Super-resolution image reconstruction employing Kriging interpolation technique

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Keywords: Super-resolution, Subpixel shift, Aliasing, Kriging interpolation, Wiener filter

Abstract: In this paper a high-resolution (HR) image is reconstructed from a sequence of subpixel shifted, aliased low-resolution (LR) frames by means of a novel nonuniform interpolation super-resolution (SR) method. A gradient-based algorithm estimates the horizontal and vertical shifts for each frame. Then, the uniformly spaced sampling points of the HR image are produced by means of Kriging interpolation. Wiener filtering is employed to deal with the restoration problem. The novelty of the proposed nonuniform interpolation approach to SR image reconstruction lies in the employment of Kriging interpolation technique. Comparisons with the original image demonstrate the superiority of our method to a conventional nonuniform interpolation one of SR image reconstruction.

1. INTRODUCTION

The employment of signal processing techniques to obtain an HR image from multiple observed LR ones is called SR image reconstruction. The goal of SR techniques is the creation of an HR image from several degraded and aliased, subpixel shifted LR images. The present work belongs to the category of nonuniform interpolation approaches to SR image reconstruction. As soon as the relative motion among the LR frames has been estimated, nonuniform interpolation is applied to produce the uniformly spaced sampling points of the HR image. Then, restoration is performed.

A nonuniform interpolation approach to SR image reconstruction is presented in [1]. A gradient-based registration algorithm calculates the shifts among frames. The frames are placed onto a uniform grid using weighted nearest-neighbor interpolation. Wiener filtering restores the blurred and noisy HR image. A set of aliased images is registered by means of a frequency domain method in [2]. Afterwards, bicubic interpolation is employed to construct an HR image. An SR image reconstruction technique is also presented in [3]. Subpixel shift information is extracted from 1-dimensional characteristic curves and adaptive subpixel interpolation leads to a uniformly spaced HR grid. Other interpolation techniques which lead to image resolution enhancement have been reported in [7-8].

In this paper an HR image is obtained exploiting the information obtained from 20 subpixel shifted, aliased LR frames. The relative motion between frames is estimated by means of a gradient-based algorithm. Kriging interpolation, a method that accepts irregularly spaced data, is employed to construct a uniformly spaced HR grid. It should be stressed that a direct reconstruction procedure is adopted, in contradiction with the employment of iterative procedures often met in the literature. Two different filters are employed to restore the resulting HR image via recursive deconvolution procedures. The novelty of the proposed approach lies in the employment of Kriging interpolation to create the uniformly spaced HR grid after registering the frames.

In Section 2 of this paper, Kriging interpolation technique is presented. Section 3 consists of a detailed description of the reconstruction procedure, while Section 4 contains the results of the conducted experiment. Some decisive aspects concerning the presented method are placed in Section 5, whereas conclusions are drawn in Section 6.

2. KRIGING INTERPOLATION

Kriging is a geostatistical interpolation technique that considers both the distance and the degree of variation between known data points when estimating values in unknown areas. A kriged estimate is a weighted linear combination of the known sample values around the point to be estimated. The unknown value of the signal $f_0 = f(x,y)$ at a given coordinate position (x_0, y_0) is expressed as

$$f(x,y) = \sum_{s=1}^{S} w_s f(x_s, y_s) \tag{1}$$

where S is the number of the known sample values of the signal. Kriging attempts to minimize the error variance and set the mean of the prediction errors to zero. Before the actual interpolation can begin, Kriging must calculate every possible distance weighting function. This is done by generating the experimental semivariogram of the data set and choosing a mathematical model which best approximates the shape of the semivariogram [4]. In our case, a Gaussian model is used (see Fig. 1). A smooth, continuous function for determining appropriate weights for increasingly distant data points is provided by the model. A semivariogram is a graph which plots the semivariance between points on the Y-axis and the distance at which the semivariance was calculated on the X-axis. The semivariance is one half the average squared difference of data values spaced a constant distance apart. As points are compared to increasingly distant points, the semivariance increases. At some distance, the semivariance will become approximately equal to the variance of the whole surface itself. This is the greatest distance over which the value at a point on the surface is related to the value at another point. This surface variance criterion leads to the selection of the appropriate known data points when estimating the value of an unknown point.

