpha_u||^2_{l_2} + ||\Sigma_{o,2}^{-1/2} \alpha_{o,2} - \Sigma_{o,2}^{-1/2} \alpha_u||^2_{l_2})$$  \hspace{1cm} (A.5.4)

Equation A.5.4 is of course based on the assumption that $\Sigma_{o,1} = \Sigma_{o,2}$, since the two Mahalanobis squared distances assume that $\Sigma_{o,1} = \Sigma_u$ and $\Sigma_{o,2} = \Sigma_u$ respectively. Therefore, for the rest of this section all the covariance matrices are designated as $\Sigma$ for simplicity.

Let now for reasons of mathematical convenience, the simple case of two dimensional parameter vectors and the following transformations:

$$a = \Sigma^{-1/2} \alpha_{o,1}, \quad a = [a_1 \ a_2]^T$$

$$\beta = \Sigma^{-1/2} \alpha_{o,2}, \quad x = [\beta_1 \ \beta_2]^T$$

$$c = \Sigma^{-1/2} \alpha_u, \quad c = [c_1 \ c_2]^T$$  \hspace{1cm} (A.5.5)
Then using Equation A.5.4 and Equation A.5.5:

\[
D = \frac{1}{2} \left( \| \mathbf{a} - \mathbf{c} \|_2^2 + \| \mathbf{b} - \mathbf{c} \|_2^2 \right) = \frac{1}{2} \left[ (a_1 - c_1)^2 + (a_2 - c_2)^2 + (\beta_1 - c_1)^2 + (\beta_2 - c_2)^2 \right] = \\
= \frac{1}{2} \left[ (a_1^2 + a_2^2) + (\beta_1^2 + \beta_2^2) + 2(c_1^2 + c_2^2) - 2a_1c_1 - 2a_2c_2 - 2\beta_1c_1 - 2\beta_2c_2 \right] = \\
= \frac{1}{2} \left[ \| \mathbf{a} \|^2 + \| \mathbf{b} \|^2 + 2(\| \mathbf{c} \|^2 - a_1c_1 - a_2c_2 - \beta_1c_1 - \beta_2c_2) \right] 
\]

Terms determining damage detection result

(A.5.6)

Evidently, only the terms containing variables \( c_1 \) and \( c_2 \) affect the damage detection result, since the variables \( a_1, a_2, \beta_1 \) and \( \beta_2 \) remain constant over different experiments.

An alternative to Equation A.5.4 is the Mahalanobis distance between the unknown parameter vector and the sample mean of the baseline healthy parameter vectors, as expressed below:

\[
D' = \frac{1}{2} \left( \| \mathbf{a} + \mathbf{b} \|_2 - \mathbf{c} \right)^2 = \frac{1}{4} \left[ (a_1 + \beta_1 - 2c_1)^2 + (a_2 + \beta_2 - 2c_2)^2 \right] = \\
= \frac{1}{4} \left[ (a_1 - c_1 + \beta_1 - c_1)^2 + (a_2 - c_2 + \beta_2 - c_2)^2 \right] = \\
= \frac{1}{4} \left[ (a_1 - c_1)^2 + (\beta_1 - c_1)^2 + 2(a_1 - c_1)(\beta_1 - c_1) + \\
+ (a_2 - c_2)^2 + (\beta_2 - c_2)^2 + 2(a_2 - c_2)(\beta_2 - c_2) \right] \text{ Terms determining damage detection result} 
\]

(A.5.7)

Through comparison of Equation A.5.6 and Equation A.5.7 it becomes evident that both equations have identical terms for determining the damage detection result (this observation of course generalizes to parameter vectors of higher dimensionality as well). Thus, the two pseudo–test statistics are equivalent in terms of detection performance, yielding identical results. Therefore, when all the parameter vectors obtained from structure(s) under various environmental/operating conditions have the same (or similar) covariance matrix (representing estimation uncertainty), then damage detection is tackled by comparing the distance metric between the unknown state vector and the sample mean of the healthy state vectors to a user defined threshold. Geometrically, this means that the healthy state parameter vectors’ subspace is represented by a hypersphere with mean the standardized baseline healthy parameter vectors’ sample mean (i.e. \( \frac{\mathbf{a} + \mathbf{b}}{2} \)) and radius equal to the threshold.
Appendix B: Chapter 4 Appendix

