Thermodynamics of Baryogenesis using the Rotating Lepton Model
Thermodynamics of Baryogenesis using the Rotating Lepton Model

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1. Το σύνολο της εργασίας αποτελεί πρωτότυπο έργο, παραχθέν από τον συγγραφέα της, και δεν παραβιάζει δικαιώματα τρίτων καθ΄ οιονδήποτε τρόπο,
2. Εάν η εργασία περιέχει υλικό, το οποίο δεν έχει παραχθεί από τον συγγραφέα, αυτό είναι ευδιάκριτο και αναφέρεται ρητώς εντός του κειμένου της εργασίας ως προϊόν εργασίας τρίτου, σημειώνοντας με παραμονής σαφή τρόπο τα στοιχεία ταυτοποίησης του, ενώ παράλληλα θεσπίζεται πως στην περίπτωση χρήσης αυτού υλικού αναπαραστάσεων, εικόνων, γραφημάτων κλπ., έχει ληφθεί η χωρίς περιορισμούς άδεια του κατόχου των πνευματικών δικαιωμάτων για την συμπερίληψη και επακόλουθη δημοσίευση του υλικού αυτού.
The fields of high energy particle physics and cosmology does not usually have a common ground and are most of the time completely separate, but they have one important common question that they both try to answer. How all the matter that we observe today was created and from what it is made.

The question of what matter is made off is an ancient one. Even from ancient Greece around the 5th century BCE Democritus first theorized that a unit of mass which cannot by further divided must exist and from which all matter is derived. He called them atoms because they cannot by divided further. Then Plato proposed the idea that all matter is made from ideal geometric forms. In this all atoms and all matter breaks down into triangles. As we will see further when explaining the concept of the Rotating Lepton Model both of them were right in a sense. Then the start of the modern atomic theory was done by Dalton in the early 1800 and for the first time it there was experimental evidence on the existence of atoms. A bit later Dimitri Mendeleyev set the basis of modern chemistry when he introduced the periodic table and the periodic law which did not only classify the known elements but manages to also predict new ones. At this point there was plenty of experimental evidence that suggested that matter was comprised from extremely small particles called atoms, but atoms themselves and their properties were largely a mystery. The next big discovery was by Tomson who discovered that inside the atom there were smaller particles with negative charge that he named electrons. Here arose the problem that even though electrons with negative charge exist inside the atom, the aforementioned is in fact neutral. The solution proposed by Rutherford was that electrons were inside a uniformly distributed positive charge. After that Rutherford with his famous gold foil experiment proved that the atom has in fact a tiny, massive nucleus where all of the positive charge is located [2].

A huge step towards understanding the structure of the atom was achieved by Bohr and his shell model of the atom. Bohr described the electrons having certain orbits of fixed size and energy around the nucleus. Applying classical Newtonian mechanics along with Planck’s quantization of energy (which would then lead to the discovery of quantum mechanics) allowed Bohr to get a very accurate description for atoms with 1 electron. Unfortunately, the theory would break down for heavier atoms. The new theories after Bohr mainly ignored the dual nature of matter and mainly focused on the wave nature of electrons to explain atomic and subatomic phenomena.

The final piece of the puzzle about the structure of the atom was given by James Chadwick with the discovery of the neutron, a neutral particle with similar mass with the proton that was also confined inside the nucleus. With this the picture of the atom was mostly complete. The protons and the neutrons were confined inside the nucleus, while the electrons were orbiting around them in specific orbits. It seemed that we had finally found the fundamental particles out of which all matter was comprised, the proton, the neutron and the electron, the atoms as Democritus had first described [2].

This would not be the case for long. Discoveries of other subatomic particles since the 1930s beside the proton the neutron and the electron along with patterns in their properties suggested that the proton and the neutron were not fundamental particles. That there were in fact even smaller particles from which the proton and the neutron were made from. This led to the development of the standard model which we will discuss in detail later to describe the properties of those particles. The proton, the neutron and particles like them were named Baryons, while the process for their formation from the elementary particles was called baryogenesis. Baryogenesis is the topic of the present work.
Before starting, some acknowledgments need to be made. First sincere thanks are expressed are to my supervising professor Dr Constantinos G. Vayenas, honorary professor at University of Patras for his extremely helpful advices and consultation and all of the discussions we had during the preparation of this work. Great thanks also go to Dionisis Tsouis, undergraduate student on University of Patras for the many helpful discussions we had while working together on the topic of the Rotating Lepton Model. Sincere thanks are also expressed to my parents Antonios and Dionysia and to all members of my family for their patience and their support during all those years.
2. SUMMARY

The purpose of this work is to study the process of baryogenesis under the scope of the Rotating Lepton Model (RLM). RLM is Bohr-type model in which leptons are trapped in a rotational bound state under their own gravitational attraction.

First, we will see how baryons and the process of baryogenesis are viewed according to the standard model of particle physics. The standard model is right now the generally accepted model that describes the behavior of particles in the microscopic scale. It describes how particles interact, how they are formed and what they are made of [13]. Then we will take the 2 simplest baryons, the proton and the neutron and discuss how they are described according to the RLM. As mentioned before according to RLM baryons are comprised from 3 leptons trapped in a rotational bound state, and in the case of baryons we have 3 neutrinos in the circular orbit [12], [7]. Furthermore, in the centre of the bound state we can also have trapped particles which in the case of the proton is a positron while in the case of the neutron can be a neutrino [6].

After that we will take the simplest of the two, that being the neutron, and starting from the forces describing the bound state we will derive the basic thermodynamic properties for the formation of the bound state as seen in [12]. According to the RLM the process of baryogenesis can be viewed as reaction of $3 \nu_e$ (neutrinos) to form a neutron. Based on that the properties that interest us are the bound energy, the Gibbs free energy, the enthalpy change, the entropy change and the transition temperature of the reaction. The transition temperature is the temperature where the change in the Gibbs free energy for the formation of the particle is 0.

Then we will describe the thermodynamic equilibrium of the reaction and produce the equilibrium diagram in which the conversion of the neutrinos when the reaction reaches equilibrium is plotted against temperature. This will give us an insight about the nature of that reaction and especially about the temperatures at which we expect that reaction to take place.

Next, we will look at the kinetics of this reaction and we will try to find an expression for the rate of that reaction. Here we will also show the very important role of electrons and or positrons as a catalyst for this reaction.

Last, we will implement the thermodynamics and the kinetics to model how the system behaves in a hypothetical adiabatic batch reactor where we will calculate the conversion of the neutrinos, the temperature change, and the time it takes for the reaction to happen. This will give us a clearer perspective about the reaction as a hole and under which conditions, we can expect it to occur.

