DIPLOMA THESIS

Modeling and Dynamic Control of Irrigation Canals

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PATRAS, February 2024
Η παρούσα διπλωματική εργασία παρουσιάστηκε

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tην Τρίτη 27 Φεβρουαρίου 2024
Η έγκριση της διπλωματικής εργασίας δεν υποδηλοί την αποδοχή των γνωμών του συγγραφέα. Κατά τη συγγραφή τηρήθηκαν οι αρχές της ακαδημαϊκής δεοντολογίας.
Μοντελοποίηση και Δυναμικός Έλεγχος Δικτύου Παροχής Νερού
Γεωργάκης Παναγιώτης

ΠΕΡΙΛΗΨΗ
Η αποτελεσματική διαχείριση των δικτύων καναλιών άρδευσης συμβάλει σημαντικά στη βελτιστοποίηση της κατανομής του νερού και στην ενίσχυση της γεωργικής παραγωγικότητας. Η ανθρώπινη διαχείριση αρδευτικών καναλιών είναι ένα πολύπολοκο έργο, το οποίο συχνά οδηγεί σε μη βέλτιστες επιδόσεις. Κατά συνέπεια, μεγάλες ποσότητες νερού σπαταλώνται, τα έξοδα συντήρησης κλιμακώνονται και οι αγρότες περιορίζονται από άκαμπτα προγράμματα άρδευσης. Η αυτοματοποίηση των αρδευτικών καναλιών κρίνεται θέμα ζωτικής σημασίας για τη μείωση των απωλειών νερού που οφείλονται σε ανθρώπινα λάθη, με μεγάλο αντίκτυπο στην παγκόσμια γεωργία και οικονομία. Η παρουσία Διπλωματική εργασία αποσκοπεί στην ανάπτυξη ενός αλγορίθμου αυτομάτου ελέγχου καναλιών άρδευσης εντός ενός περιβάλλοντος προσομοίωσης. Αναλύεται εκτενώς η μεθοδολογία ελέγχου των κατάντη επιπέδων νερού των αρδευτικών καναλιών. Χρησιμοποιούνται έναν συνδυασμό καθιερωμένης βιβλιογραφίας και εξειδικευμένων εργαλείων προσομοίωσης, διερευνάται η φυσική της ροής ανοιχτών αγωγών. Η επιλογή απλουστευμένων μοντέλων για σκοπούς ελέγχου διερευνάται με τη χρήση των εργαλείων της θεωρίας γραμμικών δυναμικών συστημάτων. Αναπτύσσεται ένας κεντρικός ελεγκτής ΜΡΣ για την αντιμετώπιση των μεταβολών του σημείου ρύθμισης του βάθους νερού και των διαταραχών λόγω βαρυτικών διακλαδώσεων. Τέλος, ο προτεινόμενος ελεγκτής δοκιμάζεται σε μια προσομοιωμένη έκδοση του καναλιού Corning στην Καλιφόρνια των ΗΠΑ, η οποία αποτελεί την 2η δοκιμασία σύμφωνα με τα κριτήρια αξιολόγησης που έχει θέσει η ASCE για την επίδοση των αλγορίθμων ελέγχου καναλιών. Τα αποτελέσματα της αξιολόγησης συζητούνται και διαμορφώνονται συμπεράσματα για την επίδοση του προτεινομένου ελεγκτή.

Λέξεις κλειδιά
[Αυτόματος Έλεγχος, Κανάλια Άρδευσης, Προσομοίωση Ροής Ανοιχτών Αγωγών, Μοντέλα για Έλεγχο Καναλιών, Γραμμικοποίηση Συστημάτων, Προβλεπτικός Έλεγχος, Έλεγχος Χωρικά Κατανεμημένων Συστημάτων, Βέλτιστος Έλεγχος]
Modeling and Dynamic Control of Irrigation Canals

Georgakis Panagiotis

ABSTRACT
Efficient management of irrigation canal networks plays a crucial role in the optimization of water allocation and the enhancement of agricultural productivity. The manual administration of irrigation canal systems is a challenging task, often resulting in suboptimal performance. Consequently, a significant amount of water is wasted, maintenance expenses escalate, and farmers are constrained by inflexible irrigation schedules. The automation of irrigation canals is crucial for reducing water losses attributed to human errors and holds significant implications for the global economy and agriculture. This thesis seeks to develop a control algorithm for irrigation canals within a simulated environment. The methodology of controlling the downstream water depths of irrigation canals is extensively analyzed. By utilizing a combination of well established literature and specialized simulation tools, the underlying physics of open channel flow are explored. The choice of simplified models for control purposes is investigated by employing the tools of linear dynamical systems theory. A centralized MPC controller is developed to handle water depth setpoint changes and gravity offtake disturbances. Finally, the proposed controller is tested on a simulated version of the Corning Canal in California, USA, which serves as Test Case 2 according to the benchmarks set by ASCE for developing canal control algorithms. The results of the benchmark are discussed and the performance of the proposed controller is evaluated.

Keywords
[Automatic Control, Irrigation Canals, Open Channel Flow Simulation, Canal Control-Oriented Models, System Linearization, Model Predictive Control, Control of Spatially Distributed Systems, Optimal Control]
Index of Tables

Table 2.1: Parameters of the two types of open channels.................................................. .18
Table 2.2: Parameters of example gate............................................................................. .30
Table 3.1: Model parameters corresponding to Canal A – Uniform Flow (H0=2.12,
Q0=14.0).......................................................................................................................... .38
Table 5.1: Parameters of ASCE Test Canal 2 - Corning Canal....................................... .54
Table 5.2: Schedules of the two benchmarks of the Corning Canal............................... .55
Table 5.3: Parameters of the MPC + Slave controller...................................................... .56
Table of Figures

Figure 1.1: Aerial photo of the Corning Canal, California, USA................................. .2
Figure 1.2: Simplified diagram of a typical irrigation canal - side profile and top view....3
Figure 1.3: Classification of Canal Control-Oriented Models........................................5
Figure 1.4: Control Variables of a Canal Reach............................................................ 9
Figure 2.1: One-dimensional open channel flow............................................................ .14
Figure 2.2: Cross section of an open channel - wetted area A(x,t), top width T(x,t) and wetted perimeter P(x,t) are clearly defined................................................................. .15
Figure 2.3: Side profile (left) and trapezoidal cross section (right) of a typical open channel with relevant geometric parameters and steady state quantities.................18
Figure 2.4: Backwater curves corresponding to different downstream depth boundary conditions – Canal A.................................................................20
Figure 2.5: Backwater curves corresponding to different downstream depth boundary conditions – Canal B.................................................................20
Figure 2.6: Frequency response due to upstream inflow discharge - Canal A...............25
Figure 2.7: Frequency response due to downstream outflow discharge - Canal A.......25
Figure 2.8: Frequency response due to upstream inflow discharge - Canal B............27
Figure 2.9: Frequency response due to downstream inflow discharge - Canal B........27
Figure 2.10: Comparison between sluice gates under free and submerged flow conditions....................................................................................................................... .30
Figure 2.11: Canal A subject to an unobstructed downstream boundary (left) and a downstream sluice gate (right)........................................................................................................... .31
Figure 2.12: Step response of Canal A subject to an unobstructed downstream boundary – pure integrator behavior...............................................................32
Figure 2.13: Step response of Canal A subject to a sluice gate at the downstream boundary – first order behavior due to local feedback................................................33
Figure 3.1: Variables relevant to the process of canal modeling.................................35
Figure 3.2: Division of a steep channel into sections as proposed by the ID model.......36
Figure 3.3: Comparison of step responses of downstream depth and outflow discharge due to inflow discharge step input - Canal A - entire timespan.................................41

Figure 3.4: Comparison of step responses of downstream depth and outflow discharge due to inflow discharge step input - Canal A - initial 1h zoom........................................42

Figure 3.5: Comparison of step responses of downstream depth and outflow discharge due to inflow discharge step input - Canal A - final 3h zoom.................................43

Figure 3.6: Comparison of magnitude responses of downstream depth due to inflow discharge - Canal B (Q0=7.0)..................................................................................44

Figure 4.1: General architecture of the proposed master-slave canal control system – communication between control layers and canal system......................................................46

Figure 4.2: Reaches interconnected through their discharges form a canal.....................47

Figure 4.3: MPC controller prediction process..................................................................51

Figure 4.4: Flowchart of the MPC algorithm....................................................................51

Figure 5.1: Side profiles of the two test canals - taken from Clemmens et al., 1998......56

Figure 5.2: Results of Benchmark 1 - Changes in the downstream target depths at 2h. 60

Figure 5.3: Results of Benchmark 1 - Changes in the downstream offtake discharges (no changes)........................................................................................................60

Figure 5.4: Results of Benchmark 1 - Responses of downstream water depths.............61

Figure 5.5: Results of Benchmark 1 - Responses of errors in the downstream water depths.........................................................................................................................61

Figure 5.6: Results of Benchmark 1 - Changes in the gate discharges................................62

Figure 5.7: Results of Benchmark 1 - Changes in the gate openings...............................62

Figure 5.8: Results of Benchmark 2 - Changes in the target downstream water depths (no changes)........................................................................................................63

Figure 5.9: Results of Benchmark 2 - Changes in the downstream offtake discharges at 2h......................................................................................................................63

Figure 5.10: Results of Benchmark 2 - Responses of downstream water depths............64

Figure 5.11: Results of Benchmark 2 - Responses of errors in the downstream water depths.........................................................................................................................64
Figure 5.12: Results of Benchmark 2 - Changes in the gate discharges

Figure 5.13: Results of Benchmark 2 - Changes in the gate openings
### Table of Abbreviations

<table>
<thead>
<tr>
<th>Abbr.</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASCE</td>
<td>American Society of Civil Engineers</td>
</tr>
<tr>
<td>SVEs</td>
<td>Saint-Venant Equations</td>
</tr>
<tr>
<td>ID</td>
<td>Integrator-Delay model</td>
</tr>
<tr>
<td>PLCs</td>
<td>Programmable Logic Controllers</td>
</tr>
<tr>
<td>IDZ</td>
<td>Integrator-Delay-Zero model</td>
</tr>
<tr>
<td>IR</td>
<td>Integrator-Resonance model</td>
</tr>
<tr>
<td>ARX</td>
<td>Auto Regressive with eXogenous inputs model</td>
</tr>
<tr>
<td>ARMAX</td>
<td>Auto Regressive Moving Average with eXogenous inputs model</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudo-Random Binary Sequence</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative control</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output system</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>LQG</td>
<td>Linear Quadratic Gaussian control</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output system</td>
</tr>
<tr>
<td>MPC</td>
<td>Model Predictive Control</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
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<td>--------------</td>
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</tr>
<tr>
<td>LMI</td>
<td>Linear Matrix Inequalities</td>
</tr>
<tr>
<td>FVM</td>
<td>Finite Volume Method</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>SWIFTEC</td>
<td>Shallow Water Integrated Framework For TEsting Controllers</td>
</tr>
<tr>
<td>ZOH</td>
<td>Zero Order Hold discretization</td>
</tr>
<tr>
<td>QP</td>
<td>Quadratic Programming</td>
</tr>
</tbody>
</table>
# Table of Nomenclature