If the semivariance calculated at a distance h is denoted by $\gamma(h)$, the relationship between the semivariogram and its corresponding covariance is given by [4]

$$\tilde{C}(h) = \begin{cases} C_0 + C_1 & \text{if } |h| = 0 \\ C_0 + C_1 - \gamma(h) & \text{if } |h| \succ 0 \end{cases} \tag{2}$$

where C_0 is the nugget effect and $C_0 + C_1$ is the sill. Theoretically $\gamma(0) = 0$, but in practice $\gamma(0) = C_0$ and this is the nugget effect. Sill is the value that the semivariance curve flattens out to. We define

$$W^T \equiv (w_1, \ldots, w_s, \mu) \tag{3}$$

where μ is the average of the known samples. Also,

$$D^T \equiv (\tilde{C}_{10}, \tilde{C}_{20}, \ldots, \tilde{C}_{s0}, 1) \tag{4}$$

where the elements $\tilde{C}_{s0}$ are the covariances between the known samples and the signal value at the coordinate position (x_0, y_0). The covariance matrix for the known samples is given by

$$C = \begin{bmatrix} \tilde{C}_{11} & \ldots & \tilde{C}_{1s} & 1 \\ \vdots & \ddots & \vdots & 1 \\ \tilde{C}_{s1} & \ldots & \tilde{C}_{ss} & 1 \\ 1 & \ldots & 1 & 0 \end{bmatrix} \tag{5}$$

The weights which are used to form the kriged estimate are computed according to

$$W = C^{-1} D \tag{6}$$

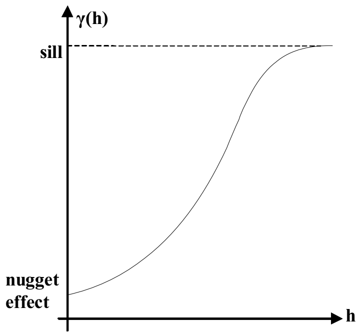


Fig. 1. Semivariogram that is modeled with a Gaussian curve.

3. RECONSTRUCTION PROCEDURE

3.1 Motion estimation

A gradient-based registration technique, that appears in [1], is employed to estimate horizontal and vertical shifts for the 20 available LR frames. In this motion estimation algorithm the first acquired frame is considered the reference frame and the shifts of the remaining frames are determined with respect to this reference frame. The resulting motion vectors help to formate a HR grid where all the available LR frames are placed. In SR image reconstruction motion estimation is of paramount importance as in practice incorrect estimates have disastrous implications. It should be noted that the motion vectors estimated by means of the frequency domain-based algorithm which is developed in [2] have also been used to form a HR grid. Nevertheless, their contribution to the SR reconstruction did not prove efficient.

3.2 Interpolation onto a uniform HR grid

After the motion estimation task has been completed, a uniform HR grid has to be created. In Fig. 2 is depicted a representative part of the nonuniform HR grid. Successive iterations of this part form the entire grid. The description of the reconstruction procedure which follows is based on the particular example grid part. The coordinate position of the first pixel of each LR frame is shown in Fig. 2. The first pixel of the reference frame is placed on the coordinate position $(0,0)$, whereas the corresponding pixels of the remaining LR frames are placed as mandated by the motion estimation vectors. In Fig. 2, each red point indicates a HR grid point. The distance between two HR grid points is equal to the width of a HR pixel. The SR reconstruction effort that is made desires to enhance the resolution by a factor of 4. Thus, 4 HR pixels form a LR pixel. The goal here is to estimate the value of each HR grid point using the pieces of information given from the 19 irregularly placed pixels of the LR frames. Kriging interpolation technique is employed to perform this task.

As far as the estimation of the value of each grid point is concerned, the following procedure, which is focused on Fig. 2, is adopted. All possible pairs of the 19 pixels are considered. The number of these pairs is 171. For every one formed pair the distance between the two LR pixels is calculated. This is the distance between the particular LR frames, as well. Thus, 171 different distances are calculated. Then, a semivariance matrix of size equal to that of each LR frame is created as far as each pair of pixels is concerned. In general, the semivariance is half the variance of the differences between all possible data points spaced a constant distance apart. In the present case, there are frames instead of points. Therefore, it is required to estimate the semivariance of the difference between two matrices. This specific semivariance is a matrix as well. At this matrix the value of the (i, j) element equals the half squared difference of the values of the (i, j) elements which belong to the two specific LR frames. So, 171 semivariance matrices are created.