During the thermodynamic and kinetic analysis, it will become clear that many similarities can be drawn between the baryogenesis process and regular chemical reactions. Thus, to expand upon those similarities we will compare the thermodynamics and the kinetics of the hadronization reaction with one of the most important and known chemical reaction. This is the synthesis of Ammonia from Hydrogen and Nitrogen [14].
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4. ΠΕΡΙΛΗΨΗ

Ο σκοπός αυτής της εργασίας είναι η μελέτη της διαδικασίας της βαρυογένεσης υπό την οπτική του μοντέλου των περιστρεφόμενων λεπτονίων (RLM). Το RLM είναι ένα μοντέλο τύπου Bohr στο οποίο τα λεπτά είναι παγιδευμένα σε μία κυκλική τροχιά λόγω της βαρυτικής τους έλξης.

Αρχικά θα μελετήσουμε πως περιγράφονται τα βαρυόνια και η διαδικασία της βαρυογένεσης σύμφωνα με το Standard μοντέλο της σωματιδιακής φυσικής. Το Standard μοντέλο είναι αυτήν την στιγμή το γενικώς αποδεκτό μοντέλο το οποίο περιγράφει τη συμπεριφορά των σωματιδίων στην μικροσκοπική κλίμακα. Περιγράφει τις αλληλεπιδράσεις μεταξύ των σωματιδίων, την δημιουργία των καθώς και από τι αποτελούνται [13]. Στη συνέχεια θα πάρουμε τα 2 απλούστερα βαρυόνια, το πρωτόνιο και το νετρόνιο, και θα δούμε πώς περιγράφονται σύμφωνα με το RLM. Όπως αναφέρθηκε και προηγουμένως σύμφωνα με το RLM τα βαρυόνια αποτελούνται από 3 λεπτόνια παγιδευμένα σε μια κυκλική τροχιά και συγκεκριμένα στην περίπτωση των βαρυνών έχουμε 3 νετρίνες στην κυκλική τροχιά [12], [7]. Στη συνέχεια θα μιλήσουμε για την θερμοδυναμική ισορροπία της αντίδρασης και θα κατασκευάσουμε το διάγραμμα της θερμοδυναμικής ισορροπίας. Σε αυτό την μετατροπή των νετρίνων όταν έχει επιτευχθεί ισορροπία σχεδιάζεται σαν συνάρτηση της θερμοκρασίας. Αυτό θα μας δώσει πληροφορίες σχετικά με την φύση της αντίδρασης και συγκεκριμένα σχετικά με την θερμοκρασία στην οποία περιμένουμε να συμβεί.

Επείτα θα μελετήσουμε την κινητική της αντίδρασης και θα προσπαθήσουμε να βρούμε μία έκφραση για τον ρυθμό της αντίδρασης. Επίσης κατά τη διάρκεια μελέτη της κινητικής θα φανεί ο σημαντικός καταλυτικός ρόλος των ηλεκτρονίων ή και των ποζιτρονίων σε αντιδράσεις αυτού του τύπου. Τέλος θα εφαρμόσουμε την θερμοδυναμική και την κινητική για να δούμε πώς θα εξελιχθεί το σύστημα μας σε έναν υποθετικό αδιαβατικό αντιδραστήρα διαλείποντος έργου. Εκεί θα υπολογίσουμε την μετατροπή των νετρίνων, την αλλαγή στην θερμοκρασία και τον χρόνο που απαιτείται για να ολοκληρωθεί η αντίδραση. Αυτό θα μας δώσει πληροφορίες σχετικά με ποιες συνθήκες αναμένουμε αυτή να λάβει χώρα. Σε όλη την διάρκεια της θερμοδυναμικής και της κινητικής ανάλυσης θα γίνει φανερό ότι υπάρχουν πολλές ομοιότητες της αντίδρασης της βαρυογένεσης με απλές χημικές αντιδράσεις. Επομένως, για να επεκταθούμε πάνω σε αυτές τις ομοιότητες θα συγκρίνουμε την θερμοδυναμική και την κινητική της αντίδρασης της βαρυογένεσης με μια από τις πιο σημαντικές και γνωστές χημικές αντιδράσεις. Αυτή η αντίδραση είναι η σύνθεση της Αμμωνίας από Υδρογόνο και Άζωτο [14].
5. SYMBOLS AND ACRONYMS

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</tr>
<tr>
<td>$n$</td>
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</tr>
<tr>
<td>$e^-$</td>
<td>electron or positron</td>
</tr>
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<td>$T$</td>
<td>Temperature</td>
</tr>
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<tr>
<td>$R_s$</td>
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8. LITERATURE REVIEW ON HADRONISATION WITH THE STANDARD MODEL AND WITH RLM

8.1 THE STANDARD MODEL AND HADRONIZATION

It is well understood by now that all atoms are made up from protons and neutrons in their core and electrons in orbitals around that [2]. Going beyond that the current understanding from theories and experiments is that all matter in the universe is made from a few basic building blocks that are called elementary particles. The standard model of particle physics is the model that describes the behavior of those particles under the influence of the fundamental forces. Out of the 4 known fundamental forces, 3 of them are incorporated in the standard model, those being the strong force, the weak force, and the electromagnetic force. The standard model does not include gravity because it is thought to be negligible at the subatomic distance. [13]. The standard model was also developed to explain how those subatomic particles (like the proton and the neutron) are made from the elementary particles. The fact that the subatomic particles are made up from even more fundamental particles is further supported by deep inelastic scattering experiments that suggest hadrons (like the proton and the neutron) are comprised of 3 smaller particles later named quarks. According to the standard model the neutron and the proton are comprised from 3 quarks that are bound together by the strong force. The neutron is comprised out of 2 down quarks and 1 up quark, while the proton is comprised from 2 up quarks and 1 down quark. Another important point that is also presented in figure 1 is that the strong force is mediated by the exchange of a particle with 0 mass and no charge termed the gluon (as in the glue that holds the hadrons together) [13]. The fact that the strong force is mediated gluons is derived from Quantum Field theory which has found great success in describing the interactions through the strong, the weak and the electromagnetic force. There interactions of particles can only happen if there is a particle to carry the force, this particle is called field quantum. For example, the electromagnetic force is carried by the photon which is the quantum of light [13].