<table>
<thead>
<tr>
<th><strong>Symbol</strong></th>
<th><strong>Definition</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(x,t)$</td>
<td>Water depth</td>
</tr>
<tr>
<td>$Q(x,t)$</td>
<td>Water discharge – flow rate</td>
</tr>
<tr>
<td>$W(t)$</td>
<td>Gate opening</td>
</tr>
<tr>
<td>$D(t)$</td>
<td>Offtake demand discharge (disturbance)</td>
</tr>
<tr>
<td>$\ldots_{u/d}$</td>
<td>Subscript indicating upstream/downstream location</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance abscissa variable</td>
</tr>
<tr>
<td>$t$</td>
<td>Time variable</td>
</tr>
<tr>
<td>$A(x,t)$</td>
<td>Wetted cross-sectional area</td>
</tr>
<tr>
<td>$V(x,t)$</td>
<td>Flow velocity</td>
</tr>
<tr>
<td>$C(x,t)$</td>
<td>Wave celerity</td>
</tr>
<tr>
<td>$I(x,t)$</td>
<td>Lateral water inflow discharge</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$S_0$</td>
<td>Slope of the channel bottom</td>
</tr>
<tr>
<td>$S_f$</td>
<td>Friction slope</td>
</tr>
<tr>
<td>$n$</td>
<td>Manning friction coefficient</td>
</tr>
<tr>
<td>$R(x,t)$</td>
<td>Hydraulic radius</td>
</tr>
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<td>Symbol</td>
<td>Description</td>
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<td>-------------</td>
</tr>
<tr>
<td>$P(x,t)$</td>
<td>Wetted perimeter</td>
</tr>
<tr>
<td>$T(x,t)$</td>
<td>Channel top width</td>
</tr>
<tr>
<td>$Fr(x,t)$</td>
<td>Froude characteristic number</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of a channel reach</td>
</tr>
<tr>
<td>$B(x)$</td>
<td>Width of the channel bottom</td>
</tr>
<tr>
<td>$(...)_0$</td>
<td>Subscript indicating steady state value</td>
</tr>
<tr>
<td>$\beta_0(x)$</td>
<td>Parameter of the linearized Saint Venant Equations</td>
</tr>
<tr>
<td>$\gamma_0(x)$</td>
<td>Parameter of the linearized Saint Venant Equations</td>
</tr>
<tr>
<td>$\kappa(x)$</td>
<td>Parameter of the linearized Saint Venant Equations</td>
</tr>
<tr>
<td>$h(x,t)$</td>
<td>Variation of water depth around a steady state $H_0(x)$</td>
</tr>
<tr>
<td>$q(x,t)$</td>
<td>Variation of discharge around a steady state $Q_0(x)$</td>
</tr>
<tr>
<td>$w(t)$</td>
<td>Variation of the gate opening around a steady state $W_0$</td>
</tr>
<tr>
<td>$d(t)$</td>
<td>Variation of the offtake discharge around a steady state $D_0(t)$</td>
</tr>
<tr>
<td>$P(s)$</td>
<td>Input-output transfer matrix of the canal system</td>
</tr>
<tr>
<td>$p_{ij}(s)$</td>
<td>Element $ij$ of the canal transfer matrix</td>
</tr>
<tr>
<td>$A(x,s)$</td>
<td>Distributed canal system matrix</td>
</tr>
<tr>
<td>$\Gamma(x,s)$</td>
<td>Distributed state transition matrix</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<td>--------------------------------------------</td>
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<tr>
<td>$C_g$</td>
<td>Gate discharge coefficient</td>
</tr>
<tr>
<td>$L_g$</td>
<td>Width of the gate opening</td>
</tr>
<tr>
<td>$k_{w/u/d}$</td>
<td>Linearized gate equation coefficients (gains)</td>
</tr>
<tr>
<td>$A_{ID}$</td>
<td>Backwater surface area corresponding to the ID model</td>
</tr>
<tr>
<td>$\tau_{ID}$</td>
<td>Transport delay corresponding to the ID model</td>
</tr>
<tr>
<td>$A_{IDZ}$</td>
<td>Backwater surface area corresponding to the IDZ model</td>
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<tr>
<td>$\tau_{IDZ}$</td>
<td>Transport delay corresponding to the IDZ model</td>
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<tr>
<td>$p_{\infty}$</td>
<td>Zeros of the IDZ model</td>
</tr>
<tr>
<td>$K$</td>
<td>Transport delay corresponding to the Muskingum model</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Inflow / outflow weighting factor of the Muskingum model</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Natural frequency of the IR model</td>
</tr>
<tr>
<td>$M_r$</td>
<td>Resonance peak of the IR model</td>
</tr>
<tr>
<td>$Q$</td>
<td>Vector of canal discharges</td>
</tr>
<tr>
<td>$H_d$</td>
<td>Vector of downstream water depths</td>
</tr>
<tr>
<td>$R$</td>
<td>Vector of target downstream water depths</td>
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<tr>
<td>$e$</td>
<td>Vector of downstream water depth errors</td>
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<tr>
<td>$P_d(s)$</td>
<td>Offtake disturbance transfer matrix</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>$A$</td>
<td>State matrix</td>
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<td>$B$</td>
<td>Input matrix</td>
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<tr>
<td>$C$</td>
<td>Output matrix</td>
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<tr>
<td>$D$</td>
<td>Feedforward matrix</td>
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<tr>
<td>$B_d$</td>
<td>Offtake disturbance input matrix</td>
</tr>
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<td>$D_d$</td>
<td>Offtake disturbance feedforward matrix</td>
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<td>State vector</td>
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<tr>
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<td>Input discharge variations vector</td>
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<td>$h$</td>
<td>Output downstream depth variations vector</td>
</tr>
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<td>$d$</td>
<td>Offtake discharge disturbance vector</td>
</tr>
<tr>
<td>$N_r$</td>
<td>Number of reaches inside the canal</td>
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<td>$T_s$</td>
<td>Controller sampling time</td>
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<td>$N_p$</td>
<td>Prediction horizon samples</td>
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<tr>
<td>$k$</td>
<td>Discrete time variable</td>
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<tr>
<td>$l$</td>
<td>Step ahead prediction samples variable</td>
</tr>
<tr>
<td>$(k + l</td>
<td>k)$</td>
</tr>
<tr>
<td>$J$</td>
<td>Linear quadratic cost function</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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</tr>
<tr>
<td>$z$</td>
<td>Vector of optimal discharge variations inputs sequence</td>
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<tr>
<td>$Q_h$</td>
<td>Downstream depth error weighting matrix</td>
</tr>
<tr>
<td>$R_{\Delta q}$</td>
<td>Discharge input control effort rate weighting matrix</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Gate opening saturation tolerance</td>
</tr>
<tr>
<td>$S$</td>
<td>Slave controller</td>
</tr>
</tbody>
</table>
# Table of Contents

1. Introduction .................................................................................................................. 1
   1.1. Background ............................................................................................................ 1
   1.2. Motivation ............................................................................................................. 4
   1.3. Objective ............................................................................................................... 4
   1.4. Literature Review ................................................................................................. 5
      1.4.1. Control-Oriented Canal Models ....................................................................... 5
         1.4.1.1. Empirical Approximations ...................................................................... 6
         1.4.1.2. Analytical Approximations of the SVEs ................................................... 6
         1.4.1.3. Data Driven Models ................................................................................. 8
      1.4.2. Control System Variables ............................................................................... 9
         1.4.2.1. Controlled Output Variables – Control Objectives ................................... 9
         1.4.2.2. Control Input Variables ............................................................................. 9
      1.4.3. Controllers ...................................................................................................... 10
         1.4.3.1. PID Controllers ....................................................................................... 10
         1.4.3.2. Optimal Controllers ................................................................................ 10
         1.4.3.3. Predictive Controllers .............................................................................. 11
   1.5. Thesis Outline ...................................................................................................... 11
   1.6. Simulation Tools .................................................................................................. 12

2. Canal Dynamics ......................................................................................................... 13
   2.1. Open Channel Flow - The Saint Venant Equations ............................................ 13
      2.1.1. Definition ....................................................................................................... 13
      2.1.2. Assumptions .................................................................................................. 13
      2.1.3. The Equations ............................................................................................... 14
      2.1.4. Initial & Boundary Conditions ..................................................................... 16
      2.1.5. Steady State Equations ................................................................................ 17
      2.1.6. Linearized Equations .................................................................................... 20
      2.1.7. Laplace Domain Representation .................................................................. 21
      2.1.8. Dynamical Characteristics of Different Canals ............................................ 23
2.1.8.1. Frequency Response of Canal A – Short and Flat Channel...23
2.1.8.2. Frequency Response of Canal B – Long and Steep Channel...25

2.2. Hydraulic Structures – Sluice Gates..............................................27
2.2.1. Non-Linear Gate Equations.......................................................28
  2.2.1.1. Free Flow Conditions.........................................................28
  2.2.1.2. Submerged Flow Conditions...............................................28
2.2.2. Linearized Gate Equation.........................................................29
2.2.3. Local Feedback Effect..............................................................29

3. Control-Oriented Models.................................................................33
3.1. Model Formulation.........................................................................33
  3.1.1. The ID Model........................................................................34
    3.1.1.1. Uniform Section..............................................................34
    3.1.1.2. Backwater Section..........................................................35
    3.1.1.3. Full ID Model.................................................................35
  3.1.2. The IDZ Model.........................................................................35
  3.1.3. The Muskingum Model............................................................36
  3.1.4. The IR Model..........................................................................37
3.2. Model Evaluation...........................................................................38
  3.2.1. Step Response Test..................................................................38
  3.2.2. Frequency Response Test.......................................................41
  3.2.3. Model Selection......................................................................42

4. Proposed Controller...........................................................................44
4.1. Master Layer................................................................................45
  4.1.1. Centralized Model – Reach Interconnection..........................45
  4.1.2. MPC Controller.....................................................................48
4.2. Slave Layer..................................................................................51

5. Controller Evaluation – ASCE Benchmark.........................................54
5.1. ASCE Test Case 2 – The Corning Canal........................................55
5.2. Description of the Proposed Benchmarks......................................56
5.3. Controller Parameters....................................................................57
5.4. Results of Benchmark 1: Reference Tracking ................................................................. .58
5.5. Results of Benchmark 2: Disturbance Rejection ......................................................... .61
5.6. Discussion .................................................................................................................... .64
6. Conclusions and Future Work ....................................................................................... .66
Bibliography ........................................................................................................................ .67
1. Introduction

1.1. Background

Irrigation is the artificial application of water to the soil to assist in the growth of crops. This is typically done in areas where there is insufficient rainfall and precipitation or where the natural water supply is not enough to support agriculture, like in long periods of drought. The pivotal role of irrigation in global agriculture is made evident by its impacts to societies and economies around the world, namely and in no order of importance:

1. **Enhanced Agricultural Productivity**: Irrigation systems allow farmers to cultivate crops consistently throughout the year, reducing their dependence on seasonal rainfall. This consistency increases agricultural productivity and leads to a more reliable food supply, contributing to global food security.

2. **Economic Stability**: The implementation and maintenance of irrigation infrastructure create a stable and diverse agricultural sector, reducing the vulnerability of economies to the fluctuations caused by short- and long-term climate variation. A reliable agricultural output, facilitated by irrigation, can contribute to the robustness of economies.

3. **Global Trade and Food Exports**: Reliable irrigation infrastructure allows regions to grow crops in larger quantities and with consistent quality, enabling them to participate in international trade markets. Countries with well-established irrigation systems often become key players in global food exports, contributing significantly to their economic development.

4. **Water Resource Management**: Irrigation systems promote efficient water use by ensuring that water is delivered precisely where and when it is needed. This contributes to sustainable water resource management, preventing over-extraction from natural water sources and minimizing the environmental impact associated with agriculture. Sustainable irrigation practices are crucial for maintaining ecosystems and ensuring long-term water availability for both agriculture and other sectors.

As of the year 2000, the overall arable land encompassed an expanse of approximately 2,788,000 square kilometers and was globally equipped with irrigation infrastructure. Approximately 68% of this landmass is situated in Asia, 17% in America, 9% in Europe, 5% in Africa, and 1% in Oceania (Siebert et al., 2006). Said infrastructure mostly comes in the form of irrigation canals.
Irrigation canals are artificial water channels specifically designed for the controlled conveyance and distribution of water to agricultural fields. These canals form a fundamental component of irrigation systems, enabling the efficient transport of water from its source, such as rivers or lakes, to the cultivated land. Their length can range from a few hundred meters to tens or even hundreds of kilometers, while their width is usually measured in the range of only a few meters. Irrigation canals are usually comprised of open channel conduits, but communication with closed channels (pipes) is not uncommon. Irrigation canals are typically situated close to farmland areas where they branch out and transport water to the crops. An example of an irrigation canal network is the Corning Canal which is situated in California, USA and is illustrated in Figure 1.1.

Most irrigation canals assume the following structure. The main canal is comprised of a series of conduits transporting water from a large inertial body of water called a reservoir. These conduits are called reaches, and are usually open channels of trapezoidal cross section. Channel reaches are connected and communicate with their immediate neighbors at points of interconnection called nodes. The nodes are the locations of the irrigation canal where hydraulic check structures are built in order to control the operation of the network. Such hydraulic structures come in the form of sluice gates, radial gates, gated weirs, etc. In irrigation canals specifically, each reach contains lateral outflow branches called offtakes, that transport water from the main canal to the crops of neighboring farmlands.
A simplified irrigation canal containing all of the aforementioned structural characteristics is illustrated in Figure 1.2. From the side profile view one can see that at each reach interconnection there is a drop in the elevation of the bottom topography. This is the case because it is quite common for hydraulic structures to also function as drop structures in order to reduce the velocity of the transported water and stabilize the flow without the risk of erosion and high energy gradients.

Most offtakes are gravity offtakes, meaning that the amount of water (flow rate) they transport to the crops is influenced by the difference between the water elevation of a reach and the water elevation of the offtake branching off of it. Thus, in order to control the amount of water supplied to farmlands, one must control the water depths of all reaches in the irrigation canal network. This is accomplished by changing the openings of the gated hydraulic structures bounding each channel reach. This task is performed either manually by operators or autonomously by a control system.
1.2. Motivation

Strategic utilization of water resources and reservoirs has emerged as a significant concern in many parts of the world. The use of freshwater has been steadily rising globally, growing by around 1% each year since 1980 and has increased by a factor of six in the last century. Nearly 69% of global water withdrawals are attributed to irrigation, contributing to a projected 40% global water deficit by 2030 (UN World Water Development Report, 2021). Therefore, the modernization of irrigation canal networks is crucial in order to minimize water waste during operation.

Irrigation canal networks are complex dynamical systems. They are characterized by very long time delays and hydraulic response times, as well as strong coupling between the interconnected reaches. This complex behavior leads to difficulties in their efficient operation. Adding to that, the fact that they are distributed along great distances means that effective communication between operators becomes difficult. One can see that the manual operation of large scale irrigation systems is a laborious and inefficient task. And yet, until at least the early 2000s, the operation of irrigation canal networks in many parts of the world remained either completely manual or, at most, open loop controlled under specified management schedules (Burt et al., 1999), (Litrico & Fromion, 2006a). Even to this day, closed loop control systems have not been universally applied, leading to water waste and suboptimal crop growth in irrigation systems around the world.

Taking into account both the difficulty and inefficiencies that come with the manual operation of irrigation canal networks, as well as the pressing need to eliminate any and all freshwater waste globally, the value in the development of ever-advancing automatic canal control algorithms becomes apparent.

1.3. Objective

The objective of this thesis is the development of a centralized model predictive controller for the autonomous and efficient operation of large scale canal networks, and the evaluation of its performance using well established performance indicators. The control system developed in this thesis is tested on a simulated version of the Corning Canal, in particular the second test case provided by ASCE (American Society of Civil Engineers) in (Clemmens et al., 1998).
1.4. Literature Review

Since the late 1990s a plethora of algorithms and frameworks have been published for the purposes of modeling and controlling irrigation canal systems. As such, it is deemed useful to categorize these publications and analyze them based on the following criteria:

1. The types of control-oriented models used to represent the irrigation canal system dynamics, and the procedures to obtain them.
2. The types of control input and controlled output variables used.
3. The types of control laws and configurations formulated.