Because of the need to work with distances and corresponding values of semivariance, the average of the values of each semivariance matrix elements is calculated. Then, using the 171 distance and semivariance pairs, a semivariogram is constructed. A Gaussian model of the form $f(x) = a_1 \exp\left(-\left(\left(x - b_1\right)/c_1\right)^2\right)$ is employed to fit the semivariogram data. The model parameters used are $a_1 = 0.03516$, $b_1 = 3.319$ and $c_1 = 1.894$. Sill is equal to 0.0232. Taking into consideration Kriging interpolation theory, the matrices C and D of sizes 20×20 and 20×1 correspondingly are calculated. It should be noted that the created semivariogram serves for the estimation of all the semivariance values that are required in the above matrix calculations. Furthermore, the number of matrices D which are constructed is 15. This happens because the representative unknown HR grid points are 15, as well. Afterwards, 15 matrices W are created. These matrices are of size 20×1 and contain the weights which will serve as coefficients for the linear combination of the LR frames. Before proceeding to the application of Kriging interpolation, it is necessary to examine which LR frames out of the 19 should be considered for the calculation of the value of each HR grid point. The surface variance criterion, mentioned earlier, is employed to reach a conclusion.

3.3 Restoration for blur and noise removal

The obtained HR image needs to be restored. Recursive Wiener filter deconvolution is employed to perform this task. It should be noted that blur and noise characteristics are considered the same for all the LR frames. It is assumed that blurring has been caused by the camera sensor PSF and relative motion between the camera and the original scene. Under the previous assumption, two different PSFs are used to obtain the restored HR image. More specifically, the first restoration employs the 2-dimensional filter $0.05ones(4,4)$ and the number of iterations is three. The resulting image is further restored by the aid of a motion filter, performing deblurring four times. This filter assumes linear motion of the camera by 6.3061 pixels with an angle of 0.6107 degrees in a counter-clockwise direction. Fig. 2 serves for the approximate estimation of the prementioned motion filter specifications. At both restoration steps additive noise, which is created by normally distributed random numbers, is taken into consideration. Smaller and greater filter structures than the 4×4 one have also been tested but did not prove efficient. Moreover, in our experience, the motion filter is indispensable to the HR image successful restoration.

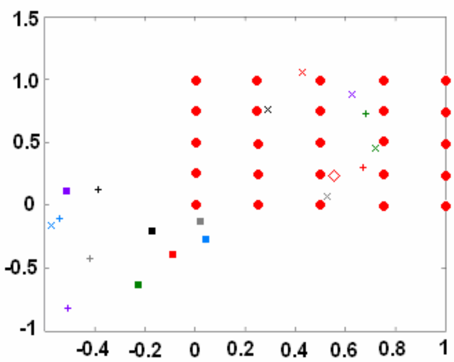


Fig. 2. Representative part of the nonuniform HR grid. Only the pixel $(1,1)$ of each frame is depicted.

4. EXPERIMENTAL RESULTS

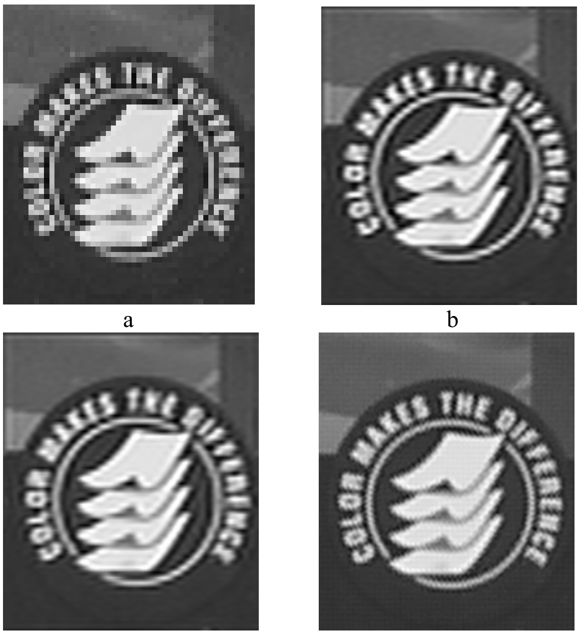
One of the LR frames is shown in Fig. 3a, while the obtained HR image is that of Fig. 3c. The SR reconstructed image proves satisfactory when compared with the original one which is shown in Fig. 3d. Besides visual comparison, the presented method is also assessed by means of mean square error (MSE) in comparison with the SR method that is described in [1]. The particular technique performs nonuniform interpolation SR image reconstruction too, but it employs the weighted nearest neighbor interpolation for the formation of a uniform HR grid. Fig. 3b shows the HR image which is constructed by means of the technique in [1]. Table 1 reports on the estimated MSE values. The proposed SR method creates a HR image which demonstrates MSE=0.0149. This value is lower than the MSE=0.0173 that is evaluated for the image resulting from the method which employs the weighted nearest neighbor interpolation. Visual comparison also proves the predominance of the proposed method. In Fig. 3b, above the letters of the word “color”, some parts of their outlines are visible. This happens for the lower part of the circle, as well. In fact, these extra outlines constitute artifacts, which do not exist in the HR image which results from the proposed method. Moreover, in the image that comes from the technique presented in [1], some sections of the circle are quite coarse and thus, differ from the original ones. However, the particular circle seems to be refined in the image of the present paper technique. Also, it should be noted that the upper outlines of the letters of the word “difference” are coarse as well and result in visible artifacts in the image which is produced by the SR method that employs the conventional interpolation technique. In this image in the interior of the depicted circle some artifacts are also observed. The prementioned artifacts are almost invisible in the HR image which is created by means of the SR method that employs Kriging interpolation technique. Fig. 4 helps to easily distinguish prementioned dissimilarities between the two differently constructed HR images.