B.1 Justification of Covariance Matrix Adjustment

Let $\hat{\alpha}_i \sim N(\mathbf{\alpha}_i, \Sigma_{\alpha})$ with $i = 1, \ldots, p_o$, identified using $p_o$ vibration response signals respectively. Let also $\hat{\mu} \sim N(\mathbf{\mu}, \Sigma_{\hat{\mu}})$ designate the sample mean of the $p_o$ model parameter vectors. Then:

$$\hat{\mu} = \frac{1}{p_o} \sum_{i=1}^{p_o} \hat{\alpha}_i \quad \text{and} \quad \mathbf{\mu} = E(\hat{\mu}) = \frac{1}{p_o} \sum_{i=1}^{p_o} \mathbf{\alpha}_i \quad \text{(B.1.1)}$$

Based on Equation B.1.1 and due to mutual independence among the parameters, the covariance matrix $\Sigma_{\hat{\mu}}$ may be expressed as:

$$\Sigma_{\hat{\mu}} = \frac{1}{p_o^2} \sum_{i=1}^{p_o} \sum_{j=1}^{p_o} E[(\hat{\alpha}_i - \mathbf{\alpha}_i)(\hat{\alpha}_j - \mathbf{\alpha}_j)^T] = \frac{1}{p_o} \sum_{i=1}^{p_o} \Sigma_{\alpha} \quad \text{(B.1.2)}$$

Assume now that an estimated parameter vector $\hat{\alpha}_i$ is centered by subtracting the sample mean $\hat{\mu}$ yielding a new random variable that follows an updated Gaussian distribution. The expressions for the mean and covariance matrix of the updated distribution are presented below:

$$E(\hat{\alpha}_i - \hat{\mu}) = \mathbf{\alpha}_i - \mathbf{\mu} = \mathbf{\alpha}_i - \frac{1}{p_o} \sum_{i=1}^{p_o} \mathbf{\alpha}_i \quad \text{(B.1.3)}$$

$$cov(\hat{\alpha}_i - \hat{\mu}) = E[(\hat{\alpha}_i - \hat{\mu} - \mathbf{\alpha}_i + \mathbf{\mu})(\hat{\alpha}_i - \hat{\mu} - \mathbf{\alpha}_i + \mathbf{\mu})^T] =$$

$$\Sigma_{\alpha} + \Sigma_{\hat{\mu}} - \frac{1}{p_o} E \left[ (\hat{\alpha}_i - \mathbf{\alpha}_i) \sum_{j=1}^{p_o} (\hat{\alpha}_j - \mathbf{\alpha}_j)^T \right] - \frac{1}{p_o} E \left[ (\hat{\alpha}_i - \mathbf{\alpha}_i) \sum_{j=1}^{p_o} (\hat{\alpha}_j - \mathbf{\alpha}_j)^T \right]^T \quad \text{(B.1.4)}$$

Equation B.1.4 may yield two possible results based on whether the parameter vector estimate $\hat{\alpha}_i$ is used for the sample mean estimation:

$$cov(\hat{\alpha}_i - \hat{\mu}) = \Sigma_{\alpha} + \Sigma_{\hat{\mu}} - \frac{2}{p_o} \Sigma_{\alpha} \quad \text{or not} \quad cov(\hat{\alpha}_i - \hat{\mu}) = \Sigma_{\alpha} + \Sigma_{\hat{\mu}} \quad \text{(B.1.5)}$$
Appendix B: Chapter 4 Appendix

The last two expressions are obtained by considering the parameter vector estimates $\hat{\alpha}$, mutual independence.

### B.2 Comparisons with the Fast Forward Selection Algorithm

In this section the postulated principal components selection, for damage detection, algorithm (denoted as postulated algorithm in the sequel) is compared to the Fast Forward Selection (FFS) algorithm utilized in [Hoell et al. 2016a]. The latter algorithm, contrary to the postulated one, assumes weakly correlated principal components. Therefore, starts the principal components selection procedure from the component yielding the best detection performance and then incorporates a new component depending on which two components combination yields the best detection performance (multiple combinations are explored to account for potential correlations among the components). The performance is measured based on Equation 4.13 and the other related equations of section 4.4, while the selected components may be non-sequential. This procedure is then repeated for increasing components number per combination, until the best principal components set in terms of detection performance is obtained. It is evident, that the fast forward selection algorithm assesses more principal components combinations, than the postulated one, to determine which one yields the best performance. Yet, given the components mutual independence, this may be an unnecessary additional computational cost.