This literature review is by no means meant to be exhaustive, but is curated in such a way as to provide insight to the current state of the art in irrigation canal control algorithms. Different methodologies are examined and compared in order to lead to a well-rounded introduction to the field before delving into the implementation presented in this thesis. For dedicated and extensive literature reviews on the subject, the reader is referred to the works of (P.-O. Malaterre et al., 1998) and (Conde et al., 2021).

1.4.1. Control-Oriented Canal Models

It is well established that the most accurate one-dimensional hydrodynamic model for open-channel conduits are the Saint-Venant Equations (SVEs) (Chow, 2009). They are a set of non-linear, hyperbolic partial differential equations that describe the evolution of the wetted cross sectional area and the water discharge along a channel’s length.
The direct use of the SVEs as a model to design canal control laws is particularly impractical due to their non-linear and distributed nature, as shown in (Liu et al., 1995). As such, most control-oriented models formulated in literature fall into three major categories, which are also illustrated graphically in Figure 1.3.

1.4.1.1. Empirical Approximations

These types of models are constructed using a combination of first principles, empirical observations and practical approximations. Although not as mathematically or physically rigorous as the analytical models described later in 1.4.1.2, the empirical models are favored in literature due to their ease of use and reliable results.

The Muskingum model, introduced in (McCarthy, 1939), is a hydrological model used for flood routing that relates the input and output discharges of an open channel, weighing the influence of the inflow over the outflow. A pure Muskingum model is unable to capture the variation in water depth, but a transfer function that relates depths and discharges can be obtained by introducing a mass balance equation downstream, forming a reservoir. It has seen use for control purposes in (Mantecón et al., 2002) and (Gómez et al., 2002), among others, but it’s drawback is that it does not capture the time delay characteristics of the mass transport.

The Integrator-Delay (ID) model, first developed in (Schuurmans, 1997), assumes a backwater profile of the depth along a channel, consisting of a uniform flow part upstream and a constant-elevation reservoir part downstream, modeled as a mass-balance equation with the addition of an inflow delay. Due to its simplicity, low order and proven reliability, the ID model is the most utilized control-oriented model for irrigation canals to date, appearing in (R. Negenborn et al., 2009), (Wahlin, 2004), (Litrico & Fromion, 2006b), (P. Overloop et al., 2008), along many others. More recently, an extension of the ID model dubbed the Integrator-Dual-Delay model was proposed in (Zhu et al., 2023).

1.4.1.2. Analytical Approximations of the SVEs

These types of models are divided into two distinct groups. The first group entails the numerical discretization of the full SVEs using the Finite Difference Method.

One common approach to this is using an explicit finite difference scheme to obtain a control-oriented model of the SVEs. This method is prevalent in many works, such as (Bonet et al., 2017), (Soler-Guitart et al., 2013), (Cen et al., 2017) and (Garcia et al., 1992). The explicit nature of the discretization provides a relatively straightforward numerical resolution of the SVEs. However, it also introduces a dependence of the resulting model’s stability to the fidelity of the discretization. This means that to ensure
internal stability, the step size of the discretization must be adequately small, resulting in a very high order model.

To combat the dependence on the discretization, implicit schemes have been implemented that introduce asymptotic stability regardless of the step size. The most prevalent of these implicit schemes is, without a doubt, the Preissman Box Scheme, first formulated for control purposes in (Liu et al., 1998). Works utilizing the Preissman scheme include but are not limited to (P.-O. Malaterre & Khammash, 2000), (Silva et al., 2007) and (Figueiredo et al., 2013).

The second group of models involve the linearization of the SVEs, as well as the concentration of their distributed characteristics to the upstream and downstream portions of the modeled reach, forming linear lumped parameter models. Unlike the finite difference approximations, the models of the second group are all continuous by default and exhibit adequately low orders. Their simplicity relative to the first group makes them quite desirable, as they can more easily be implemented in low-level industrial Programmable Logic Controllers (PLCs).

The Hayami model, developed by (Hayami, 1951), is one of the earliest known models obtained by the linearization of the SVEs with the purpose of analyzing the propagation of flood waves in rivers. It essentially reduces the SVEs to a diffusive wave model, neglecting transport terms and assuming a time-invariant structure.

The Corriga model, formulated later by (Corriga et al., 1980), is the first model developed by taking the Laplace transform of the linearized SVEs, and one of the first models of this group to be used for the purposes of controlling irrigation canals. The model is comprised of transfer functions with delays that simulate the relationship between upstream and downstream variables. Works utilizing it include (Qiao & Yang, 2010) and (Corriga et al., 1982).

The Integrator-Delay-Zero (IDZ) model developed by (Litrico & Fromion, 2004a) forms similar relationships between the depths and discharges upstream and downstream of the reach. Its low frequency behavior is almost identical to that exhibited by the ID model. However, the IDZ model captures high frequency hydraulic phenomena much more accurately than the ID and Corriga models due to the presence of a zero in the numerator of its transfer functions. Its improved accuracy in high frequencies was established in (Litrico & Fromion, 2004c), where its frequency response was compared to that obtained by the Preissman discretization of the SVEs. The IDZ model is, to date, one of the best compromises between model order economy and physical consistency in both low and high frequencies. It has seen extensive use in literature, such as in (Horváth et al., 2014), (Segovia et al., 2018), (Guanghua &
Menghao, 2020) and (Segovia et al., 2019). A higher order IDZ model was alternatively employed in (Liao et al., 2018).

Finally, for shorter, flat canals prone to hydraulic resonance phenomena, the Integrator-Resonance (IR) model was proposed in (van Overloop et al., 2010), accurately capturing the first mode of a hydraulic oscillator. However, its effectiveness is reduced when modeling longer canals with considerable slope, particularly due to the absence of time delays in the model.

1.4.1.3. Data Driven Models

These types of models are obtained by the use of measured data, rather than first principles. By measuring the input and output data of a canal system, one can perform system identification techniques in order to obtain a data-driven model. These models are separated into two categories.

Black-Box models are data-driven models that characterize a canal system based solely on its input-output behavior, without explicit knowledge of the system’s internal structure and dynamics. Black-Box modeling is a very common and quite effective procedure for obtaining control-oriented models for irrigation canals through system identification. In literature, these models are usually parametric, either of the Auto Regressive with eXogenous inputs (ARX) or of the Auto Regressive Moving Average with eXogenous inputs (ARMAX) categories. ARMAX models are favored more in literature because they tend to model disturbances better, while ARX models are more economic, requiring less calculations for online-identification strategies. Both models are typically obtained by feeding the irrigation system (or a simulation thereof) with inputs that are Pseudo-Random Binary Sequences (PRBS). This choice of input is made in order to excite dynamics across a wide bandwidth, imitating white noise. A common alternative is to simply excite the system using step inputs and measuring their step responses, essentially modeling only the low frequency dynamics. Examples of Black-Box models in literature are (Rivas-Perez et al., 2008), (Rivas-Perez et al., 2003) and (Bolea & Puig, 2016).

Grey-Box models are data-driven models whose structure is predefined using first principles, while their parameters are estimated using system identification. In (Weyer, 2000), a Grey-Box model is proposed, whose structure is that of a non-linear mass-balance law. The parameters are identified using a prediction error method of quadratic cost. Similar strategies have been adopted in (Ooi & Weyer, 2005) and (Bedjaoui & Weyer, 2009).
### 1.4.2. Control System Variables

In this section, the variables relevant to the control of an irrigation system are discussed. All the relevant variables of a typical canal reach are illustrated in Figure 1.4.

![Control Variables of a Canal Reach](image)

**Figure 1.4: Control Variables of a Canal Reach**

#### 1.4.2.1. Controlled Output Variables – Control Objectives

As mentioned in 1.1, the main objective of a typical irrigation canal control system is the retention of the water depth $H_d$ downstream of each reach at a specified target value based on the offtake demand $D$. Naturally, the canal must never overflow. This control objective is prevalent in nearly all literature on the subject. The control of the upstream depth $H_u$ is usually seen as redundant, since the offtakes are commonly situated downstream and it holds that $H_d > H_u$ for non-flat canals. Alas, example works that include it in the control objectives are (Horváth, Galvis, Valentín, & Benedé, 2015) and (Segovia et al., 2019), which entail the control of short, flat in-land navigation canals.

#### 1.4.2.2. Control Input Variables

An irrigation canal network operation objective is achieved by utilizing a controller that acts on the gate openings of the hydraulic structures situated at the nodes. Thus, the actual control input variables of each reach are the upstream and downstream gate openings $W_u$, $W_d$. However, the actual implementation of the gate openings as control inputs in a model-based control system is difficult due to the non-linear interactions between reaches. This choice of inputs is usually reserved for data driven models described in 1.4.1.3. A much more common approach in literature is the use of the upstream and downstream discharges $Q_u$, $Q_d$ as control input variables from a master controller, and the subsequent transformation of these discharges to required
gate openings by a set of slave controllers. Most models described in 1.4.1.1 and 1.4.1.2 adopt this approach, with (Litrico et al., 2008) describing various slave controller architectures. In (Horváth, Galvis, Valentín, & Benedé, 2015), the pros and cons in the choice between discharges and gate openings as control inputs are discussed in detail.

1.4.3. Controllers

1.4.3.1. PID Controllers

The most common control algorithm used both in canal control literature and in the process control industry is Proportional-Integral-Derivative (PID) control. The reason for the prevalence of PID controllers is their simplicity, and the fact that they can be tuned without the need for proper control-oriented models, rather with industrial auto-tuners. For canal control specifically, PI controllers are utilized to maintain a constant water depth downstream of each channel reach. Since PID controllers are Single-Input Single-Output (SISO) systems, they can only act on a single canal reach at a time. Therefore, they are used either in distributed or decentralized control architectures. Examples of PI controllers in literature include (Litrico et al., 2003), (Litrico & Fromion, 2006b) and (Bolea, Puig, & Grau, 2014). In (P. Overloop et al., 2005), a PI controller with a filter was examined for the control of resonant canals.

1.4.3.2. Optimal Controllers

A common approach to achieve optimal control in irrigation canals is by the use of Linear Quadratic Regulators (LQR). LQR controllers can naturally handle Multiple-Input Multiple-Output (MIMO) systems, and are thus typically used in centralized architectures. In the work (P. O. Malaterre, 1998) an LQR controller was first used in an irrigation canal application. Later in (Faruk Durdu, 2006) an LQR controller was supplemented with a Kalman Filter, forming a Linear Quadratic Gaussian (LQG) controller. This was done for the purposes of comparing different state observers. A comparison between an infinite horizon LQR controller and a finite horizon unconstrained Model Predictive Controller (MPC) is discussed in (P.-O. Malaterre & Rodellar, 1996). The results of this work showed that MPC controllers exhibit considerably improved control capabilities.

An alternative application of optimal control in irrigation canals concerns the optimal tuning of decentralized or distributed PI controllers, rather than the use of MIMO controllers. The works (P. Overloop et al., 2005) and (Arauz et al., 2020) address this method. In the latter work specifically, a Linear Matrix Inequalities (LMI) optimal control problem is solved, resulting in feedback that tunes the PI controllers.
1.4.3.3. Predictive Controllers

Predictive controllers have gained considerable appeal in process control over the years as computational power steadily increases. Irrigation canal control applications are no different. MPC control specifically has become prevalent in literature, appearing in (Wahlin, 2004), (R. Negenborn et al., 2009), (Segovia et al., 2018), (Cen et al., 2017) and (P. J. Overloop et al., 2014), among others. MPC control is compatible with both centralized and decentralized or distributed architectures. Distributed MPC control is particularly desirable in large scale, multiple reach irrigation networks, as it is far less computationally intensive in comparison to its centralized counterpart. MPC controllers are also particularly effective in controlling systems that exhibit considerable time delays. Another benefit of MPC over the aforementioned control strategies is the ability to enforce hard constraints on the control inputs and controlled outputs. However, the effectiveness of the MPC controller in disturbance rejection suffers when they are not measured accurately, as illustrated in (Horváth, Galvis, Valentín, & Rodellar, 2015).

The Smith Predictor is an alternative control strategy that enables the use of non-predictive controllers in systems that include time delays, without taking said delays into account in the main controller tuning process. Example works include (Bolea, Puig, & Blesa, 2014) and (Bolea & Puig, 2016).

1.5. Thesis Outline

The structure of this thesis is organized as follows:

• Chapter 1 served as a brief introduction to the irrigation canal control problem. Additionally, a small part of past and current literature was examined, reviewing prevalent control-oriented models, control variables and control strategies.

• Chapter 2 illustrates the complete dynamics of an irrigation canal system. First, the complete hydrodynamic behavior of the open channel reaches is analyzed. Subsequently, the non-linear coupling between the canal reaches due to the presence of hydraulic structures is considered.

• Chapter 3 investigates the various control-oriented models presented in 1.4.1. The models are subsequently compared against the full canal model over their time and frequency responses.

• Chapter 4 presents the proposed irrigation canal control system structure. The overall control architecture and its installation on the canal is illustrated. Additionally, both the master and slave control layers are extensively analyzed.
Chapter 5 introduces the benchmarks introduced by ASCE for the purposes of gauging the effectiveness of canal control algorithms. The proposed controller is then tested on said benchmarks and its performance is evaluated using well established performance indicators.

1.6. Simulation Tools

Since a laboratory canal is not available for the purposes of testing the effectiveness of the proposed predictive controller, a fluid simulation is utilized instead. In the student thesis (Georgakis, 2023) a Computational Fluid Dynamics (CFD) canal simulator was developed that solves the full transient SVEs over a network comprised of reaches and gated hydraulic structures. The numerical scheme employs a combination of the Finite Volume Method (FVM), Riemann solvers and strong stability preserving time integrators. It was first programmed in Julia, but later ported to MATLAB in order to take advantage of the extensive control framework it provides. The combined CFD and control system simulator is named SWIFTEC (Shallow Water Integrated Framework For TEsting Controllers).
2. Canal Dynamics

2.1. Open Channel Flow - The Saint Venant Equations

2.1.1. Definition

The Saint-Venant equations (SVEs), named after Adhémar Jean Claude Barré de Saint-Venant and first formulated in (Saint-Venant, 1871), are a set of hyperbolic partial differential equations used to model the flow of water in open channel conduits. The SVEs can be derived by section-averaging the Shallow Water Equations in one dimension, which, in turn, are derived from the fundamental principles of fluid mechanics, the conservation of mass and momentum. The Saint-Venant equations are commonly employed in hydraulic engineering for analyzing and simulating the behavior of water in rivers, streams, and man-made canals. They form the basis for various numerical models that help engineers predict and manage water flow in different hydraulic scenarios.