![Figure 1 Schematic representation of the proton and the neutron according to the standard model.](image-url)
According to the standard model there are 12 elementary, or fundamental particles of spin ½ that are called fermions as seen on figure 2. Fermions are subdivided in 2 categories, the quarks that interact mainly via the strong force because of their property of color change. In addition to that quarks carry electric charge and weak isospin ant thus they can also interact via the weak and electromagnetic force. The other category of fermions, the leptons does not carry color change and cannot interact via the strong force. The top 3 leptons as seen in figure 2 carry electric charge and can thus interact via the electromagnetic and weak force. The 3 bottom leptons, the 3 neutrinos, does not carry charge and thus can only interact via the weak force. Here it is also important to note that every particle in the fermion family has an antiparticle with exactly the same properties but with different sign its charge that is not shown here in figure 2. It is not entirely sure if this also applies for the 3 neutrinos. In the most cases we will see neutrinos and anti-neutrinos treated separately but there is a suspicion that neutrinos and their antiparticles might be the same. Here we will treat them as being separate as it is common. Besides the fermion family there are the 5 fundamental bosons which are responsible for the mediation of the forces. The gluon is the mediator of the strong force, the photon is the mediator of the electromagnetic force, the $W^\pm$ and the $Z$ are the mediators of the weak force and the recently discovered Higgs boson is responsible for the intrinsic mass of particles.

2 very important properties of the strong force, which is the force that holds the subatomic particles together, are confinement and asymptotic freedom. Confinement is the property of the strong force to increase with distance. The quarks are confined inside the atom and cannot be separated. This is a very interesting property because it means that quarks cannot be isolated from the atom, and thus cannot be studied directly. Asymptotic Freedom is the fact that the strong force decreases below a certain distance of 0.5 fm [13].

As seen in figure 3 baryons belong in the hadron family. Baryons are the group with the heaviest subatomic particles, and they are comprised of an odd number of quarks, at least 3. The most familiar baryons are the proton and the neutron that make almost all of the matter that we encounter in
our daily lives. Baryogenesis which is the topic of the present work is the process with which baryons are formed from their constituent particles.

Figure 3 Elementary particle boson, hadron and fermion families.

Right now, one of the best ways to study the subatomic particles is in particle colliders like the Large Hadron Collider in CERN. Experiments at very high collision energies at particle colliders suggest the existence of a deconfined phase of matter existing at extremely high temperatures and pressure. This state of matter which is called Quark Gluon Plasma, is thought to have existed for a few microseconds after the big bang. After that the universe cooled bellow the temperature at which free quarks and gluons could exist. After that the quarks were confined to form hadrons and all the matter that exists in the universe. When matter transitions to that state the quarks and gluons that are usually confined inside the atoms break that confinement and can move freely much like in normal plasma where all of the electrons are striped from the atoms and those too can move freely. Under this scope the process for the formation of the quark gluon plasma and then (when it cools) the process of hadronization is viewed as a phase transition not unlike the phase transitions we encounter with normal matter like the transition from solid to liquid or from hot gas to plasma. This phase transition is estimated to happen at temperatures of 173 MeV while more recent studies calculate a transition temperature closer to 190 MeV [11], [4].

In this study the kT unit system is used for temperature. In this all temperatures are multiplied by the Boltzmann Constant $k_b = 8.617 \cdot eV K^{-1}$ in order to work with more convenient energy units. For reference 100 MeV are a little over 1 trillion K ($1.16 \cdot 10^{12}$ K).

Another way to describe the process of hadronization is with jet fragmentations theories. According to those, the hadronization process is viewed more like a reaction where we use the fragmentation functions to describe the outcome. After a high energy collision, we can use the fragmentation fractions to calculate the probability of a hadron to be produced. Fragmentation fractions are functions of the energy scale at which the patron is produced and of momentum fraction of the light cone caried by the produced hadron. Generally, patrons can hadronize to any hadron that is not kinematically forbidden, but the most stable and thus most likely hadrons produced are $\pi$ (pions), K (Kaons), $p$ (protons), and $n$ (neutrons). Because of that their fragmentation functions are the most well described [1]. Even though fragmentation theories can predict with relative precision the outcome of hadronization reactions given the initial conditions, they still require the use of adjustable parameters [3].
8.2 NEUTRINOS

Neutrinos are very light fundamental particles that have no charge (they are neutral and from this property also arises their name). A very crucial property of neutrinos is that they do not interact through the strong force but only through the weak force and gravity. This makes their detection very difficult and only recently we have started trying to detect and measure their properties directly. In fact neutrinos are so light and interact so weakly that until very recently it was thought that they were massless [8]. Furthermore, neutrinos are the second most abundant particle in the observable universe after photons. There are estimated to be $10^{30}$ neutrinos compared to protons and neutrons that are $10^{78}$ particles. Despite that due to their miniscule mass and their extremely weak interactions neutrinos very rarely interact with normal matter, usually passing right through it. For context somewhat of $6\times10^{10}$ neutrinos pass through every cm$^2$ of earth every second. These neutrinos are ones mainly created in nuclear reaction inside the sun's core like the reaction presented in figure 4 left. There 2 protons fuse together to form a deuterium atom and release 1.442 MeV. During the fuse one of the protons undergoes an inverse beta decay to become a neutron and releases a positron and a neutrino. In figure 4 right we see the reaction that was used to detect the solar neutrinos in water Cherenkov detector experiments. This reaction proceeds through the weak force and involves charge currents in the case of the $W^+$ boson and neutral currents in the case of the $Z^0$ boson.

![Feynman diagrams for charged and neutral electron – neutrino scattering.](image)

As seen in figure 2 there are three types of neutrinos called flavors, the electron neutrino, the muon neutrino, and the tau neutrino one for every of the top 3 leptons in the standard model. Another important fact is that neutrinos cannot be accounted as having mass in the context of the standard model. The standard model was developed for decades before it was known that neutrinos had mass and thus it incorporated them as massless inside the model [Griffiths]. Right now, it is not known if the standard model can change in order to incorporate the neutrinos as particles with mass. The fact that neutrinos actually had mass was first proposed when it was tried to measure the electron neutrinos produced through nuclear reactions in the sun like the one presented in figure 4 left. There we found a deficit in the electron neutrinos detected compered with what we would expect. This could be explained with the idea of neutrino oscillations. This is that neutrinos could change flavors, for example a neutrino could oscillate between an electron neutrino and a muon neutrino. It can be proven that for this process to occur the neutrinos must have mass. To be more accurate if this process is what really happening, we cannot actually talk about the mass of the neutrinos, like the mass of the electron neutrino because it does not have a well-defined mass or energy. What we talk about when referring to the mass of a neutrino are the mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$ with masses $m_1$, $m_2$ and $m_3$. The fact that Neutrinos have mass was proven beyond doubt when Takaaki Kajita measured the atmospheric neutrino oscillations for which he received the Nobel prize in
2015[17]. There pions and muons are produced when cosmic rays hit air molecules in the upper atmosphere. Then muon and electron neutrinos are produced from the decay of those pions and muons. The ration of the produced electron and muon neutrinos can be estimated and compared with the detected neutrinos. Neutrinos produced directly overhead that traveled a smaller distance from the source to the detector were detected in the expected ratio. Neutrinos on the other hand that came from bigger angles and had to travel bigger distances were detected at different ratios. This had proven that neutrino’s oscillations were true and that neutrinos had mass [13]. Although neutrinos were confirmed to have mass, determining with relative precision the mass eigenvalues was still a major problem. This is because the detectors are sensitive to the difference in the squared mass eigenvalues. Right now, the best approximation calculated from tritium beta decay for the mass of the heaviest neutrino in the normal hierarchy has a value of 0.05 eV/c² [10].