The two algorithms comparison is performed, per supervised method, in terms of computational efficiency, as well as damage detection performance. All the results are based on the experimental set-up of section 4.3 and the assessment procedure of subsection 4.5.1. Toward this end, Figure B.2.1 depicts the number of FFS algorithm iterations, per supervised damage detection method and employed set of baseline beams, required for selecting the 'best' combination of principal components. Evidently, the S–nPCA–MM–AR and S–PCA–MM–AR methods require about 1500 iterations, while the S–PCA–AR method requires about 700 iterations on average. On the other hand, the postulated algorithm requires only 57 iterations for the S–nPCA–MM–AR method and 56 for the other two methods (see section 4.4). Taking into account that the two algorithms follow the same methodology per iteration, it becomes evident that the postulated algorithm is significantly faster in terms of computational time, yielding a significantly shorter training period.

The principal components selected via the FFS algorithm are shown per supervised method in Figure B.2.2, in the form of bar diagrams with each bar depicting how many times a specific component is included in the set defining the damage subspace (frequency of use), for all 50 employed sets of baseline beams. Figure B.2.2(a) shows that the first component is always selected in the S–nPCA–MM–AR method, while most of the remaining components are used in most of the 50 employed sets of baseline beams. Similar results are obtained for the S–PCA–MM–AR method in Figure B.2.2(b), with the first component being the basic difference since this is rarely included in the set defining the damage subspace. On the other hand, the S–PCA–AR method exhibits an approximately uniform use of all the components in Figure B.2.2(c), with the last half of the components depicting somewhat higher utilization frequency than the first half. Comparing Figure B.2.1 to its counterpart Figure 4.7 obtained using the postulated algorithm, it becomes evident that the FFS algorithm does not select all the last $m$ components, but only a subset of them, per employed set of baseline beams. On the other hand, the postulated algorithm selects a significant part of the last components, for all the employed sets of baseline beams.

The two algorithms are additionally compared in terms of damage detection performance via the ROC curves of Figure B.2.3. This compares the two algorithms per supervised method and under OC PA, depicting that the postulated algorithm is better than the FFS algorithm when the S–nPCA–MM–AR method is used, while the opposite is observed for the other two methods. Among all the methods and the two algorithms the best performance is achieved by the S–nPCA–MM–AR method that uses the postulated algorithm.

When damage scenarios are missing from the baseline phase, yet are included in the inspection phase (OC SA), the postulated algorithm outperforms its FFS counterpart for the S–nPCA–MM–AR...
and S–PCA–MM–AR methods. On the other hand, when the S–PCA–AR method is considered the FFS algorithm outperforms the postulated one, although the latter yields improved performance and the former decreased performance compared to the results obtained under OC PA\textsubscript{all}.

Summarizing, the postulated algorithm is significantly faster than its FFS counterpart and yields improved performance, when the S–nPCA–MM–AR method is considered. On the other hand, the FFS algorithm yields improved performance, when the S–PCA–AR method is considered, while both algorithms may yield good performance when the S–PCA–MM–AR method is considered. Of course, the best performance is achieved when the postulated algorithm is combined with the S–nPCA–MM–AR method.
Fig. B.2.2. Principal component frequency of use in the set of selected by the FFS algorithm components: (a) S–nPCA–MM–AR, (b) S–PCA–MM–AR and (c) S–PCA–AR \([n_a = 57, \nu_o = 15, \nu_d = 8]\).
B.2 Comparisons with the Fast Forward Selection Algorithm

Fig. B.2.3. Comparative assessment of the FFS and the postulated principal components selection algorithms, per supervised damage detection method, under OC PA (all damage scenarios included in the baseline and inspection phases), using ROC curves. \([n_a = 57, \nu_o = 15, \nu_d = 8]\)

Fig. B.2.4. Comparative assessment of the FFS and the postulated principal components selection algorithms, per supervised damage detection method, under OC SA (all damage scenarios included in the inspection phase), using ROC curves. \([n_a = 57, \nu_o = 15, \nu_d = 4]\)
B.3 Supervised Principal Components Selection

Figure B.3.1 depicts in terms of histograms how many times a design parameter $m$ value is selected by the postulated principal components selection algorithm for the 50 employed sets of healthy baseline beams and per supervised method. It is evident that the two PCA–MM–AR based methods exhibit similar $m$ values that are concentrated within the range $[45, 56]$, while the S–PCA–AR method exhibits values in the range $[16, 56]$.