2.1.2. Assumptions

When utilizing the SVEs to model open channels, the following physical assumptions ought to be made:

1. The flow of water is one-dimensional, with a uniform velocity profile lateral to each flow cross section.
2. The pressure along the channel is hydrostatic, assuming vertical accelerations are negligible in comparison to horizontal advection.
3. The flow is approximately laminar, with turbulence and friction phenomena being grouped in appropriate resistance laws valid for both steady and unsteady flow.
4. Variations of the width of the channel along the direction of flow are small.
5. The slope of the bottom topography of the channel is small.

If assumptions 1 or 4 are violated, the flow can no longer be considered one-dimensional. In that case, the 2D Shallow Water Equations should be solved instead. If assumptions 2 or 5 do not hold, then either the Boussinesq or Euler equations should be used in order to account for the vertical acceleration components. Finally, if assumption 3 fails and the flow is strongly turbulent, then the full Navier-Stokes equations should be used to model the channel hydrodynamics. In this thesis, all five assumptions are considered valid.
2.1.3. The Equations

As previously stated, the SVEs are a set of coupled hyperbolic partial differential equations. The first equation describes the conservation of mass along a channel:

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = I(x,t)$$  (2.1)

The second equation describes the conservation of momentum along a channel:

$$\frac{\partial Q(x,t)}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2(x,t)}{A(x,t)} \right) = gA(x,t) \left( S_0 - \frac{\partial H(x,t)}{\partial x} - S_f \right)$$  (2.2)

Although stated to be conservation laws, Eqs. (2.1) and (2.2) are actually balance laws, due to the presence of the source terms acting as sources or sinks of mass and momentum respectively.

The variables relevant to the SVEs are as follows:

- $x \ (m)$: The abscissa along the flow direction
- $t \ (s)$: The time variable
- $A(x,t) \ (m^2)$: The wetted cross-sectional area of the channel. For a given cross sectional geometry, a relation $A(x,t) = f(H(x,t))$ can be defined to obtain closure of the SVEs.
- $H(x,t) \ (m)$: The water depth
- $Q(x,t) \ (m^3/s)$: The water discharge, given by $Q(x,t) = A(x,t) V(x,t)$

Figure 2.1: One-dimensional open channel flow
Modeling & Dynamic Control of Irrigation Canals

Panagiotis Georgakis

- $V(x,t) \, (m/s)$: The flow velocity
- $C(x,t) \, (m/s)$: The wave celerity
- $I(x,t) \, (m^2/s)$: The lateral water inflow discharge
- $g \, (m^2/s)$: The gravitational acceleration
- $S_0$ (-): The slope of the bottom topography
- $S_f$ (-): The friction source term

The friction source term $S_f$ accounts for momentum losses due to the friction of the flowing water with the bottom of the channel. Manning’s friction law, formulated in (Manning et al., 1890) and shown in Eq. (2.3), is one of the most prevalent friction models in open channel literature.

$$S_f = \frac{n^2 Q^2}{A^2 R^{3/4}}$$

(2.3)

Where:
- $n \, (sm^{-1/3})$ : The Manning’s friction coefficient
- $R \, (m)$ : The hydraulic radius, defined as $R = A(x,t)/P(x,t)$
- $P \, (m)$ : The wetted perimeter, illustrated in
- $T \, (m)$ : The top width of the wetted channel cross section

![Diagram](Figure 2.2: Cross section of an open channel - wetted area $A(x,t)$, top width $T(x,t)$ and wetted perimeter $P(x,t)$ are clearly defined)
2.1.4. Initial & Boundary Conditions

In order for the SVEs to be fully defined, they ought to be complemented with physically consistent initial and boundary conditions. Enforcing initial or boundary conditions that violate the physics of open channel flow can lead to, at best, numerical artifacts and inconsistencies, and at worst, solutions that fail to converge. The analysis of the choice of initial and boundary conditions presented here is taken from the complementary student thesis (Georgakis, 2023).

The initial conditions come in the form of initial water depth and discharge profiles $H(x,0)$ and $Q(x,0)$ respectively. For simulation and control purposes, these are often selected to be possible steady state profile configurations of the channel. This practice ensures smooth solutions, barring unwanted numerical artifacts potentially produced by the scheme chosen to solve the SVEs.

To obtain physically consistent boundary conditions of the SVEs, one must take into account the various flow regimes of open channel flow. For that, it is necessary to define the Froude number, shown in Eq. (2.4).

$$Fr(x,t) = \frac{V(x,t)}{C(x,t)} = \frac{V(x,t)}{\sqrt{\frac{g}{A(x,t)}}}$$

(2.4)

The Froude number is a characteristic dimensionless number that describes the strength of the inertial forces relative to the forces of gravity acting on a channel. Depending on the values of the Froude number different kinds of flow regimes are exhibited along a channel reach. The choice of boundary conditions depends on said flow regime configurations.

- If $Fr < 1$ at a boundary, then the flow at that boundary is said to be subcritical. When the flow is subcritical, the velocity of wave propagation is greater than the mean flow velocity. This means that waves can travel both upstream and downstream. Subcritical flow tends to be smooth and slow, usually found in deep slow-moving rivers, gentle-sloped man-made canals, and large reservoirs. At a boundary where the flow is subcritical, exactly one boundary condition must be enforced, either depth $H$ or discharge $Q$.

- If $Fr > 1$ at a boundary, then the flow at that boundary is said to be supercritical. When supercritical flow occurs, the velocity of wave propagation is smaller than the mean flow velocity. Any wave produced during the flow regime is only able to travel towards the direction of flow, downstream. Supercritical flow is usually fast and erratic, typically found in steep mountain rivers and waterfalls. At a
boundary where flow is supercritical, both depth $H$ and discharge $Q$ boundary conditions need to be enforced.

- Finally, if $Fr \approx 1$, then the flow is said to be transcritical, thus approaching criticality ($Fr = 1$). Transcritical flow regimes are violent and discontinuous, characterized by the presence of hydraulic jumps. This type of flow regime is often found at weirs, dams, and right before spillways. It is uncommon for such regime to occur at a channel boundary. Transcritical flow usually occurs at a location where the flow changes from super- to subcritical.

Taking the physics of the various possible flow regimes into consideration, one can select appropriate boundary conditions upstream and downstream of a channel reach to complete the SVEs.

For the purposes of modeling, simulating and controlling irrigation systems, it should be noted that most canals operate at subcritical flow regimes. This means that two boundary conditions are needed for a single channel reach, one for each boundary. The most common physically consistent boundary condition configuration seen in open channel hydraulics literature is an inflow hydrograph $Q(0, t)$ upstream and a rating curve $H(L, t) = f(Q(L, t))$ downstream, where $L$ the length of the reach. However, in the book (Litrico & Fromion, 2009), it was proven that the application of discharge boundary conditions $Q(0, t)$ and $Q(L, t)$ upstream and downstream respectively is able to converge to a correct solution. This information is useful, as it enables control engineers to act on boundary discharges to control the boundary depths, as discussed in 1.4.2.

2.1.5. Steady State Equations

Most linear models mentioned in 1.4.1 are obtained through linearization around a steady state solution of the canal. Although this methodology neglects the modeling of transient shock phenomena, such as hydraulic jumps, it enables control engineers to utilize the conventional tools and strategies of linear systems and control theory. As such, it is deemed useful to illustrate some of the key characteristics of the steady state profiles an open channel reach can assume.

The steady state solutions of the SVEs can be obtained by neglecting the partial time derivative terms of Eqs. (2.1) and (2.2). Let $(H_0(x), Q_0(x))$ be a steady state profile of a channel reach, with subscript $(\cdots)_0$ indicating the steady value of a flow variable. Specifically, the steady state depth profiles of a channel reach are called backwater curves. The steady state form of the SVEs is shown in Eqs. (2.5) and (2.6). The mass source term (lateral water inflow) is omitted for simplicity.
\[
\frac{dQ_0(x)}{dx} = 0 \Rightarrow Q_0(x) = Q_0 \\
\frac{dH_0(x)}{dx} = \frac{S_0 - S_{f0}(x)}{1 - F_0^2(x)}
\]

A notable case of steady state solutions can be obtained from the homogeneous form of the steady SVEs. In that case, the steady state equations produce a constant steady profile \((H_0(x), Q_0(x))\) along the whole length of the channel reach. The homogeneous momentum equation can then be reduced to Eq. (2.7),

\[
S_0 = S_{f0}(x)
\]

the solution of which produces a constant depth \(H_0(x) = H_n\), called the normal depth. The normal depth corresponds to the uniform flow regime, where the depth of flow is constant and equal to the normal depth along the channel. The normal depth and the uniform flow regime are of particular interest for control engineers since they can be used to formulate linear control-oriented models.

Using the normal depth as a reference, it is useful to examine the steady backwater curves produced by different boundary conditions. For this purpose, two distinct types of open channels with different dynamical characteristics are examined. Their parameters are illustrated in Figure 2.3 and catalogued in Table 2.1.

<table>
<thead>
<tr>
<th>Table 2.1: Parameters of the two types of open channels</th>
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<tbody>
<tr>
<td><strong>Canal A</strong></td>
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<tr>
<td>(L)</td>
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<tr>
<td>(m)</td>
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<tr>
<td>(B)</td>
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<td>(Q_0)</td>
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Figure 2.3: Side profile (left) and trapezoidal cross section (right) of a typical open channel with relevant geometric parameters and steady state quantities
Canal A is a shorter, relatively flat channel with considerable water storage. In contrast, Canal B is a long and steep channel with a shallow water profile. The normal depths $H_n$ listed in Table 2.1 correspond to the selected steady input discharges $Q_0$. Note that a channel’s normal depth and steady input discharge form unique pairs.

By steadily supplying channels A and B with constant discharges $Q_0$, and enforcing different boundary conditions for the downstream water depth $H_0(L)$, it is possible to obtain different backwater profiles for the two types of channels.

- If $H_0(L) > H_n$, then the water flowing through the channel is decelerating. This often occurs when the water meets an obstruction like a hydraulic structure downstream. This is the most common backwater profile and the one that is of interest for canal control purposes. A decelerating curve could alternatively be obtained by enforcing $H_0(L) = H_n$ and lowering the input discharge.

- If $H_0(L) < H_n$, then the water flowing through the channel is accelerating. This is usually the case when there is a sudden slope increase downstream of the channel. An accelerating curve could alternatively be obtained by enforcing $H_0(L) = H_n$ and raising the input discharge.

- If $H_0(L) = H_n$, then it is obvious that the flow is uniform, and $H_0(x) = H_n$.

![Figure 2.4: Backwater curves corresponding to different downstream depth boundary conditions – Canal A](image)
In Figures 2.4 – 2.5, all of the possible backwater configurations are illustrated by enforcing downstream depth boundary conditions \( H_0(L) = H_n \times [0.8, 1.0, 1.2] \). The different backwater profiles are computed either by simulating the full SVEs using SWIFTEC and letting the simulation reach steady flow conditions, or by numerically integrating the steady SVEs (2.5) and (2.6). Both methods yield identical profiles.

### 2.1.6. Linearized Equations

As discussed in 2.1.5, one can linearize the SVEs around a steady state operating point \((H_0(x), Q_0(x))\) in order to gain access to the powerful tools developed for the analysis of linear dynamical systems. This is done by studying the evolution of the variations of the flow variables around a steady state, rather than the flow variables themselves. Let \(h(x, t)\) and \(q(x, t)\) be said variations of the water depth and discharge variables respectively, such that:

\[
H(x, t) = H_0(x) + h(x, t) \quad (2.8)
\]
\[
Q(x, t) = Q_0(x) + q(x, t) \quad (2.9)
\]

By substituting Eqs. (2.8) and (2.9) to the full SVEs (2.1) and (2.2) and neglecting derivative terms higher than first order, one can obtain the linear approximation of the SVEs shown in Eqs. (2.10) and (2.11).
\[ T_0(x) \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \]  
(2.10)

\[ \frac{\partial q}{\partial t} + 2V_0 \frac{\partial q}{\partial x} + (C_0^2(x) - V_0^2(x)) T_0 \frac{\partial h}{\partial x} - \beta_0(x) q - \gamma_0(x) h = 0 \]  
(2.11)

The parameters of the linearized SVEs are calculated by the expressions below, where spatial dependence is omitted for readability.

\[ \beta_0 = -\frac{2g}{V_0}(S_0 - \frac{\partial H_0}{\partial x}) \]  
(2.12)

\[ \gamma_0 = V_0^2 \frac{dT_0}{dx} + g T_0 \left[ (1+\kappa)S_0 - (1+\kappa-(\kappa-2)F_0^2) \frac{dH_0}{dx} \right] \]  
(2.13)

\[ \kappa = \frac{7}{3} - \frac{4A_0}{3T_0P_0} \frac{\partial P_0}{\partial H} \]  
(2.14)

The linearized open channel system is complemented with discharge boundary conditions \( q(0,t) = q_0(t) \) and \( q(L,t) = q_L(t) \) at the upstream and downstream boundaries respectively. For detailed derivation of the linearized SVEs, the reader is referred to (Litrico & Fromion, 2004b).