8.3  ROTATING LEPTON MODEL RLM

The Rotating Lepton Model (RLM) is a Bohr-type rotating particle model that can be used to describe the structure and dynamics of subatomic particles. In RLM at least 2 leptons are trapped in a rotating state bound by their own gravitational attraction acting like the strong force. As we explained before neutrinos indeed have mass and thus they can interact through the gravitational force. The reason that the gravitational force is so strong for the leptons that have miniscule mass is due to relativistic effects because the particles have very high velocities. Then to get a unique solution for the mass and the energy of the system the De Broglie equation for the quantization of angular momentum is used. [12], [8], [7]

According to RLM the proton and the neutron are comprised by 3 neutrinos in a circular orbit that are bound together by their gravitational attraction. The difference between the proton and the neutron is that the former has a positron trapped in its centre, while the later has an antineutrino trapped in its centre. The structure of the proton and the neutron are presented in figure 5 along with the electric dipoles induced in the neutrinos. The dipoles are the reason for the small difference in mass between the proton and the neutron while the dipoles in the neutron and the antineutrino in its centre play a part in the explanation of the inverse beta decay process [6].
First before we describe the equations that govern the bound state an important note must me made about the concept of mass in general. There are 2 ways to define mass that lead to 2 different masses. First is the mass defined by Newtons gravitational law which is by definition the mass that we use in the newtons gravitational law, equation (8.1) to compute the correct force between 2 objects with mass. The other is from Newtons 2nd law of motion, equation (8.2) in which the inertial mass is the measure of resistance to changes in the motion state of an object. Inertial mass in an inherent property of matter [12].

\[ F = G \frac{m_1 m_2}{r^2} \]  \hspace{1cm} (8.1)

\[ F = \frac{dp}{dt} = m_i \frac{dv}{dt} \]  \hspace{1cm} (8.2)

According to the Equivalence Principle that has been confirmed thousands of times [18] the inertial mass and the gravitational mass are equivalent, equation (8.3)

\[ m_i = m_g \]  \hspace{1cm} (8.3)

One other topic that we must discuss before we proceed with the RLM is about Special Relativity. Einstein’s theory of special relativity is based upon 2 core principles. First that the laws of physics are the same for all inertial observers. And second that the electromagnetic wave travels at the same speed, the speed of light, in vacuum regardless of the motion of its source. Direct results of those 2 postulates are time dilation and length contraction. These mean that observers moving with different speeds relative to each other will measure time passing slower or faster and the same object as being longer or shorter. The more the difference in speed between the 2 observers approaches the speed of light the more these effects become significant [15]. Another result of special relativity in context
with RLM is that an observer in an inertial reference frame in the lab and an observer in a reference frame moving along a neutrino in figure 5, will measure different forces acting upon the neutrino. In relativistic mechanics equation (8.2) takes the form seen in equation (8.4)

\[ F = \frac{dp}{dt} = \gamma m_0 \frac{dv}{dt} + \gamma^3 m_0 \frac{1}{c^2} (v \cdot \frac{dv}{dt}) v \]  \hspace{1cm} (8.4)

With \( \gamma \) being the Lorentz factor. The Lorentz factor is a measure of how close the speed between 2 reference frames is to the speed of light. For velocities we encounter in our everyday lives the Lorentz factor is 1 and we can ignore it. When however, we are dealing with speeds close to the speed of light the Lorentz factor increases rapidly becoming very important as we will see when we compute the mass on the bound state.

\[ \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \]  \hspace{1cm} (8.5)

\( m_0 \) is the rest mass, that being the mass measured when the object is at rest relative to the observer. For colinear \( \mathbf{F} \) and \( \mathbf{v} \) equation (8.4) after some simple algebra becomes

\[ F = \gamma^3 m_0 \frac{dv}{dt} \]  \hspace{1cm} (8.6)

From equation (8.6) for relativistic motion the inertial mass as seen in equation (8.2) is \( \gamma^3 m_0 \) and thus due to the equivalence principle

\[ m_g = m_i = \gamma^3 m_0 \]  \hspace{1cm} (8.7)

The above proof for the connection of inertial and rest mass in equation (8.6) can also be extended to apply for an arbitrary motion of any particle under an instantaneous velocity \( \mathbf{v} \) [8]. Thus equation (8.6) also applies for the circular motion of the 3 relativistic particles seen in figure 6.

\( \gamma m_0 \) is the relativistic mass that is corresponding to the total particle energy.

\[ E = \gamma m_0 c^2 \]  \hspace{1cm} (8.8)

From energy conservation follows that the composite energy is 3 times the energy of every neutrino, equation (8.8).

\[ E_c = m_c c^2 = 3 \gamma m_\nu c^2 \]  \hspace{1cm} (8.9)

And from equation (8.9) it follows that,

\[ m_c = 3 \gamma m_\nu \]  \hspace{1cm} (8.10)
In table 2 we have the different masses we that we use for a particle. Every time we write an equation, we must be careful of what mass we need to use. If the particle is at rest relatively to the observer, then we use the rest mass. If we want to calculate the Energy of the particle like in equation (8.9) we use the relativistic mass. And as we mentioned before when we want to calculate the gravitational attraction, we must use the gravitational mass. Note that the Inertial mass and the gravitational mass are the same according to the equivalence principle.