Image Origin	MSE
Proposed SR method	0.0149
SR method of [1]	0.0173

Table 1: Results in terms of MSE for the SR reconstructed image.

5. DISCUSSION

Experimentation proved that results do not change if at the motion estimation procedure any other LR frame apart from the first one is considered the reference frame. Furthermore, according to Kriging interpolation theory, Kriging technique is employed only when the number of data points with known values is more than 5. Thus, in the present paper, where there are frames instead of points, 6 frames should at least be used for the calculation of the value of each HR grid point. Indeed, forming the kriged estimate using only 3 frames did not lead to a satisfactory HR image. Additionally, the surface variance criterion assessed that at least 18 frames should be employed at Kriging interpolation procedure. Furthermore, knowing or approximating the true PSF which has caused the blurring and the existence of noise is of paramount importance to the successful execution of the HR image restoration. Experimentation proved that a combination of PSFs accounting for the induced blurring should be employed for restoration. In fact, one of the employed filter structures should be a motion filter. Moreover, lower MSE values than that of Table 1 were achieved using different filter structures than the ones mentioned in Section 3.3. However, the resulting images were not visually satisfactory. The computational complexity of the proposed method is of higher order than that of the technique in [1], as far as the nonuniform interpolation step is concerned. With reference to our method, the value of each HR grid point is calculated employing 18 or 19 frames. On the contrary, the SR method in [1] uses only 3 frames for the particular purpose. Furthermore, the weights which are employed by the method in [1] are simply distances of frames from the HR grid points. Regarding the weights used at the proposed method, they come from the application of Kriging technique. The computational complexity of the registration and restoration steps of the proposed method is approximately of the same order as that of the corresponding steps of the method in [1].



a b c d

Fig. 3. (a) The 4th frame of the LR sequence. (b) The HR image coming from the SR method of [1]. (c) The HR image coming from the proposed SR method. (d) The original HR image.

a b c d

Fig. 4. (a) The word “difference” in the HR image of the proposed SR method. (b) The word “difference” in the HR image of the SR method of [1]. (c) The word “color” in the HR image of the proposed SR method. (d) The word “color” in the HR image of the SR method of [1].

It should be noted that the HR image coming from the SR method described in [5] and implemented by means of the software described in [6] is considered the original HR image with which comparisons are made. The prementioned paper presents a hybrid technique whose performance is superior to that of the nonuniform interpolation ones. So, as there is no original image available, it is considered that the hybrid method-originated image can successfully represent the desired result of the SR reconstruction effort that is made. The programming tool used is Matlab.

6. CONCLUSIONS

In this work a nonuniform interpolation SR method creates a HR image from 20 subpixel shifted, aliased LR frames. Resolution is enhanced by a factor of 4. The frames shifts are determined by a gradient-based registration algorithm and Kriging interpolation is employed to perform the task of interpolation onto a uniform HR grid. Mapping to the HR grid is a direct procedure of reconstruction. Recursive Wiener filter deconvolution performs the restoration of the obtained HR image. Visual comparison and MSE values have proved that the proposed method predominates over the nonuniform interpolation SR method which constructs the HR grid by means of the weighted nearest neighbor interpolation that is a conventional technique. The present SR method creates a HR image which satisfactorily approaches the original one. The proposed method is novel as it embodies, for the first time, Kriging interpolation technique in the nonuniform interpolation approach to SR image reconstruction. Altering the application of Kriging technique used in the proposed method in order to obtain more powerful SR results is the scope of future research.

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