### 2.1.7. Laplace Domain Representation

One of the purposes of linearizing the SVEs is to obtain an input-output transfer matrix \( P(s) \) in the Laplace domain that relates the boundary water depths to the boundary discharges, or variations thereof. This relation is exhibited in Eq. (2.15).

\[
\begin{bmatrix}
  h(0,s) \\
  h(L,s)
\end{bmatrix} = \begin{bmatrix}
  p_{11}(s) & p_{12}(s) \\
  p_{21}(s) & p_{22}(s)
\end{bmatrix} \begin{bmatrix}
  q(0,s) \\
  q(L,s)
\end{bmatrix}
\]

(2.15)

Where \( s \) is the complex Laplace variable. To obtain this transfer matrix, it is necessary to apply a Laplace transform to the linearized SVEs (2.10) and (2.11). Doing so and reordering the terms leads to the distributed system of ordinary differential equations (2.16).

\[ \frac{\partial}{\partial x} \begin{bmatrix}
  q(x,s) \\
  h(x,s)
\end{bmatrix} = A(x,s) \begin{bmatrix}
  q(x,s) \\
  h(x,s)
\end{bmatrix} \]  
(2.16)
Where $A(x, s)$ the distributed system matrix which is defined as:

$$
A(x, s) = \begin{bmatrix} 0 & -B_0(x) s \\ -s + \beta_0(x) & 2 V_0(x) B_0(x) s + \gamma_0(x) \\ B_0(x) (C_0^2(x) - V_0^2(x)) & B_0(x) (C_0^2(x) - V_0^2(x)) \end{bmatrix}
$$

Integrating this linear system along the length of the canal would result in a solution of the following form:

$$
\begin{bmatrix} q(x, s) \\ h(x, s) \end{bmatrix} = \Gamma(x, s) \begin{bmatrix} q(0, s) \\ h(0, s) \end{bmatrix} = \begin{bmatrix} y_{11}(x, s) & y_{12}(x, s) \\ y_{21}(x, s) & y_{22}(x, s) \end{bmatrix} \begin{bmatrix} q(0, s) \\ h(0, s) \end{bmatrix}
$$

(2.17)

Where $\Gamma(x, s)$ the distributed state transition matrix. By evaluating this transition matrix, one can compute the required transfer matrix from Eq. (2.15) as follows:

$$
p_{11}(s) = -\frac{y_{11}(L, s)}{y_{12}(L, s)}
$$

$$
p_{12}(s) = \frac{1}{y_{12}(L, s)}
$$

$$
p_{21}(s) = y_{21}(L, s) - y_{22}(L, s) \frac{y_{11}(L, s)}{y_{12}(L, s)}
$$

$$
p_{22}(s) = \frac{y_{11}(L, s)}{y_{12}(L, s)}
$$

(2.18)

Since the matrix $A(x, s)$ exhibits spatial dependence there exists no closed form solution of (2.16). The only exception to this is when the flow is uniform, in which case $A$ no longer depends on $x$, as demonstrated in (Litrico & Fromion, 2009). In the same work, an economic numerical integration scheme was developed to solve for the unknown state transition matrix by approximating non-uniform flow as uniform for each spatial increment, and interconnecting the obtained solutions.

One can see that the solution of the characteristic equation $y_{12}(L, s) = 0$ yields the poles of the canal system. These poles are infinite in number due to the distributed nature of the hydrodynamic system. However, for any prospective control-oriented model, the low frequency poles are the most dominant and therefore the ones that must be accurately modeled. In particular, $y_{12}(L, s)$ always contains a first order integrator that is responsible for modeling the mass balance phenomena of the hyperbolic system, regardless of the canal geometry. This free integrator constitutes a marginally stable system.
2.1.8. Dynamical Characteristics of Different Canals

A linear canal model in the Laplace domain allows for the examination of the open channel flow characteristics in the frequency domain. This is done by calculating the frequency response of the canal system.

The frequency response or Bode plot is a graphical representation of the relationship between the input and output of a system as a function of frequency. The Bode plot typically consists of two plots: the magnitude response and the phase response. The magnitude response shows the ratio of the output magnitude to the input magnitude (usually in decibels) as a function of frequency, while the phase response shows the phase shift between the input and output signals as a function of frequency. The horizontal frequency axis is usually plotted on a logarithmic scale, in decades or octaves of radians per second (rad/s) or Hz. The magnitude axis is typically represented in decibels (dB) while the phase in degrees or radians.

To obtain the frequency response of a canal, or a simulation of the SVEs, one would have to excite it using sinusoidal input discharges of rising frequency in order to capture a large enough dynamics bandwidth. This is an arduous task, so a good compromise is to simply calculate the frequency response analytically using the linearized SVEs. This is done by simply substituting \( s = j \omega \) in the obtained transfer matrix \((2.15)\) and \((2.17)\), and separating the magnitude and phase of the resulting complex valued function of frequency. Then the magnitude and phase functions are plotted against frequency for a predetermined bandwidth.

Examining the Bode plot can give great insight on the dynamics of different types of canals at different frequencies. Additionally, having access to the frequency response of the SVEs can provide a good reference for the full dynamics of an open channel, which can then be compared to the dynamics of control-oriented models in order to gauge their validity. For these reasons, the frequency responses of canals A and B from 2.1.5 will be plotted and analyzed.

Because the downstream depth of a channel is the main variable to be controlled, the Bode plots will contain only the responses of the downstream depth \( h(0, s) \) due to upstream \( q(0, s) \) and downstream \( q(L, s) \) input discharge excitations. A frequency bandwidth of \( 10^{-5} - 10^{-1} \) Hz is deemed sufficiently large enough for both canals.

2.1.8.1. Frequency Response of Canal A – Short and Flat Channel

The Bode plots of the uniform, accelerating and decelerating backwater profiles assumed by Canal A are presented in Figure 2.6 and Figure 2.7.
Figure 2.6: Frequency response due to upstream inflow discharge - Canal A

Figure 2.7: Frequency response due to downstream outflow discharge - Canal A
By examining the above frequency responses of Canal A, various remarks can be made about the dynamics of short and flat canals.

- At low frequencies ($10^{-5} - 10^{-3}$ Hz) the canal exhibits pure integrator behavior, the gain of which is purely dependent on the inflow and outflow discharges. This means that the canal behaves like a reservoir at low excitation frequencies, wherein mass balance phenomena dominate over advection. Additionally, the free integrator means that the hydraulic system (barring the presence of hydraulic structures) is marginally stable, in the sense that a step input discharge excitation would result in a ramp response.

- At frequencies $10^{-3} - 3 \times 10^{-3}$ Hz the slope of the magnitude plot decreases, indicating the presence of a zero in the corresponding transfer functions. This zero relates to the propagation of waves along the channel due to abrupt changes in the inflow or outflow discharges.

- At high frequencies the oscillation of both magnitude and phase response plots becomes apparent. Each peak of the oscillating profiles corresponds to a resonant mode of the channel. Short and flat channels are prone to hydraulic resonance phenomena, in the sense that generated waves constantly propagate back and forth after bouncing off of the boundaries. The higher the frequency of resonance, the smaller the length of the propagating waves. One can also see that larger discharge inflows or outflows correspond to more pronounced resonant peaks. This inclination to resonance is well-known by control engineers and as such, short and flat channels are generally considered more difficult to control by the community than their longer and steeper counterparts.

- The decreasing values of the phase response due to the upstream inflow discharge $q(0,s)$ indicate that there exists a time delay. This means that changes in the inflow discharge take some time (deadtime) to affect the downstream depth. This does not occur for the downstream outflow discharge $q(L,s)$, which means that it affects the downstream depth instantly, as expected.

2.1.8.2. Frequency Response of Canal B – Long and Steep Channel

In the same manner as with Canal A, the Bode plots of the uniform, accelerating and decelerating backwater profiles assumed by Canal B are presented in Figure 2.8 and Figure 2.9.
Figure 2.8: Frequency response due to upstream inflow discharge - Canal B

Figure 2.9: Frequency response due to downstream inflow discharge - Canal B
Again, by examining the above frequency responses of Canal B, various remarks can be made about the dynamics of long and steep canals.

- At low frequencies \((10^{-5} - 10^{-3} \text{ Hz})\) the canal exhibits pure integrator behavior just like Canal A. This means that at low frequencies, canals behave as marginally stable reservoirs regardless of their geometric characteristics.

- At frequencies \((10^{-3} - 5 \times 10^{-3} \text{ Hz})\) the slope of the magnitude plot increases, indicating the presence of a second pole. Soon after, the slope decreases and flattens, indicating the presence of zeros. This series of characteristics models the development and propagation of waves along the channel.

- At high frequencies no resonance peaks are visible in the frequency plots. This suggests that long and steep canals do not allow propagating waves to bounce off of the boundaries. This makes sense, as the large length of these channels means that waves dissipate before they reach the boundaries. The steeper slope also contributes to this, as waves traveling upstream require considerably more energy to overcome gravity, as indicated by the flatter shape of the magnitude plot Figure 2.9 due to the downstream outflow discharge \(q(L, s)\).

- The decreasing values of the phase response due to the upstream inflow discharge \(q(0, s)\) indicate that there exists a time delay, just like with Canal A. This means that all canals exhibit deadtimes, regardless of geometry. Time delays are, in fact, a property shared by all hyperbolic systems, since the velocity in which information propagates is finite.

2.2. Hydraulic Structures – Sluice Gates

Although the SVEs are a very accurate model for unobstructed open channel flow, they do not hold at canal nodes where hydraulic check structures are situated. As such, in order to obtain a complete canal model, it is necessary to also model the physics of hydraulic structures.

Hydraulic structures come in many forms. Some examples concerning irrigation canals specifically include sluice gates, weirs, culverts, spillways and dams, among others. However, in the present thesis special focus is given on sluice gates, since they are the structures exclusively used in the control benchmarks presented in Chapter 5. For extensive information on the mechanisms of hydraulic structures, the reader is referred to (Chow, 2009) and the (HEC-RAS Hydraulic Reference Manual, n.d.).
2.2.1. Non-Linear Gate Equations

As with most hydraulic structures, sluice gates are modeled as non-linear relationships between the flow rate through them $Q$, the gate control opening $W$ and the neighboring water depths $H_{u/d}$. The structure of this relationship depends on the flow condition in which the gate operates.

2.2.1.1. Free Flow Conditions

Free flow conditions occur when the head difference between the water depths upstream and downstream of the gate is large. This leads to the water entering the gate to exit it unobstructed. Free flow conditions are typically found in shallow channels with negligible backwater effects. The equation of a sluice gate under free flow conditions is given in Eq. (2.19), where $C_g$ is a gate discharge coefficient and $L_g$ the width of the gate opening.

$$Q(t) = f(W, H_u) = C_g L_g W(t) \sqrt{2gH_u(t)}$$  \hspace{1cm} (2.19)

2.2.1.2. Submerged Flow Conditions

Submerged flow conditions occur when the head difference between the water depths upstream and downstream of the gate is small. This leads to the water entering the gate to crash into backwater upon exit. Submerged flow conditions are usually found in flatter channels with significant storage that are fully submerged in backwater. The equation of a sluice gate under submerged flow conditions is given in Eq. (2.20).

$$Q(t) = f(W, H_u, H_d) = C_g L_g W(t) \sqrt{2g(H_u(t) - H_d(t))}$$  \hspace{1cm} (2.20)

Figure 2.10 illustrates the difference between free and submerged flow.
2.2.2. Linearized Gate Equation

In the same manner as with the SVEs, it is useful to linearize the gate equations around a steady operating point in order to formulate linear models. For this, small variations of the gate discharge \( q \), gate opening \( w \) and neighboring water depths \( h_u \) and \( h_d \) around a steady state are considered. Thus, the linearized gate equation is presented in Eq. (2.21).

\[
q(t) = k_u h_u(t) + k_w w(t) + k_d h_d(t)
\]

The linearization coefficients are calculated by differentiating the non-linear gate equations around a steady operating point, as shown in Eq. (2.22).

\[
k_u = \frac{\partial f}{\partial H_u}(W_0, H_{u0}, H_{d0})
\]

\[
k_w = \frac{\partial f}{\partial W}(W_0, H_{u0}, H_{d0})
\]

\[
k_d = \frac{\partial f}{\partial H_d}(W_0, H_{u0}, H_{d0})
\]

If the flow under the gate is free then \( k_d = 0 \).

2.2.3. Local Feedback Effect

As discussed in 2.1.7-2.1.8, a channel reach without the presence of hydraulic structures exhibits a delayed pure integrator behavior, meaning the transfer matrix \( P(s) \) in Eq. (2.18) has a marginally stable structure. However, this is no longer the case when hydraulic structures are present at the boundaries of the open channel. To prove this, the following example is presented, illustrated in Figure 2.11.

![Figure 2.11: Canal A subject to an unobstructed downstream boundary (left) and a downstream sluice gate (right)](image-url)
Let there be an open channel with either an unobstructed downstream boundary or a downstream sluice gate, wherein water flows out with a discharge variation $q_2$. In the case of the sluice gate, there is an inert reservoir situated downstream of the gate. The upstream reach boundary always remains unobstructed and is subject to an inflow discharge variation $q_1$. The parameters of the channel are taken to be those of Canal A in Table 2.1. The parameters relevant to the gate are given in Table 2.2.

<table>
<thead>
<tr>
<th>Gate Coefficient</th>
<th>Gate Width</th>
<th>Gate Opening</th>
<th>Reservoir Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_g$</td>
<td>$L_g$</td>
<td>$W_0$</td>
<td>$H_{res}$</td>
</tr>
<tr>
<td>0.65</td>
<td>7.0</td>
<td>0.44</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The responses of both downstream cases following a step input variation of $q_1 = 0.5 \text{ m}^3/\text{s}$ are shown in Figure 2.12 and Figure 2.13. The simulations are carried out by solving the full SVEs in SWIFTEC. As expected, the case of no hydraulic structures downstream shows a delayed integrator behavior, indicated by the ramp response to a step input. However, in the presence of a sluice gate downstream, the step response indicates a first order system behavior.
To explain this behavior, the linearized formulations of both the SVEs and the gate equation described in this chapter are utilized.