### Table 2 Different masses of a particle

<table>
<thead>
<tr>
<th>Mass Type</th>
<th>Mass Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rest mass</td>
<td>$m_0$</td>
</tr>
<tr>
<td>Relativistic mass</td>
<td>$\gamma m_0$</td>
</tr>
<tr>
<td>Inertial mass</td>
<td>$\gamma^3 m_0$</td>
</tr>
<tr>
<td>Gravitational mass</td>
<td>$\gamma^3 m_0$</td>
</tr>
</tbody>
</table>

As seen on figure 6 the 3 particles are trapped in a circular motion bound together under their own gravity acting as the centripetal force, thus the relativistic equation of circular motion (8.11) can be produced in equations (8.11). The $\sqrt{3}$ in the denominator is due to the geometry of the composite state. Note that here on the right term where we have the gravitational attraction we use the gravitational mass, while on the left term when calculating the centripetal force, we use the relativistic mass.

$$F = \frac{\gamma m v^2}{r} = \frac{G m_0^2 v^6}{\sqrt{3} r^2}$$  \hspace{1cm} (8.11)

Equation (8.11) then is coupled with the de Broglie wavelength equation for the quantization of angular momentum (8.12).

$$L = \gamma m v r = \hbar (2n - 1)$$  \hspace{1cm} (8.12)
Note again here the use of the relativistic mass in equation (8.12). For our example we will take the first state where n=0 (for excited states where n > 0 we get states that have the properties of heavier particles [7]). If equations (8.11) and (8.12) are solved for $v \approx c$ then a unique solution is obtained for $\gamma$, the energy and the composite mass seen in equations (8.13), (8.14) and (8.15)

$$\gamma_c = 3^{1/12}(m_{PL}/m_\nu)^{1/3}$$  \hspace{1cm} (8.13)

$$E_c = 3\gamma_c m_\nu c^2 = 3^{13/12}m_{PL}^{1/3}m_\nu^{2/3}c^2$$  \hspace{1cm} (8.14)

$$m_c = E_c/c^2 = 3\gamma_c m_\nu = 3^{13/12}m_{PL}^{1/3}m_\nu^{2/3}$$  \hspace{1cm} (8.15)

In equation (8.15) if we substitute the composite mass for the mass of the neutron $m_c = m_n = 939.5$ MeV / $c^2$ and solve for the mass of the rotating particle we have $m = m_\nu = 0.0437$ eV / $c^2$ which is very close to the mass of the heaviest neutrino on the normal hierarchy. Furthermore, if equation (8.11) or (8.12) is solved for $r$ then the result is $r = 0.630$ fm which is an excellent approximation for the radius on the neutron.

At this point the effect of the electric dipoles seen in figure 5 can be further explained. As seen in more detail in figure 7 in the proton the positron in the center induces a dipole charge in the neutrinos trapped in the circular orbit. Dipole-charge and dipole-dipole interactions are by their nature attracting and thus an attractive force is produced. As mentioned before the system can only occupy specific states as specified by the quantization of angular momentum (De Broglie equation) and the total centripetal force can take only certain values. For example, for the case of the proton and neutron where n=0 we have a specific $\gamma$, and thus a specific radius, mass, and gravitational force. If the attraction dipole-charge forces are introduced, then the system in order to be again in the equilibrium state will need to have a smaller gravitational attraction and thus a smaller $\gamma$. Having a smaller $\gamma$ as we see from equations (8.13), (8.14) and (8.15) results in a system with lower mass and smaller radius in comparison with a system without the induced dipoles. The auto-induced dipoles in the case of the neutron are much weaker than the dipoles in the proton and this is the reason that the mass on the proton is smaller than the mass of the neutron. In the present analysis the auto-induced dipoles in the neutron will be ignored as they are very weak.
If we rewrite equation (8.11) including the electrostatic attraction we get equation (8.16).

\[
\gamma_c m_v v^2 \frac{r}{r} = \left( \frac{G m_v^2 y^6 - Q e^2 / \varepsilon}{\sqrt{3} r^2} \right)
\]

(8.16)

Where,

\[
Q = q_1 q_2 + q_1 q_3
\]

(8.17)

And after solving equation (8.16) we get equation (8.17)

\[
R = \left( \frac{R_s}{2\sqrt{3}} \right) \left( \frac{y^2}{y^2 - 1} \right) \left[ y^5 - \frac{QR_c}{y R_s} \right]
\]

(8.17)

Where \( R_s \) and \( R_c \) are the Schwarzschild and Coulomb radius.

\[
R_s = \frac{2Gm_v}{c^2}
\]

(8.18)

\[
R_c = \frac{2e^2}{\varepsilon m_v c^2}
\]

(8.19)

Then if we solve equation (8.17) together with the de Broglie wavelength equation (8.12) the final state depends only on \( Q \). The mass of the proton is acquired if the 3 charges of the hadron constituents are \( q_1 = 2e \) \( q_2 = -e \) \( q_3 = 0 \) or if \( q_1 = -e \) \( q_2 = e \) \( q_3 = e \). These charges also provide an excellent fit to the proton’s magnetic moment [12].
9. THERMODYNAMICS OF HADRONISATION WITH RLM

As seen previously according to RLM the neutron is comprised by 3 neutrinos. Thus, the process for the formation of the neutron i.e. the process of hadronization can be written as the reaction of 3 neutrinos to form a neutron, equation (9.1). Here the reactants are the free neutrinos traveling through space that interact through their relativistic gravitational attraction to form the bound state, which is the neutron. Furthermore as any regular reaction we can associate the rate constant for the forward reaction (the reaction of baryogenesis).

\[ 3\nu_e \xleftrightarrow{k} n \]  \hspace{1cm} (9.1)

By starting from the gravitational force holding the 3 neutrinos together we can calculate the thermodynamical properties for this reaction as seen in [12], beginning from the binding energy calculation. The gravitational force acting upon the bound state constituents is given by equation (9.2).

\[ F_G = \frac{Gm_{\nu}c^6}{\sqrt{3}R^2} \]  \hspace{1cm} (9.2)

\[ R = \frac{R_s}{2\sqrt{3}}v^5 \]  \hspace{1cm} (9.3)

\[ R_s = \frac{2Gm_{\nu}}{c^2} \]  \hspace{1cm} (9.4)

By substituting equations (9.3) and (9.4) in (9.2) we get the gravitational force, equation (9.5) that is only dependent on radius and thus it can be considered as a conservative force. The force vector has a direction always pointing in the center of rotation and so we can define a conservative vector field \( V_G(R) \) as seen in equation (9.6)

\[ F_G = -m_{\nu}c^2 \left( \frac{2\sqrt{3}}{R_s} \right)^{1/5} \frac{1}{R^{4/5}} \]  \hspace{1cm} (9.5)

\[ V_G(R) - V_G(R_{min}) = \int_{R_{min}}^{R} F_G dR' = -5m_{\nu}c^2 \left( \frac{2\sqrt{3}}{R_s} \right)^{1/5} \left( R^{1/5} - R_{min}^{1/5} \right) \]  \hspace{1cm} (9.6)