![Histograms of selected design parameter $m$ for S–nPCA–MM–AR, S–PCA–MM–AR, and S–PCA–AR](image)

**Fig. B.3.1.** Histograms of the selected design parameter $m$, for the 50 employed set of healthy baseline beams: (a) S–nPCA–MM–AR, (b) S–PCA–MM–AR and (c) S–PCA–AR [$n_d = 57$, $v_0 = 15$, $v_d = 8$].

B.4 Damage Detection Results using One/two Damage Scenarios in the Baseline Phase

The S–PCA–AR and S–nPCA–MM–AR methods robustness to the utilization of only one or two damage scenarios in the baseline phase, as well as to the type of this/these damage(s), is assessed in this section. The assessment is performed in terms of damage detection performance via ROC curves and involves comparisons between the methods, as well as with their unsupervised counterparts.
B.4 Damage Detection Results using One/two Damage Scenarios in the Baseline Phase

B.4.1 The Assessment Procedure

The methods damage detection performance is assessed through the systematic procedure presented in subsection 4.5.1, while additional tests are performed via rotations between the damaged beams of the baseline and inspection phases. In detail, sets of \( p_d = \{1, 2\} \) damaged beams are included in the baseline phase, while different combinations of the available damage scenarios are considered. These are referred to as the employed sets of baseline damage scenarios. Damage detection tests are performed per such set in order to investigate the methods sensitivity to the number and type of baseline damage scenarios. For each number of baseline beams and baseline damage scenarios considered, the respective number of inspection beams, employed sets of baseline damage scenarios, inspection experiments, employed sets of baseline beams and aggregate inspection experiments are provided in Table B.4.1.

In addition, the methods test pseudo–statistic \( D \) is normalized by following the procedure described in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b], while their performance is assessed using ROC curves and the Area Under the Curve (AUC) metric (see [Fawcett 2006; K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]).

Table B.4.1: Number of baseline and inspection beams, employed sets of baseline damage scenario, inspection experiments, employed sets of baseline beams, and aggregate inspection experiments per number of damage scenarios included in the baseline phase.

<table>
<thead>
<tr>
<th>Number of baseline beams ( (\nu_o) )</th>
<th>Number of inspection beams ( (i) )</th>
<th>Number of employed sets of baseline damage scenarios</th>
<th>Number of Inspection experiments (per set of baseline beams; ( z = 7 \cdot i ))</th>
<th>Number of employed sets of baseline beams ( (h) )</th>
<th>Number of aggregate inspection experiments ( (t_i = h \cdot z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>23</td>
<td>8</td>
<td>161</td>
<td>50</td>
<td>8 050</td>
</tr>
<tr>
<td>9</td>
<td>21</td>
<td>8</td>
<td>147</td>
<td>50</td>
<td>7 350</td>
</tr>
<tr>
<td>11</td>
<td>19</td>
<td>8</td>
<td>133</td>
<td>50</td>
<td>6 650</td>
</tr>
<tr>
<td>13</td>
<td>17</td>
<td>8</td>
<td>119</td>
<td>50</td>
<td>5 950</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>8</td>
<td>105</td>
<td>50</td>
<td>5 250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of baseline beams ( (\nu_o) )</th>
<th>Number of inspection beams ( (i) )</th>
<th>Number of employed sets of baseline damage scenarios</th>
<th>Number of Inspection experiments (per set of baseline beams; ( z = 7 \cdot i ))</th>
<th>Number of employed sets of baseline beams ( (h) )</th>
<th>Number of aggregate inspection experiments ( (t_i = h \cdot z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>22</td>
<td>4</td>
<td>154</td>
<td>50</td>
<td>7 700</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>4</td>
<td>140</td>
<td>50</td>
<td>7 000</td>
</tr>
<tr>
<td>11</td>
<td>18</td>
<td>4</td>
<td>126</td>
<td>50</td>
<td>6 300</td>
</tr>
<tr>
<td>13</td>
<td>16</td>
<td>4</td>
<td>112</td>
<td>50</td>
<td>5 600</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
<td>4</td>
<td>98</td>
<td>50</td>
<td>4 900</td>
</tr>
</tbody>
</table>