The downstream depth variation of the channel is given by the second row of the transfer matrix $P(s)$ in Eq. (2.15), that is:

$$h(s) = p_{21}(s) q_1(s) + p_{22}(s) q_2(s)$$

(2.23)

In the case of the sluice gate being present, the downstream outflow discharge variation $q_2$ is given by the linearized gate equation (2.24).

$$q_2(t) = k_u h(t)$$

(2.24)

Note that the gate opening and gate downstream depth variation terms are omitted. This is because the gate opening and water depth downstream of the gate are assumed to be constant. For this example, $k_u = 5.64$. Plugging Eq. (2.24) in Eq. (2.23) results in the following expression for the case of a sluice gate downstream:

$$h(s) = \frac{p_{21}(s)}{1 - p_{22}(s) k_u} q_1(s)$$

(2.25)
From Eq. (2.25) it becomes apparent that the system no longer contains free integrators, as the gate equation introduces local feedback downstream of the channel, due to the gate discharge $q_2$ being a function of the depth $h$. This is the reason why real canal networks can actually reach steady flow conditions after sudden changes in demand. This also further illustrates that the SVEs cannot accurately capture the full canal dynamics on their own. As such, careful consideration of the gate dynamics is taken in the following chapters.
3. Control-Oriented Models

The linearized model discussed in 2.1.7 cannot be efficiently utilized for control purposes because it is distributed, which means it possesses infinite poles and parameters. As such, it is necessary to formulate simplified, lumped parameter models for the channel reaches.

In this chapter, a portion of the canal control-oriented models introduced in the literature review 1.4.1 are properly defined and analyzed. The models are then compared in the time and frequency domain in order to gauge their accuracy and choose the most fitting for the canal control system. The model comparisons are carried out on Canal A.

3.1. Model Formulation

The models discussed in the later sections should be of the same structure as the linearized Laplace domain representation of the SVEs. In particular, the second row of the Saint-Venant transfer matrix $P(s)$ in Eq. (2.15) is employed, just like in 2.2.3, since the goal of the control system is the maintenance of the downstream water depth.

All simplified models employ variations of the flow variables around steady state operating points, rather than the flow variables themselves. Again, the variations of the flow variables are denoted using lower case letter versions of said flow variables.

$$h(s) = p_{21}(s) q_1(s) + p_{22}(s) q_2(s) + p_{23}(s) d(s)$$

Figure 3.1: Variables relevant to the process of canal modeling

The simplified control-oriented models are essentially approximations of the distributed transfer functions $p_{21}(s)$ and $p_{22}(s)$ that relate the upstream and downstream discharges to the downstream depth.
Here, the offtake discharge $D$ is assumed to be located very close to the downstream boundary, enough so that the offtake discharge variation $d$ affects the downstream depth in the same way as the outflow discharge. To avoid redundancy, offtakes are omitted in this chapter.

### 3.1.1. The ID Model

The Integrator-Delay (ID) model was first proposed by (Schuurmans, 1997). It works by splitting an open channel into two sections, an upstream uniform section and a downstream flat backwater section, as illustrated in Figure 3.2.

![Figure 3.2: Division of a steep channel into sections as proposed by the ID model](image)

Some types of open channels, like with the case of Canal A in 2.1.5, are completely submerged in backwater and exhibit no uniform section. Alas, the ID model attempts to captures their dynamics as well, albeit less accurately than with steep channels like Canal B.

#### 3.1.1.1. Uniform Section

In this section, the flow of water is assumed to be uniform. Additionally, it is assumed that formed waves are only allowed to travel downstream, as advection phenomena dominate this section. The time in which a wave can travel the length of the uniform section is given by Eq. (3.2).

$$
\tau_{ID} = \frac{L}{C_0 + V_0} \quad (3.2)
$$

Where $L$ the length of the channel, $C_0$ the wave celerity and $V_0$ the flow velocity at steady state. This wave travel time constitutes a time delay for the proposed model.
that affects the relationship between the inflow and outflow discharges of the uniform section. This relationship is shown in Eq. (3.3) in both the time and Laplace domains.

\[ q_2(t) = q_1(t - \tau_{ID}) \Leftrightarrow q_2(s) = q_1(s) e^{-\tau_{ID} s} \quad (3.3) \]

### 3.1.1.2. Backwater Section

Backwaters are difficult to model because their dynamics are characterized by resonance phenomena in high frequencies, as demonstrated in 2.1.8.1. However at low frequencies, backwaters behave like inert reservoirs characterized by simple mass-balance relationships. The mass-balance dynamics are shown in Eq. (3.4) in both the time and Laplace domains.

\[ A_{ID} \frac{dh(t)}{dt} = q_1(t) - q_2(t) \Leftrightarrow h(s) = \frac{1}{A_{ID} s} (q_1(s) - q_2(s)) \quad (3.4) \]

Where \( A_{ID} = T_0 L_d \) the total area assumed by the backwater reservoir, \( L_d \) the length of the backwater and \( T_0 \) the top width of the channel surface downstream.

### 3.1.1.3. Full ID Model

The full ID model can be obtained by joining the uniform and backwater sections. This is accomplished by simply assuming that the outflow discharge of the uniform section is equal to the inflow discharge of the backwater section. This produces the ID model shown in Eq. (3.5), represented in the Laplace domain.

\[ h(s) = \frac{1}{A_{ID} s} e^{-\tau_{ID} s} q_1(s) - \frac{1}{A_{ID} s} q_2(s) \quad (3.5) \]

The calculations of the ID model parameters require the evaluation of the abscissa where the two sections meet, which is the heart of the ID model. For the detailed derivation, the reader is referred to (Schuurmans, 1997).

### 3.1.2. The IDZ Model

The IDZ Model, first developed by (Litrico & Fromion, 2004c), aims to be a direct improvement of the ID model at high frequencies. It assumes the same basic principles as the ID model, in the sense that it splits an open channel into uniform and backwater sections. Where the IDZ model differs from the ID model is the addition of a zero in the numerator of its transfer functions to approximately model high frequency dynamics, something that the ID model fails to do so. The IDZ model assumes the form shown in Eq. (3.6) in the Laplace domain.
Where $P_{21 \infty}$ and $P_{22 \infty}$ are the added zeros. The parameters of the IDZ model excluding the zeros are calculated in the same manner as with the ID model, but the interconnection of the uniform and backwater sections is much more sophisticated. The lengthy derivations of the IDZ model are out of the scope of this thesis, so the reader is referred to (Litrico & Fromion, 2004c) for more information.

3.1.3. The Muskingum Model

The Muskingum hydrological model, developed by (McCarthy, 1939), aims to model the relationship between inflow and outflow discharges in a channel by considering only the effects of water transport and storage. The Muskingum transport model is shown in Eqs. (3.7) and (3.8).

$$u_s(t) = K (\chi q_1(t) + (1 - \chi)q_t(t))$$  \hspace{1cm} (3.7)

$$\frac{du_s(t)}{dt} = q_1(t) - q_t(t)$$  \hspace{1cm} (3.8)

Where $u_s(t)$ is the storage volume variation, $K = \tau_{ID}$ is the transport delay, $\chi$ is the weighting factor between inflow and transport discharges, and finally, $q_t(t)$ is the transport discharge variation. The transport model does not contain a reach outflow discharge variation term $q_2(t)$. In order to introduce it and complete the model, one must complement the transport model with a storage mass-balance equation (3.9).

$$A_s \frac{dh(t)}{dt} = q_t(t) - q_2(t)$$  \hspace{1cm} (3.9)

Where $A_s = T_0 L_d = A_{ID}$ the downstream backwater storage area. By combining Eqs. (3.7)-(3.9) and taking the Laplace transform of the results, one can derive the complete Muskingum model that relates the downstream depth to the inflow and outflow discharges, as shown in Eq. (3.10).

$$h(s) = \frac{1 - K \chi s}{A_s s + K (1 - \chi) A_s^2} q_1(s) - \frac{1}{A_s} q_2(s)$$  \hspace{1cm} (3.10)

Note that the Muskingum model does not model the transport delay using a time delay exponential in the Laplace domain. Instead it opts for a second order model. Although, at a glance, the Muskingum model looks less economic than the control-
oriented models discussed before it, the absence of an explicit time delay results in a much more compact discrete state space. This is due to the fact that discrete delays are absorbed as poles into their systems, increasing the model order. However, one must also note that the Muskingum model is non-minimum phase as it contains a positive zero \( s = 1/K \chi > 0 \) in its transfer function.

### 3.1.4. The IR Model

The Integrator-Resonance (IR) model, first developed by (van Overloop et al., 2010), aims to capture the backwater and resonance phenomena of short and flat canals, considering transport phenomena to be negligible. The inflow transfer function of the IR model consists of two parts:

- A free integrator to model the backwater of area \( A_s = T_0 L = A_{ID} \), with gain \( 1/A_s \)
- A damped hydraulic oscillator with natural frequency \( \omega_0 \) and resonance peak \( M_r \).

The natural frequency \( \omega_0 \) and resonance peak \( M_r \) are there to model the first resonant mode of the canal. They are either estimated using non-parametric identification procedures, or can be calculated using the following equations:

\[
\omega_0 = \frac{2\pi}{L \left( \frac{C_0 + V_0}{C_0 - V_0} \right)}
\]

\[
M_r = \frac{R_0^{4/3} H_0}{ng Q_0 L n^2}
\]

The full model is presented in Eq. (3.13).

\[
h(s) = \frac{1}{A_s s^2} \cdot \frac{\omega_0^2}{s^2 + 1/M_r - s + \omega_0^2} q_1(s) - \frac{1}{A_s s} \cdot \frac{2 s^2 + \frac{2}{A_s M_r}}{s^2 + \frac{1}{M_r} s + \omega_0^2} q_2(s)
\]

Note that since the IR model neglects transport phenomena (absence of a delay term), its accuracy suffers in the case of longer canals.
3.2. Model Evaluation

In order to gauge the accuracy of the presented control-oriented models, their time and frequency responses are to be compared against those produced by the full SVEs. The parameters of Canal A are employed to obtain both time and frequency responses. The parameters of the control-oriented models corresponding to Canal A are catalogued in Table 3.1.

Table 3.1: Model parameters corresponding to Canal A – Uniform Flow (H0=2.12, Q0=14.0)

<table>
<thead>
<tr>
<th>ID</th>
<th>IDZ</th>
<th>Muskingum</th>
<th>IR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ID}$</td>
<td>$A_{IDZ}$</td>
<td>$\tau_{ID}$</td>
<td>$\tau_{IDZ}$</td>
</tr>
<tr>
<td>($m^2$)</td>
<td>($m^2$)</td>
<td>(s)</td>
<td>(s)</td>
</tr>
<tr>
<td>4e+4</td>
<td>696.83</td>
<td>3.65e+4</td>
<td>707.72</td>
</tr>
</tbody>
</table>

3.2.1. Step Response Test

Like in the experiment in 2.2.3, a sluice gate is placed downstream of Canal A in order to obtain the step response of the full SVEs, as well as the control-oriented models. The parameters of the downstream gate are also kept the same. The step input discharge variation is, again, $q_1 = 0.5 \ m^3/s$ while the step responses are of the downstream depth $H$ and the outflow discharge $Q_2$. The responses of the SVEs are simulated using SWIFTEC, while the responses of the control-oriented models are calculated from Eqs. (2.24) and (2.25). The simulation time is 10 hours. The step responses for both depth and discharge are presented in Figure 3.3-Figure 3.5.

At a first glance, it becomes obvious that all of the proposed models adequately capture the steady state of the full SVEs. Zooming in at Figure 3.5, it becomes apparent that all models exhibit small steady state errors relative to the SVEs, specifically 0.21% and 0.02% for the downstream depth and outflow discharge respectively. Such steady state errors are expected due to model inaccuracies. Additionally, one must take into account the fact that the models are built around linearizations around steady state operating points. If the response strays too far from said operating points, then errors arise. This is the major weakness of non-adaptive linearization.

Zooming in at Figure 3.4, one can see that the IDZ and IR models best follow the transient response of the SVEs. This is interesting, considering that the IR model does not model the deadtime caused by advection. This is the case because the Canal A is completely submerged in backwater. The ID and IDZ models exactly capture the delay.
Figure 3.3: Comparison of step responses of downstream depth and outflow discharge due to inflow discharge step input - Canal A - entire timespan

Figure 3.4: Comparison of step responses of downstream depth and outflow discharge due to inflow discharge step input - Canal A - initial 1h zoom
3.2.2. Frequency Response Test

To gauge the effectiveness of the proposed models at high frequencies, a frequency response test on Canal A is conducted. To avoid cluttering, only the magnitude responses of the transfer functions \( h(s)/q_1(s) \) are plotted. The magnitude responses of the control-oriented models are compared against the responses calculated according to 2.1.8.1. To give emphasis to the resonance behavior, the steady discharge is selected to be \( Q_0 = 7.0 \text{ m}^3/\text{s} \). This will, of course, alter the model parameters of Table 3.1.

The magnitude responses are showcased in Figure 3.6. From there, one can observe that all models follow the low frequency response adequately well, which is logical considering the results of the step response test in 3.2.1. At high frequencies where resonance phenomena are prevalent, the ID and Muskingum models completely diverge from the SVEs. This is because these models do not take resonance into account. On the other hand, the IDZ model manages to capture the mean values of the resonance peaks due to its transfer functions containing a zero. Finally, the IR model attempts to capture the first resonance peak of the SVEs, but ultimately fails. This is the case because although Canal A is relatively flat, its length is large enough so that transport phenomena, which the IR model neglects, cannot be ignored.
3.2.3. Model Selection

Taking into account the results of both 3.2.1 and 3.2.2, the IDZ model exhibits the responses closest to the ones produced by the full SVEs, both in low and high frequencies. As such, for the rest of this thesis the IDZ model will be used for the purposes of developing a canal control system.
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4. Proposed Controller

The majority of irrigation canal control systems developed in literature assume a two-layer control architecture, forming a master-slave relationship between the controllers, as shown in Figure 4.1.