But \( R_{min} = 2.343R_s \) is in the order of magnitude of \( 10^{-63} \) for the neutrino and \( R \) is in the order of magnitude of \( 10^{-15} \) so \( R_{min} \approx 0 \) resulting in equation (9.7)

\[ V_G(R) = -5m_{\nu}c^2 \left( \frac{2\sqrt{3}}{R_s} \right)^{1/5} \left( \frac{R}{R_s} \right)^{1/5} = -5\gamma m_{\nu}c^2 \]  \hspace{1cm} (9.7)
The kinetic energy $E_K$ is given by equation (9.8), [12]

$$E_K = 3(\gamma - 1)m_\nu c^2 \tag{9.8}$$

Now the change in the Hamiltonian energy for the formation of the bound state can be computed from equation (9.10) in which the Hamiltonian energy is given by equation (9.9)

$$H = E_{\text{relativistic}} + V_G = 3m_\nu c^2 + E_K + V_G \tag{9.9}$$

$$\Delta H = H_f - H_i = -(2\gamma + 3)m_\nu c^2 \tag{9.10}$$

Thus, the change in Hamiltonian energy is found to be

$$\Delta H = -2m_\nu c^2 \tag{9.11}$$

And by using equation (9.12) the change in Hamiltonian can be easily computed in equations (9.13) and (9.14), [12]

$$m_c = 3\gamma m_\nu \tag{9.12}$$

$$\Delta H = -\frac{2}{3}m_n c^2 \tag{9.13}$$

$$\Delta E^0 = -\left(\frac{3}{2}\right)m_n c^2 = -626.4 \text{ MeV/n} \tag{9.14}$$

From equation (9.14) we see that the reaction for the formation of the bound state i.e. the hadronization reaction, is highly exothermic according to RLM. To get a better feel about the energy being released we can compare the energy being released from a very famous reaction, the formation of ammonia. For the reaction we have $\Delta H = -45.86 \text{ kJ/mol}$, while for the neutron synthesis we have $\Delta H = -6.036 \times 10^{10} \text{ kJ/mol}$. This is about 10 orders of magnitude higher.

$$\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow NH_3 \tag{9.15}$$

The entropy change for the formation of the neutron from 3 neutrinos can be defined as seen in equation (9.16). The sign of $\Delta S$ is negative because the neutrinos lose degrees of freedom after the formation of the bound state [8].

$$\Delta S^0 = -3k_b \ln 3 \tag{9.16}$$

The Gibbs free energy is given from thermodynamics as
Thermodynamics of Baryogenesis using the Rotating Lepton Model

\[ G = E - TS \]  \quad (9.17)

From equation (9.17) we can define the reference temperature \( T_0 \) which is the temperature where we have \( \Delta G^0 = 0 \) as seen in equation (9.18), [8]

\[ T_0 = \frac{\Delta E^0}{\Delta S^0} = 2.206 \cdot 10^{12} K = 190.1 \frac{MeV}{k_b} \]  \quad (9.18)

| Table 3 Comparison of the basic thermodynamic properties between the ammonia synthesis and the neutron synthesis reaction. \( T_{cr} \) is the temperature where \( \Delta G = 0 \). |
|---|---|---|---|
| Reaction | \( \Delta H \) (kJ/mol) | \( \Delta S \) (J/mol K) | \( T_{cr} \) (K) |
| \( \frac{1}{2} N_2 + \frac{3}{2} H_2 \rightarrow NH_3 \) | -45.8 | -99.1 | 4.6 \cdot 10^2 |
| \( 3\nu_e \leftrightarrow n \) | 6.06 \cdot 10^{10} | -9.20 | 6.56 \cdot 10^{12} |

And then the energy and the entropy at any other temperature as seen in equations (9.19) and (9.20), [9]

\[ E = E_0 + c_p (T - T_0) \]  \quad (9.19)

\[ S = S_0 + c_p \ln \left( \frac{T}{T_0} \right) \]  \quad (9.20)

And from equation (9.17) by substituting equations (9.19) and (9.20) we can compute the Gibbs free energy at any temperature (which is the chemical potential), equation (9.21)

\[ G = E_0 + c_p (T - T_0) - TS_0 - c_p T \ln \left( \frac{T}{T_0} \right) \]  \quad (9.21)

Here at \( T = T_0 \) it is \( G_0 = 0 \) and using equation (9.16) we get.

\[ T_0 S_0 = E_0 \]  \quad (9.22)

Now \( S_0 \) can be eliminated from equation (9.21) giving the Gibbs free energy as a function of temperature in equation (9.23), [9].

\[ G = E_0 \left( 1 - \frac{T}{T_0} \right) + c_p (T - T_0) - c_p T \ln \left( \frac{T}{T_0} \right) \]  \quad (9.23)

Where

\[ T_0 = 190.1 \frac{MeV}{k_b} \]  \quad (9.24)
\[ E_0 = 1415\ MeV \]  \hspace{1cm} (9.25)

And for monoatomic ideal gas [9] we have

\[ C_p = \frac{5}{2} k_b \] \hspace{1cm} (9.26)

Now the graph of the thermodynamic equilibrium can be produced. This is the graph that shows us how many neutrinos react before the reaction reaches equilibrium for any given temperature. First the change in Gibbs free energy \( \Delta G \) needs to be defined before and after the formation of the bound state. This can be defined as shown in equation (9.27).

\[ \Delta G = \Delta E_0 \left( 1 - \frac{T}{T_0} \right) + \Delta C_p (T - T_0) - \Delta C_p T ln \left( \frac{T}{T_0} \right) \] \hspace{1cm} (9.27)

Were, \( \Delta E_0 \) is the energy difference between the bound state and the free neutrinos, that being the binding energy calculated before in equation (9.14).

\[ \Delta E_0 = 626.4\ MeV \] \hspace{1cm} (9.28)

\( \Delta C_p \) is the change in specific heat capacity during the formation of the bound state.

\[ \Delta C_p = C_{p,n} - C_{p,\nu e} \] \hspace{1cm} (9.29)

The specific heat under constant volume \( C_V \) is given by equation (9.30)

\[ C_V = f \frac{k_b}{2} \] \hspace{1cm} (9.30)

Where \( f \) are the degrees of freedom. \( C_p \) and \( C_V \) are connected via equation (9.31)

\[ C_p = C_V + k_b \] \hspace{1cm} (9.31)

Before the formation of the bound state, we have 3 neutrinos. Every neutrino has \( \frac{1}{2} \) degrees of freedom because they move along a straight path and only in one direction. Thus, the total degrees of freedom are \( 3/2 \). As for the neutron, the neutrinos are trapped in this bound state and according to the center of rotation the neutrinos have 0 degrees of freedom. By substituting equations (9.30) and (9.31) into (9.29) we get that

\[ \Delta C_p = - \frac{3}{4} k_b \] \hspace{1cm} (9.32)

\( \Delta G \) is connected with the equilibrium constant with equation (9.33)
\[ \Delta G = -k_B T \ln(K_a) \]  \hfill (9.33)

By solving equation (9.33) for \( K_a \) we get equation (9.34) that gives the equilibrium constant dependence on temperature.

\[ K_a = \exp \left( -\frac{\Delta G}{k_B T} \right) \]  \hfill (9.34)

Now for the reaction for the neutron synthesis we denote \([\nu_e]\) the neutrino concentration (number density), \([\nu_{e,0}]\) the initial neutrino concentration and \(x\) is the total neutrino conversion into neutrons.