B.4.2 Damage Detection Results and Comparisons

The values of the test pseudo–statistic \( D \) per structural state, corresponding to all the aggregate inspection experiments (see Table B.4.2) are shown for the ‘optimal’ design parameters of the S–nPCA–MM–AR and S–PCA–AR methods (see Table B.4.2) in Figure B.4.1(a) and (b). It is evident that the S–nPCA–MM–AR method distinguishes the damaged from the healthy state test pseudo–statistics, for most of the inspection experiments. On the other hand, the S–PCA–AR method depicts difficulty in separating the small damages of low impact energy (i.e. AL, CL) from the healthy state. These observations are verified through Figure B.4.1(c) and (d), which present the same test pseudo–statistics by
Fig. B.4.1. Experimental assessment of the S–nPCA–MM–AR and S–PCA–AR methods via: (a),(b) the test pseudo–statistic $D$ and (c),(d) the respective box plots, for all the damage scenarios and aggregate inspection experiments (see Table B.4.2; the top and bottom of each box are the 25th and 75th percentiles; the red line in the middle of each box is sample median; the lines extending above and below each box are called whiskers and are drawn from the ends of the interquartile ranges, with length 1.5 times the interquartile range; the red crosses represent observations out of the whiskers range; the black horizontal line is defined by the top whisker of the healthy state and separates the healthy and damaged states).

Table B.4.2: ROC curves construction details and ‘optimal’ design parameters per method.

<table>
<thead>
<tr>
<th>Method</th>
<th>No. of tested decision thresholds (points on the curve)</th>
<th>No. of Inspection experiments</th>
<th>$n_d$</th>
<th>$\gamma$(%)</th>
<th>$v_o$</th>
<th>Baseline Damage Scenario(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S–PCA–AR</td>
<td>4852</td>
<td>5250</td>
<td>67</td>
<td>–</td>
<td>13</td>
<td>Damage DL</td>
</tr>
<tr>
<td>U–PCA–AR</td>
<td>5552</td>
<td>5600</td>
<td>57</td>
<td>90</td>
<td>15</td>
<td>–</td>
</tr>
<tr>
<td>S–nPCA–MM–AR</td>
<td>4852</td>
<td>4900</td>
<td>57</td>
<td>–</td>
<td>15</td>
<td>Damages DL &amp; DH</td>
</tr>
<tr>
<td>U–nPCA–MM–AR</td>
<td>5552</td>
<td>5600</td>
<td>57</td>
<td>99.4</td>
<td>15</td>
<td>–</td>
</tr>
</tbody>
</table>

means of box plots. As shown in the latter figure, when the S–nPCA–MM–AR method is considered, the damaged states box plots exceed the black continuous line (with a small exception made for the damage CL) which separates the healthy and damaged states with false alarm rate $\leq 3\%$. On the other hand, when the S–PCA–AR method is considered the box plots corresponding to damages AL and CL do not exceed the black line.

Further comparison between the two supervised methods is provided via Figure B.4.2 that depicts the utilization frequency (i.e. number of utilizations) of each principal component in the employed sets of baseline beams (see Table B.4.1). This is presented per method, for the respective ‘optimal’ design parameters provided in Table B.4.2. As it is shown, the S–PCA–AR method uses all of the principal
used Principal Components Histogram: S-nPCA-MM-AR

(a)

Used Principal Components Histogram: S-PCA-AR

(b)

Fig. B.4.2. Principal component frequency of use: (a) the S–nPCA–MM–AR method ($n_a = 57$, $\nu_o = 15$; baseline damages: DL & DH), and (b) the S–PCA–AR method ($n_a = 57$, $\nu_o = 15$; baseline damage: DL).

components except for the first one that is always removed. Thus, implicitly accounts for most of the information included in the model parameter vector, while indicates that only the first component is significantly affected by uncertainty. On the other hand, the S–nPCA–MM–AR method may not use in between 1 and 15 principal components, depending on the employed set of baseline beams, thereby depicting that uncertainty affects more than one components in this case. Main cause of this discrepancy in the number of utilized components is the utilization of centered PCA in the former method and non–centered in the latter. These two PCA versions lead to two completely different transformations of the model parameter vector, and thus a significant discrepancy in the damage–sensitive principal components. Further details and interpretations of the two PCA versions are provided in the main text and in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b].