The first control layer, dubbed the master layer, utilizes the control-oriented models formulated in Chapter 3 to generate the required control input signals. Said control signals come in the form of discharge trajectories \( Q = [Q_1, Q_2, \ldots] \) that are required to maintain the water depths \( H_d = [H_d^{(1)}, H_d^{(2)}, \ldots]^T \) downstream of each reach. The master controller is developed and tuned based exclusively on the dynamics of the channel reaches. It is completely agnostic to the physics of the gated hydraulic structures that connect the reaches and the non-linear interactions caused by them. This is the case because the development of a centralized model that considers the gate openings as control input variables is a much harder task.

The second control layer, dubbed the slave layer, is needed because the actual manipulated variables of a real irrigation system are the openings \( W = [W_1, W_2, \ldots]^T \) of the gated hydraulic structures connecting the reaches. Thus, the slave control layer
is responsible for generating gate openings such that the required discharge trajectories \( Q = [Q_1, Q_2, \ldots]^T \) generated by the master layer are successfully delivered at each reach interconnection.

To summarize, the master control layer generates discharges \( Q = [Q_1, Q_2, \ldots]^T \) at the sluice gates to control the downstream water depths \( H_d = [H_d^{(1)}, H_d^{(2)}, \ldots]^T \) while the slave control layer acts on the sluice gate openings \( W = [W_1, W_2, \ldots]^T \) to deliver the generated discharges.

### 4.1. Master Layer

#### 4.1.1. Centralized Model – Reach Interconnection

As discussed above, the master layer utilizes control-oriented models. However, the models formulated in Chapter 3 all refer to the case of a single channel reach. A centralized controller needs to have access to the variables of the whole canal system. As such, a corresponding centralized canal model must be constructed to complement the controller. In order to do so, one must interconnect the control-oriented reach models to form a complete canal.

The Laplace domain representation of the dynamics of an open channel reach \( j \), that is, the relationship between the downstream depth variation \( h_d^{(j)}(t) \), the upstream and downstream discharge variations \( q_j(t) \) and \( q_{j+1}(t) \) respectively, and the offtake discharge variation \( d^{(j)}(t) \), is shown in Eq. (4.1).

\[
    h_d^{(j)}(s) = p_{21}^{(j)}(s) q_j(s) + p_{22}^{(j)}(s) q_{j+1}(s) + p_{23}^{(j)}(s) d^{(j)}(s), \quad j = [1, 2, \ldots, N_r]
\]  (4.1)

![Figure 4.2: Reaches interconnected through their discharges form a canal](image-url)
Where $N_r$ the total number of reaches comprising the canal. Note that the offtakes are considered close to the downstream boundaries, and are thus modeled using the same transfer function $p_{22}^{(i)}(s)$ as the downstream outflow.

In order to connect the reaches of the canal, one must simply observe that the outflow of a reach is equal to the outflow of the next reach. This steady continuity relation between reaches is an approximation of reality, since there always exist discharge losses between reaches, even without offtake branches. Alas, for the purposes of control, it is deemed sufficient. Therefore, the centralized canal model in the Laplace domain is shown in Eq. (4.2).

$$
\begin{bmatrix}
    h_d^{(1)}(s) \\
    h_d^{(2)}(s) \\
    \vdots \\
    h_d^{(N_r-1)}(s) \\
    h_d^{(N_r)}(s)
\end{bmatrix}
= 
\begin{bmatrix}
    q_1(s) \\
    q_2(s) \\
    \vdots \\
    q_{N_r}(s) \\
    q_{N_r+1}(s)
\end{bmatrix}
+ 
\begin{bmatrix}
    d^{(1)}(s) \\
    d^{(2)}(s) \\
    \vdots \\
    d^{(N_r-1)}(s) \\
    d^{(N_r)}(s)
\end{bmatrix}
+ 
\begin{bmatrix}
    p_{21}^{(1)}(s) & p_{22}^{(1)}(s) & 0 & \cdots & 0 & 0 \\
    0 & p_{21}^{(2)}(s) & p_{22}^{(2)}(s) & \cdots & 0 & 0 \\
    0 & 0 & p_{21}^{(3)}(s) & \cdots & 0 & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & 0 & \cdots & p_{21}^{(N_r)}(s) & p_{22}^{(N_r)}(s)
\end{bmatrix}
\begin{bmatrix}
    p_{21}^{(1)}(s) \\
    p_{22}^{(1)}(s) \\
    0 \\
    0 \\
    0 \\
    \vdots \\
    0 \\
    0 \\
    \vdots \\
    0 \\
\end{bmatrix}
\begin{bmatrix}
    p_{22}^{(1)}(s) \\
    0 \\
    0 \\
    0 \\
    0 \\
    \vdots \\
    0 \\
\end{bmatrix}
$$

By observing the sparse structure of the canal transfer matrices, it becomes apparent that a channel reach can only influence its immediate neighbors.

In order for the centralized canal model of Eq. (4.2) to become usable in a model predictive controller, it must first be converted to a discrete state space form. The discretization of the model can easily be accomplished through a Zero Order Hold
(ZOH) or Tustin method with sampling time $T_s$. The conversion from the Laplace domain to a state space form is equally easily accomplished through the use of canonical forms. The whole procedure is performed in MATLAB, using the `c2d` and `ss` functions. The obtained state space is shown in Eqs. \((4.5)-(4.6)\).

\[
\begin{align*}
x(k + 1) &= A x(k) + B q(k) + B_d d(k) \\
h(k) &= C x(k) + D q(k) + D_d d(k)
\end{align*}
\tag{4.5}
\tag{4.6}
\]

Eq. \((4.5)\) is the state equation and Eq. \((4.6)\) is the output equation. The denoted variables are as follows:

- $k$ : The current time step
- $x(k) \in \mathbb{R}^{n_x}$ : The state vector
- $q(k) \in \mathbb{R}^{n_q}$ : The discharge variance control input vector
- $d(k) \in \mathbb{R}^{n_d}$ : The offtake variance disturbance input vector
- $h(k) \in \mathbb{R}^{n_h}$ : The downstream water depth variance output vector
- $A \in \mathbb{R}^{n_x \times n_x}$ : The state matrix
- $B \in \mathbb{R}^{n_x \times n_q}$ : The control input matrix
- $B_d \in \mathbb{R}^{n_x \times n_d}$ : The disturbance input matrix
- $C \in \mathbb{R}^{n_h \times n_x}$ : The output matrix
- $D \in \mathbb{R}^{n_h \times n_q}$ : The control input feedforward matrix
- $D_d \in \mathbb{R}^{n_h \times n_d}$ : The disturbance input feedforward matrix

Note that the existence of input feedforward matrices implies that the discharge outflows and offtakes can directly influence the downstream depths. This is line with the principles of channel reach modeling.

Also note that the number of states, inputs and outputs is raised due to the presence of input delays. The delays are absorbed inside the model as additional poles during the discretization process. The number of extra poles is equal to:

\[
\text{number of added poles} = \sum_{j=1}^{N_t} \left\lfloor \frac{T_j}{T_s} \right\rfloor
\]
Where $\tau_j$ is the time delay corresponding to each channel reach $j$. The delays must be rounded up or down because the number of poles is an integer. MATLAB normally rounds up to obtain the "worst case" scenario of additional poles.

### 4.1.2. MPC Controller

Model Predictive Control (MPC) is a control strategy used in engineering and industrial processes to optimize the performance of dynamical systems. Unlike traditional reactive control methods, MPC utilizes a predictive model of the system to calculate optimal control actions over a finite time horizon. By considering future system behavior and constraints, MPC can generate control inputs that minimize a defined cost function, thereby achieving desired performance while adhering to said operational constraints. This predictive nature allows MPC to handle time delays, model uncertainties, constraints and non-linearities more effectively, making it a powerful control strategy for the automatic operation of irrigation canals, which contain all of these characteristics.

In the MPC methodology, the present control action is calculated in real-time rather than relying on a pre-calculated, offline control law. An MPC controller utilizes the canal model $P$, along with its current state $x$ and measurements of inputs $q$, outputs $h$, and disturbances $d$, at each sampling instant $k$ to accomplish the following tasks:

1. To calculate a future control sequence $q(k + l|k), l \in [0, N_p]$ based on a current state $x(k)$ over a finite prediction horizon $N_p$ in an optimal manner. This is done while enforcing constraints on the control inputs and controlled outputs.
2. To employ the first control action $q(k + 1|k)$ in the sequence as the next input for the canal system, thus updating the state of the canal to the next time instance $x(k + 1)$.
3. To feedback the updated (now current) state $x(k + 1) \rightarrow x(k)$ either through sensor feedback or state estimation and repeat the prediction process.

The MPC prediction process described in 1 is illustrated graphically in Figure 4.3. Here, $h(k + l|k)$ is the predicted canal depth response due to a future control sequence $q(k + l|k)$ at prediction instance $l$. 
The structure of the complete MPC controller described in steps 1, 2 and 3 is shown in Figure 4.4 below.

Where $r(k)$ is simply the vector of the variations of the water depth reference values $R(k) = [R_{1}(k), R_{2}(k), ...]^T$. 

Figure 4.3: MPC controller prediction process

Figure 4.4: Flowchart of the MPC algorithm
Because the state space model in Eqs. (4.5)-(4.6) is obtained through the use of canonical forms, the state variables are not physical, measurable quantities. Thus, the state vector $x(k)$ at the current time step cannot be measured, even in a perfect simulated setting. As such, a Kalman Filter is employed as a state estimator in order to reconstruct the unmeasurable states $x(k)$ using the known discharge variations $q(k)$ and measured downstream depths $h(k)$. This is facilitated by calling MATLAB’s built-in `kalman` function inside the MPC loop, where the Kalman covariance matrices are calculated automatically based on the state space model.

The calculation of the optimal control input discharge variations $q(k)$ is accomplished by minimizing the following linear quadratic criterion in (4.7).

$$ J(z(k)) = \sum_{l=1}^{N_p} [e^T(k+l|k) Q_h e(k+l|k)] + \sum_{l=0}^{N_p-1} [\Delta q^T(k+l|k) R_{\Delta q} \Delta q(k+l|k)] $$

(4.7)

Where:

- $J$: The linear quadratic cost function to be minimized.
- $z(k)$: The optimal control input sequence, given by:
  $$ z(k) = [q(k|k), q(k+1|k), \ldots, q(k+N_p-1|k)]^T $$
- $e(k)$: The error of the water depth from the reference (or variations thereof). It holds that $e(k) = R(k) - H_d(k) = r(k) - h(k)$.
- $\Delta q(k)$: The difference between successive control input discharge variations, given by: $\Delta q(k) = q(k) - q(k-1)$
- $Q_h$: Weighting matrix that penalizes deviations of the canal downstream depths from their respective reference values. The larger its weights, the more emphasis given on reference tracking.
- $R_{\Delta q}$: Weighting matrix that penalizes changes of the control input discharge variations. The larger its weights, the less aggressive the changes in the control input discharges.

Both $Q_h$ and $R_{\Delta q}$ are positive semi-definite matrices, and all of their weights are located in their main diagonals.

The minimization of the linear quadratic cost $J$ is subjected to the following operation constraints:
• $0 < H_{\min} \leq H_d(k) \leq H_{\max}$ knowing that $h(k) = H_d(k) + H_0$

• $0 < Q_{\min} \leq Q(k) \leq Q_{\max}$ knowing that $q(k) = Q(k) + Q_0$

In order to minimize the linear quadratic cost $J$ and obtain the optimal control sequence $z(k)$, one must formulate the constrained optimization problem as a Quadratic Programming (QP) problem. The calculation of the QP matrices is performed automatically by defining an $\text{mpc}$ controller object in Matlab. The optimal control sequence is then obtained by solving the QP problem using $\text{quadprog}$. At each time step, only the first element of the optimal control sequence vector $z(k) = [q(k|k), q(k+1|k), \ldots, q(k+N_p-1|k)]^T$ is utilized as a control input vector for the canal system. Note that the discharge variations $q(k)$ must be first converted to absolute discharges $Q(k)$ before being fed into the slave layer.

The selection of the weights of the $Q_h$ and $R_{\Delta q}$ matrices, as well as the length of the prediction horizon $N_p$ is the tuning process necessary for the MPC controller to be performed as desired for the canal system. The input discharge and output depth constraints need also be specified.

4.2. Slave Layer

Once the optimal discharge trajectories $Q = [Q_1, Q_2, \ldots, Q_{N_r+1}]^T$ are generated by the master layer, the slave layer is responsible for calculating the gate openings $W = [W_1, W_2, \ldots]^T$ such that said discharges are actually delivered. To accomplish this, it is necessary to invert the gate equation given by Eq. (2.20), assuming that the gates are submerged. For a given sluice gate $j$, the required gate opening would be calculated as follows:

$$W_j(k) = \frac{Q_j(k)}{C_{gj} L_{gj}^2 g \left( \frac{1}{2} \left( H_d^{(j-1)}(k) - H_u^{(j)}(k) \right) \right)} \quad (4.8)$$

The above operation is performed for every slave controller, denoted as $S_j$.

This strategy is called static inversion gate control in canal control literature. It is one of the simplest slave control laws and is used in most canal control systems around the world. However, (Litrico et al., 2008) showed that this strategy is suboptimal, because if fails to take the dynamical characteristics of the reach interactions into

\footnote{Note that the gate opening vector $W = [W_1, W_2, \ldots]^T$ has no defined length, since it is not known how many sluice gates the canal contains. It is not necessary that $\text{length}(W) = \text{length}(Q) = N_r$.}
account. In the same work, dynamic inversion controllers are proposed instead. For simplicity, the static inversion controller or Eq. (4.8) will be used from here on out.