\textbf{Table 4 Neutrino reaction stoichiometry}

<table>
<thead>
<tr>
<th>(3\nu_e)</th>
<th>(\rightarrow)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>([\nu_{e,0}])</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(<a href="1-x">\nu_{e,0}</a>)</td>
<td>([\nu_{e,0}]\frac{x}{3})</td>
<td></td>
</tr>
<tr>
<td>\textbf{Total:}</td>
<td>([\nu_{e,0}](1 - \frac{2x}{3}))</td>
<td></td>
</tr>
</tbody>
</table>

Now the molar fraction for every for the neutrinos and the neutrons are computed in equations (9.35) and (9.36) along with the molar fraction equilibrium constant in equation (9.37)

\[ y_{\nu_e} = \frac{[\nu_e]}{[\nu_{e,0}](1 - \frac{2x}{3})} = \frac{(1-x)}{(1 - \frac{2x}{3})} \]  \hfill (9.35)

\[ y_n = \frac{[n]}{[\nu_{e,0}](1 - \frac{2x}{3})} = \frac{x}{(1 - \frac{2x}{3})} \]  \hfill (9.36)

\[ K_y = \frac{y_n}{y_{\nu_e}^3} = \frac{x}{(1-x)^3} \frac{(1 - \frac{2x}{3})^2}{(1 - \frac{2x}{3})^3} \]  \hfill (9.37)

For ideal gasses \(K_a\) and \(K_y\) are connected via equation (9.38)

\[ K_a = K_y P^{\Sigma n_i} \]  \hfill (9.38)

From reaction (9.1) we have \(\Sigma n_i = -2\), where \(P\) is the dimensionless pressure given by equations (9.39), (9.40) and (9.41)
\[ P = \frac{P}{P_0} \] (9.39)

\[ P = ck_bT \] (9.40)

\[ P_0 = c_0k_bT \] (9.41)

Where \( c \) is the total particle concentration and \( c_0 \) is the initial particle concentration. By substituting equations (9.40), (9.41) and the total particle concentration from table 1 in equation (9.39) we get equation (9.42).

\[ P = \frac{ck_bT}{c_0k_bT} = \frac{c}{c_0} = \frac{n}{n_0} = \frac{[\nu_{\mu,0}]}{[\nu_{\tau,0}]} \left( 1 - \frac{2x}{3} \right) = \left( 1 - \frac{2x}{3} \right) \] (9.42)

Now we can substitute the pressure from equation (9.42) and \( K_y \) from equation (9.37) into equation (9.38) resulting in equation (9.43).

\[ K_a = \frac{x}{(1 - x)^3} \] (9.43)

By replacing \( K_a \) in equation (9.43) with the expression from equation (9.34) we can calculate for every temperature the neutrino conversion where the reaction reaches equilibrium.
10. KINETICS OF HADRONISATION WITH RLM

10.1 KINETICS AND CONSTANTS

Now after studying the thermodynamics of the reaction for the formation of the neutron, we will try to find an expression for the kinetics of the reaction (9.1) written again here in (10.1)

\[ 3\nu_e \leftrightarrow n \]  

(10.1)

Where the division of the kinetic constant of the reaction with the kinetic constant of the backwards reaction gives the equilibrium constant, equation (10.2)

\[ \frac{k}{k_{-1}} = K_{eq} \]  

(10.2)

A possible expression for the kinetics of the reaction (10.1) is presented in equation (10.3)

\[ r = k[k_{\nu_e}]^2 - k_{-1}[\nu_e] = k\left[[\nu_e]^2 - \frac{[n]}{K_{eq}[\nu_e]}\right] \]  

(10.3)

Where from table 4 for the concentrations of neutrinos and neutrons we have

\[ [\nu_e] = [\nu_{e,0}](1 - x) \]  

(10.4)

\[ [n] = [\nu_{e,0}] \frac{x}{3} \]  

(10.5)

Equation (10.3) was structured as to give the equilibrium constant when the rate is 0, \( r=0 \) in order to agree with the thermodynamics that were analysed earlier as seen in equation (10.6)

\[ r = 0 \Rightarrow K_{eq} = \frac{[n]}{[\nu_e]^3} \]  

(10.6)

From previous studies [5] we have an expression for the rate constant for the hadronization reaction \( k \) in equation (10.3) as seen in equation (10.7)

\[ k = \sigma_{AA}c \exp\left(\frac{-E_a}{k_b T}\right) / [\nu_{e,0}] \]  

(10.7)

Where, the activation energy \( E_a \) for the reaction is
\[ E_a = 251 \text{ Mev} \] (10.8)\

Here the activation energy is the minimum energy that the neutrinos can have in order to form the bound state. This is because the neutrinos must have a velocity and thus a Lorentz factor at least equal to those of the rotational bound state when the state is formed. If the velocity is smaller the required force to keep the neutrinos trapped in the bound state will not be enough. \( \sigma_{AA} \) is the reaction cross section given by equation (10.9) and \( c \) is the speed of light.

\[ \sigma_{AA} = \pi (r_1^2 + r_2^2) \] (10.9)

Where \( r_1 \) and \( r_2 \) are the mean distance between the reacting particles. To determine this value, we know that approximately \( 6 \times 10^{10} \) neutrinos are crossing every cm\(^2\) per s on earth. From this we can determine the mean distance to be

\[ r_1 = r_2 = 10^{-3} \text{ m} \] (10.10)

Here the equilibrium constant \( K_{eq} \) is given by equation (10.30) in which \( \Delta G \) is again calculated using equation (10.27)

\[ K = \exp \left( -\frac{\Delta G}{k_b T} \right) / [\nu_{e,0}]^2 \] (10.11)

To get a better understanding about the results of our findings we will now try to implement our model in a hypothetical adiabatic batch reactor that will give us insight about the behavior of the model. The equations for mass and heat equilibrium are given in equations (10.12) and (10.13). In the heat equilibrium equation (10.13) the heat dissipation is ignored because the reactor is taken to be adiabatic.

\[ \frac{dx}{dt} = \frac{V}{N_0} r \] (10.12)

\[ \frac{dT}{dt} = -\frac{(\Delta H_R)V}{m_e C_V} r \] (10.13)