The performance of the S–PCA–AR, S–nPCA–MM–AR, U–PCA–AR, and U–nPCA–MM–AR methods is comparatively assessed in Figure B.4.3, through ROC curves and respective AUC values (see construction details in Table B.4.2). The comparison is based on the use of a set of ‘optimal’ design parameters per method (see Table B.4.2), which lead to their ‘best’ damage detection performance, as shown in B.4.3 and in [K. J. Vamvoudakis–Stefanou, J. S. Sakellariou, et al. 2018b]. Figure B.4.3 shows that both supervised methods achieve improved damage detection performance over their unsupervised counterparts. In detail Figure B.4.3 depicts the following:

- The S–nPCA–MM–AR method achieves the best performance among the four methods in the [0%, 3.7%] false alarm rate (false positive rates) range and the highest overall performance (among all methods) with AUC = 0.99426.
- The U–nPCA–MM–AR method achieves the second best performance, for false alarm rates below 1%, and the second highest overall performance with AUC = 0.9937.
- The S–PCA–AR method achieves the best performance for false alarm rates over 3.7%, while achieves the third highest overall performance with AUC = 0.99063.
- The worst performance is depicted for the U–PCA–AR method, for all the false alarm rates, with
Fig. B.4.3. Experimental assessment of the U–nPCA–MM–AR, S–nPCA–MM–AR, U–PCA–AR and S–PCA–AR methods via (a) ROC curves, and (b) AUC measure. ‘Optimal’ design parameters are employed - details in Table B.4.2).

AUC = 0.9644.

B.4.3 Sensitivity to the Low Number and the Type of Damage Scenarios Included in the Baseline Phase

The sensitivity of the S–PCA–AR and S–nPCA–MM–AR methods to: (a) the AR model order, (b) the number of baseline beams, and (c) the number and type of baseline damage scenarios is assessed in terms of the AUC metric in Figure B.4.5 and Figure B.4.6 respectively. In particular, Figure B.4.5 depicts that the damage detection performance of the S–PCA–AR method is significantly susceptible to the selection of the baseline damage scenarios. It is evident that the best performance is achieved when damage DL is included in the set of baseline damage scenarios (AUC values: 0.99063 and 0.98992 - sets of baseline damage scenarios: DL, DL & DH respectively). As shown in Figure B.4.4, the FRF magnitude estimate obtained from damage DL has the lowest Euclidean distance (as compared to the other damage scenarios) from the mean FRF magnitude obtained from all the damage scenarios. This indicates that damage DL is the most suitable to represent all the other damage scenarios in the training phase and explains the improved performance of the method. In addition, the method performance is affected by the number of baseline beams and the AR model order, as shown in Figure B.4.5. Increased number of baseline beams leads to high AUC values, while very low or high AR model orders lead to low AUC values.

Similar sensitivity to the design parameters is depicted by the S–nPCA–MM–AR method, as shown in Figure B.4.6. In detail, inclusion of damage DL in the set of baseline damage scenarios, yields the best damage detection performance, with AUC values 0.99426 and 0.99425 (sets of baseline damage scenarios: DL, DL & DH respectively). In addition, increase in the number of baseline beams and/or the AR model order, leads to improved damage detection performance. The S–nPCA–MM–AR method AUC values for all the combinations of its design parameters, lie in the range [0.8985, 0.99426] which is wider as compared to its unsupervised counterpart ([0.9217, 0.9937]), thus indicating higher sensitivity of the method.
B.4 Damage Detection Results using One/two Damage Scenarios in the Baseline Phase

Fig. B.4.4. Euclidean distances between the Welch–based FRF magnitude estimate of each damage scenario and the mean FRF magnitude estimate obtained from all the damage scenarios per frequency value.

former method to its design parameters.