Note also, that the calculated gate openings $W = [W_1, W_2, \ldots]^T$ may lead to the sluice gates rising from the water in an attempt to deliver the required gate discharges. The complete emergence of the gates is generally something to be avoided, so conditional static corrections to the gate openings are enforced, as shown in the pseudocode below. This strategy is essentially a form of saturation control.

$$\text{if } W_j \leq H_u^{(j)} \Rightarrow W_j = H_u^{(j)} - \varepsilon$$

Where $\varepsilon$ is a predefined gate dryness tolerance. The gate condition check is performed only on the depth downstream of the gate $H_u^{(j)}$, since it holds that $H_u^{(j)} < H_d^{(j-1)}$. The selected tolerance value must not be too large, as this can cause the delivered discharge to fail to meet the operation demands. In this thesis, the tolerance is chosen to be $\varepsilon = 0.01 \text{ m}$. 
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5. Controller Evaluation – ASCE Benchmark

In the work (Clemmens et al., 1998), two test canal cases are formulated and presented by the American Society of Civil Engineers (ASCE). These two test cases were constructed for the purposes of evaluating the performance of different control algorithms on semi-realistic simulated conditions, before deploying them on real canals. The two test cases are presented in Figure 5.1.

Test case 1 corresponds to the WM lateral canal in central Arizona. It is a particularly steep canal, notable for its very shallow waters and high Froude numbers. Its target depths are greater than the normal depths corresponding to each of its reaches. Test canal 2 is modeled after the upstream portion of the Corning canal in California. Compared to test case 1, the Corning Canal is much flatter, longer and possesses much greater storage volumes in its reaches. Its target operating range is the normal depth for each of its reaches.

The canal control community generally agrees that flat resonant canals that are operated close to their normal depths are noticeably more difficult to control than their steeper, shallower counterparts (Clemmens et al., 1998). In other words, test case 2 is considered a more interesting, albeit challenging control problem. For this reason, test canal 2 is chosen for benchmarking the controller proposed in Chapter 4.
### 5.1. ASCE Test Case 2 – The Corning Canal

The geometrical characteristics and general parameters of the Corning Canal are given in Table 5.1.

<table>
<thead>
<tr>
<th>Reach Number</th>
<th>Reach Length $L_j$ (m)</th>
<th>Bottom Width $B_j$ (m)</th>
<th>Reach Capacity $H_{\max}^{(j)}$ (m)</th>
<th>Gate Width $L_{gj}$ (m)</th>
<th>Target Depth $R^{(j)}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7000</td>
<td>7.0</td>
<td>2.5</td>
<td>7.0</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>7.0</td>
<td>2.5</td>
<td>7.0</td>
<td>2.1</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
<td>7.0</td>
<td>2.5</td>
<td>7.0</td>
<td>2.1</td>
</tr>
<tr>
<td>4</td>
<td>4000</td>
<td>6.0</td>
<td>2.3</td>
<td>6.0</td>
<td>1.9</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>6.0</td>
<td>2.3</td>
<td>6.0</td>
<td>1.9</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
<td>5.0</td>
<td>1.9</td>
<td>5.0</td>
<td>1.7</td>
</tr>
<tr>
<td>7</td>
<td>2000</td>
<td>5.0</td>
<td>1.9</td>
<td>5.0</td>
<td>1.7</td>
</tr>
<tr>
<td>8</td>
<td>2000</td>
<td>5.0</td>
<td>-</td>
<td>-</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Additionally, the bed slope of the canal is $S_0 = 0.0001$, with an elevation drop of 0.2m at each gate. The channel’s Manning’s roughness coefficient is $n = 0.020$.

Note that the Corning Canal contains sluice gate structures at the upstream boundary of each of its reaches. Upstream of the first sluice gate lies an inert reservoir. A pump is situated at the downstream boundary of Reach 8, pumping water out of the canal. This discharge is uncontrollable, so it is considered a measurable disturbance just like the gravity offtakes located downstream of each reach. This means that there are 8 downstream depth controlled variables $H_d^{(j)}(t)$, 8 control input gate discharges $Q_j(t)$ and 9 measurable disturbances: 8 gravity offtake discharges $D^{(j)}(t)$ and the pump outlet discharge $Q_9(t)$.

The equality between controlled output variables and control input variables from a first glance constitutes a fully actuated system. However, this is only the case for the control-oriented canal model. The actual canal system is distributed in space, thus exhibiting an infinite amount of degrees of freedom. Any prospective canal control system would thus be considered underactuated. This fact highlights the difficulties of designing a control system for the irrigation canal operation.
5.2. Description of the Proposed Benchmarks

In order to evaluate the performance of the developed control system on the Corning Canal, two benchmarks are proposed that test different aspects of the control system:

1. A *reference tracking benchmark*, which entails the changing of the downstream target depths at certain reaches of the canal. Such a benchmark would evaluate the ability of the proposed control system to adapt to changes in the control reference, while also gauging how well it performs once the system begins straying off of the operating point it was designed around.

2. A *disturbance rejection benchmark*, which entails the variation of the gravity offtake discharge values at certain reaches of the canal. Such a benchmark would gauge the ability of the control system to maintain its target state despite changes in water demand.

Both benchmarks are run on a 24h simulation of the Corning Canal, as instructed by ASCE. The simulation is performed using SWIFTEC. The schedules of the proposed benchmarks are exhibited below. The changes of both benchmarks are spread out along the canal so as to not show bias towards the upstream or downstream portion. The downstream water depths, errors, discharges, gate openings, offtake discharges and reference changes will be plotted.

Table 5.2: Schedules of the two benchmarks of the Corning Canal

<table>
<thead>
<tr>
<th>Reach Number</th>
<th>Operating Target Depth ( (j) ) ((m))</th>
<th>Operating Boundary Discharge ( Q_j ) ((m^3/s))</th>
<th>Operating Offtake Discharge ( D^{(j)} ) ((m^3/s))</th>
<th>Benchmark 1 - Changes in the Target Depths at 2h</th>
<th>Benchmark 2 - Changes in the Offtake Discharges at 2h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1</td>
<td>11.0</td>
<td>1.0</td>
<td>+ 0.1</td>
<td>+ 1.5</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>10.0</td>
<td>1.0</td>
<td></td>
<td>+ 1.5</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>9.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td>8.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.9</td>
<td>7.0</td>
<td>1.0</td>
<td>− 0.2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.7</td>
<td>6.0</td>
<td>1.0</td>
<td>+ 1.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.7</td>
<td>5.0</td>
<td>1.0</td>
<td></td>
<td>+ 1.0</td>
</tr>
<tr>
<td>8</td>
<td>1.7</td>
<td>4.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pump</td>
<td>-</td>
<td>3.0</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.3. Controller Parameters

Each reach of the Corning Canal is modeled using the IDZ model. The parameters chosen for the MPC & Slave controller after sensible tuning are kept the same for both benchmarks and catalogued in Table 5.3.

Table 5.3: Parameters of the MPC + Slave controller

<table>
<thead>
<tr>
<th>Controller Sampling Time $T_s$ (min)</th>
<th>Prediction Horizon $N_p$</th>
<th>Weights of the output error matrix $Q_{hi}$</th>
<th>Weights of the actuation effort matrix $R_{\Delta q_{ii}}$</th>
<th>Gate opening saturation tolerance $\varepsilon$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>20</td>
<td>10</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

- The chosen sampling time is enforced by ASCE due to the operational constraints of the Corning Canal, so there was no freedom in its selection. This choice is restrictive, since it fails to capture some of the dynamics caused by the system time delays.

- The prediction horizon samples were selected by continuously raising them until it no longer made a difference to the resulting system response. The minimum prediction samples were set equal to the maximum delayed samples (additional poles) caused by the absorption of the discretized time delays. These were calculated to be only 1 sample, due to the large sampling time.

- The weights of the output error and actuation effort matrices were selected by trial and error and were made to be constant along their main diagonals. This makes sense, since the reach dynamics are similar and the controlled outputs describe the same physical quantities – the downstream depths.

- The gate opening saturation tolerance was chosen using trial and error. Although smaller tolerance values are ideal, the gates can not make infinitesimal movements due to the inertia of the gate AC motors.

- Maximum depth constraints were set equal to the corresponding reach capacities in Table 5.1.
5.4. Results of Benchmark 1: Reference Tracking

Figure 5.2: Results of Benchmark 1 - Changes in the downstream target depths at 2h

Figure 5.3: Results of Benchmark 1 - Changes in the downstream offtake discharges (no changes)
Figure 5.4: Results of Benchmark 1 - Responses of downstream water depths

Figure 5.5: Results of Benchmark 1 - Responses of errors in the downstream water depths
Figure 5.6: Results of Benchmark 1 - Changes in the gate discharges

Figure 5.7: Results of Benchmark 1 - Changes in the gate openings
5.5. Results of Benchmark 2: Disturbance Rejection

Figure 5.8: Results of Benchmark 2 - Changes in the target downstream water depths (no changes)

Figure 5.9: Results of Benchmark 2 - Changes in the downstream offtake discharges at 2h
Figure 5.10: Results of Benchmark 2 - Responses of downstream water depths

Figure 5.11: Results of Benchmark 2 - Responses of errors in the downstream water depths
Figure 5.12: Results of Benchmark 2 - Changes in the gate discharges

Figure 5.13: Results of Benchmark 2 - Changes in the gate openings
5.6. Discussion

By examining the plotted results of both benchmarks, one can see that the proposed MPC controller is able to adequately respond to both changes in the target depths, and changes in the offtake disturbance discharges (changes in water demand).

According to the results of Figure 5.4 and Figure 5.5 in Benchmark 1, the reference tracking capabilities of the controller are excellent. After expected spikes in the error values due to sudden reference changes, the controller converges to the new canal steady state with a maximum steady state error of 0.01m (0.45%) corresponding to Reach 1. The maximum error caused by the change in reference at 2h corresponds to Reach 5 with a value of -0.2m (10%). This large value is expected due to the large reference variation. From Figure 5.6 and Figure 5.7, one can see that the actuation profiles are similar between reaches and that the discharge profiles are consistent with the corresponding gate opening profiles, due to the small selected value of the saturation tolerance. The error values are well within the error ranges found in relevant literature, such as in (Clemmens et al., 1998) and (Horváth, Galvis, Valentín, & Rodellar, 2015). The deviations from the linearization operating point do not seem to greatly affect the controller performance.

Looking at the results of Figure 5.10 and Figure 5.11 in Benchmark 2, the disturbance rejection capabilities of the controller seem comparable those corresponding to reference tracking. More specifically, the maximum steady state error corresponds to Reach 1 with a value of, again, 0.01m (0.45%). The other reaches show steady state errors in the order of 0.001m. The maximum error caused during the demand changes at 2h corresponds to Reach 1 and has a value of 0.1m (4.5%). Again, both the maximum error and the maximum steady state error values are comparable to the ones obtained in literature.

Although the error values for both benchmarks are considered acceptable, the fact that Reach 1 is consistently the most error-prone canal location is notable. The reason this is the case is probably because the inert reservoir located upstream of Reach 1 creates a noticeable and constant flow rate gradient between the upstream and downstream portions of Reach 1. This constant discharge difference could be considered a form of unmeasured disturbance, which acts as an additional source of error for Reach 1 not found in the other reaches.

Finally, the very large response times of the Corning Canal need to be taken into account. The work of (Clemmens et al., 1998) clearly illustrates the difference in response times between Test Case 1 and 2, with Case 1 showing response times in the range of 6-7h compared to the 22h+ of the Corning Canal. This further illustrates the difficulty in controlling resonant canals with significant storage.
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6. Conclusions and Future Work

In this thesis, the process of modeling irrigation canals and devising control laws for their automatic operation has been thoroughly explored. An extensive literature review on the subject of canal modeling and canal control algorithm development was made in order to provide insight on the state of the art. The underlying physics of open channel flow were briefly explored, which give insight to the steps necessary to model the canal operation. In particular, the Saint-Venant Equations were used in order to formulate a basis for the construction of linear control-oriented models. The responses of the Saint-Venant Equations in both the time and frequency domain were calculated and analyzed, shedding light into the dynamical characteristics of different types of open channels in the scope of linear time invariant systems theory. Using the knowledge provided by the canal system analysis, control-oriented open channel models were formulated, and their time and frequency responses were compared with the ones obtained by the full Saint Venant Equations. Based on how well the control-oriented models captured the dynamical characteristics of open channel flow, a choice of model was made in the IDZ model, which exhibited excellent accuracy in low frequencies and moderate performance in high frequencies. The models of each reach were interconnected to form a complete discharge-oriented model in the Laplace and state space domains. Utilizing the developed canal model, a two-layer canal control system was developed with a Model Predictive Controller serving as the depth-controlling master layer, and a gate control network serving as the discharge controlling slave layer. The slave controllers employ a static inversion of the sluice gate equations in order to regulate the structures. The complete control system was tested on a simulated version of the Corning Canal, corresponding to the well established ASCE Test Case 2. Specifically, two benchmarks were devised, testing both the reference tracking and disturbance rejection capabilities of the proposed control system. The results show promise, with errors in the same magnitudes as the relevant literature.

In future work, improvements on aspects of the master-slave canal controller configuration are to be explored. For the MPC controller, the employment of linear parameter varying models is ought to be examined, as the current controller should not stray away too much from the operating point of the linearized model. Additionally, the addition of integral action to the state space canal model needs to be explored for the purposes of improving step-input disturbance rejection capabilities. Finally, for the slave controllers, a dynamic inversion of the gate equations based on the models of the reaches is something that should be analyzed.
Bibliography