In addition, Figure B.4.5 and Figure B.4.6 show reduced damage detection performance for both methods (with few exceptions), when two damage scenarios (instead of one) are used in the set of baseline damage scenarios. This is the case, since the one damage scenario often yields reduced performance compared to the other, when used individually in a set. Therefore, their combination in the set of two damage scenarios in the baseline, yields reduced performance compared to the latter single damage scenario counterpart. It is noted that the two methods’ AUC values corresponding to the combinations \( \nu_o = 7 \) with \( n_a = \{57, 67, 77\} \), \( \nu_o = 9 \) with \( n_a = \{67, 77\} \) and \( \nu_o = 11 \) with \( n_a = 77 \) are not presented, since the number of the available experiments is lower than the considered AR order violating the condition \( p > n_a \) (see subsection 4.4.2, subsection 4.4.3, and Table B.4.1).

B.4.4 Discussion

The best performance between the assessed supervised and unsupervised methods, for the ‘optimal’ set of design parameters, is achieved by the S–nPCA–MM–AR method, while both supervised methods outperform their unsupervised counterparts. The S–PCA–AR method achieves very good performance as well, while when small damages and/or false alarm rates are not of interest it may be considered as the best choice.

In terms of training complexity, the supervised methods have an advantage over their unsupervised counterparts, due to the automated selection of the principal components that define the damage subspace. A general guideline for the determination of the remaining design parameters is the use of as many sample structures from the population as possible, a sufficiently high AR model order, and damage scenarios in the baseline phase with dynamics that may approximate the other scenarios as well.
Fig. B.4.5. Sensitivity analysis for the S–PCA–AR method in terms of AR order $n_d$, number of baseline beams $\nu_o$, and set of baseline damage scenarios, by means of the AUC metric. (50 baseline sets; the ‘optimal’ performance point is indicated by a magenta circle and an arrow; points with AUC values close to the ‘optimal’ are indicated by black arrows.)
B.4 Damage Detection Results using One/two Damage Scenarios in the Baseline Phase

Fig. B.4.6. Sensitivity analysis for the S–nPCA–MM–AR method, in terms of AR order \( n_a \), number of baseline beams \( \nu_o \), and set of baseline damage scenarios, by means of the AUC metric. (50 baseline sets; the ‘optimal’ performance point is indicated by a magenta circle and an arrow; points with AUC values close to the ‘optimal’ are indicated by black arrows.)
C.1 Hyper--Parameters Selections & Damage Detection Results.

Figure C.1.1 depicts the selected, by means of the Bayesian Information Criterion (BIC), number of Gaussian mixture components, per employed set of baseline structures, corresponding to the case study A. Most of the selected values are greater than the value 20, thus indicating a compact healthy subspace geometry, of low complexity.

Fig. C.1.1. Case study A: Selected U–EM–GM–AR method’s hyper--parameter j (corresponds to the GM components number), per employed set of baseline structures.

Figure C.1.2 and Figure C.1.3 provide the Area Under the ROC Curve (AUC) values for the U–DP–GM–AR method, per strength parameter $\alpha$ value, and per low or high impact energy damages. These, demonstrate the method’s significant sensitivity to the strength parameter $\alpha$ selection, thus indicating that great caution is required by the user when training the method.
Fig. C.1.2. Case study A: Hyper-parameter $\alpha$ of the U–DP–GM–AR method, versus detection performance presented via AUC values.

Fig. C.1.3. Case study B: Hyper parameter $\alpha$ of the U–DP–GM–AR method, versus detection performance presented via AUC values.

Figure C.1.4 and Figure C.1.5 depict the U–cGM–AR test pseudo–statistics $D$ values for the experimental case studies A and B respectively. These are therein compared with the respective values obtained from five other powerful state–of–the–art methods, and illustrate the U–cGM–AR method’s superiority.
C.1 Hyper-Parameters Selections & Damage Detection Results.

Fig. C.1.4. Case study A: Scatter plots of the methods’ test pseudo-statistics $D$. 
Fig. C.1.5. Case study B: Scatter plots of the methods’ test pseudo–statistics $D$. 