\( \frac{N_0}{V} \) is the initial neutrino concentration \( [\nu_{e,0}] \) which has a mean value of

\[ [\nu_{e,0}] = 336 \cdot 10^6 \nu_e m^{-3} \] (10.14)

\[ C_V = \frac{3}{2} k_b \] (10.15)

\[ \Delta H_R = -626.4 \text{ MeV} \nu_e^{-1} \] (10.16)
The term \( \frac{m_t}{V} \) is the total mass inside the reactor divided by the volume of the reactor if \( C_V \) is given in \( \frac{J}{kgK} \). In our case where \( C_V \) is in \( \frac{MeV}{\nu_e K} \), thus \( \frac{m_t}{V} \) is the total number of particles inside the reactor divided by the volume, which is the neutrino concentration (10.36)

\[
\frac{m_t}{V} = [\nu_e]
\]

(10.17)

### 10.2 Electron Catalysis

It has been shown in previous work that the rate of the reaction for the formation of the neutron can be greatly increased if electrons and, or positrons are present. \( e^\pm \) act as a catalyst by greatly reducing the activation energy of the reaction from 251MeV to 10MeV in equation (10.7) [5]. This is again very similar with the case of ammonia synthesis from Nitrogen and Hydrogen. On its own, without the presence of a catalyst, the reaction for the synthesis of ammonia is very difficult to achieve in industrial scale and it is very inefficient. Then the Haber-Bosch process was developed which uses an iron-based catalyst to lower the activation energy of the reaction. This is because the catalyst lowers the dissociation energy of the \( N \equiv N \) bond which is the strongest bond among diatomic molecules [14].

In the case of hadronization the presence of \( e^\pm \) decreased the activation energy approximately 25 times. This has a huge effect in the rate of the reaction because the activation energy is inside the exponential. We can compare the rate constant with and without \( e^\pm \) catalysis for any given temperature using equation (10.7) and that will give the increase in the rate of the reaction. For example the rate of the reaction for a temperature of 10 MeV is \( 2.9 \cdot 10^{10} \) times (10 orders of magnitude) bigger when we have \( e^\pm \) catalyzing the reaction. In figure 8 we have the rate enhancement ratio dependance on temperature. We can see that for lower temperatures where the hadronization reaction is favored because it is exothermic the presence of the catalyst has a huge effect in the rate of the reaction. For higher temperatures due to the nature of the Arrhenius equation the rate enhancement tends towards 1. We see that happening at a temperature of 100 MeV where the presence of the catalyst does not have significant difference.

![Figure 8 Hadronization rate enhancement ratio dependance on temperature.](image)
Here the rate of the reaction that was previously given by equation (10.3) will be modified slightly to include the presence of the electrons or positrons as seen in equation (10.18).

\[
\begin{align*}
  r &= k[e][v_e]^2 - k_{-1}[e]\left[\frac{n}{v_e}\right] = k[e]\left[[v_e]^2 - \frac{[n]}{K[v_e]}\right] \\
  \text{(10.18)}
\end{align*}
\]

Like before we have 

\[ [v_e] = \left[v_{e,0}\right](1 - x) \quad \text{and} \quad [n] = \left[v_{e,0}\right]\frac{x}{3} \]

And for the electron concentration we have

\[ [e] = 0.25 \text{ e} \cdot \text{m}^{-3} \quad (10.19) \]

which is the mean value for the vacuum of space.
11. RESULTS AND DISCUSSION

11.1 THERMODYNAMIC EQUILIBRIUM

Using equations (9.43) and (9.34) the graph of the thermodynamic equilibrium can be produced. As seen on figure 9 for lower temperatures most of the neutrinos react before the reaction reaches equilibrium and we have a high neutrino conversion. Then as the temperature increases the equilibrium is moved more to the left for the reaction and thus the conversion is lower. This is behavior and the shape of the graph are expected from an exothermic reaction such as the one that is studied here.

![Thermodynamic Equilibrium Graph](image)

*Figure 9 Thermodynamic equilibrium for the reaction of the neutron synthesis from 3 neutrinos for the temperature range 0-2T₀ (0-381 MeV)*

For comparison we have a very famous exothermic reaction which is Ammonia synthesis from Nitrogen and Hydrogen. By following the same process as in the neutrino reaction we can create the graph of the thermodynamic equilibrium for the ammonia synthesis reaction, see appendix A. A comparison of the 2 graphs is seen on figure 10. Both graphs begin with total conversion of the reactants for lower temperatures that begins to drop as the temperature increases. This is due to the exothermic nature of both reactions, that as the temperature increases the equilibrium is moved to the left side of the reaction leading to lower reactant conversion. Another similarity is that for high temperatures the conversion tends towards 0, meaning that only the reverse reaction is favored, and the equilibrium has moved entirely to the left.
Figure 10 Thermodynamic equilibrium for the reaction of the neutron synthesis from 3 neutrinos for the temperature range $0-4T_0$ (0-762 MeV) left and comparison with the thermodynamic equilibrium for the ammonia synthesis from Nitrogen and Hydrogen for the temperature range 200-700 K.

11.2 KINETICS

Using equations (10.3) to (10.17) the system of the two differential equations (10.12) and (10.13) can be solved numerically with initial conditions, see appendix B

\[ t_0 = 0 \]  \hspace{1cm} (11.1)

\[ x_0 = 0 \]  \hspace{1cm} (11.2)

\[ T_{(0)} = T_{in} \]  \hspace{1cm} (11.3)

We have chosen $T_{in}$ to be 1% of $T_0$.

In figure 11 the neutrino conversion is plotted against time. The most important observation from figure 5 is that the reaction happens very slowly. It takes $10^{40}$ s for a very small fraction of the neutrinos to react and form neutrons which is an unreasonable amount of time. This means that for this reaction to be realistically possible we need to find a way to speed up the rate of the reaction.
As it was mentioned previously electrons and positrons can be used as a catalyst for the neutrino reaction. Replacing equation (10.3) with (10.18) and then using equations (10.4) to (10.17) the system of the two differential equations (10.12) and (10.13) can be solved numerically with initial conditions see appendix B

\[ t_0 = 0 \]  \hspace{1cm} (11.4)

\[ x_0 = 0 \]  \hspace{1cm} (11.5)

\[ T_0 = T_{in} \]  \hspace{1cm} (11.6)

And then we produce the diagrams

- x-T neutrino conversion with Temperature
- x-t neutrino conversion with time
- T-t Temperature with time

One factor that we can change is the initial temperature \( T_0 \). In the following diagrams we will also study the dependence on the initial temperature. On figure 12 left the neutrino conversion is plotted against time for 4 different initial temperatures from 0.007\( T_0 \) to \( T_0 \). For all of the graphs the conversion initially starts at 0, which was set as the initial condition, and then after some time it increases until it reaches equilibrium at which point the reaction stops. The first observation here is that when
the initial temperature increases then the time that the reaction for the reaction to reach equilibrium decreases, the reaction is completed faster. In addition to that when the initial temperature increases the equilibrium conversion decreases. The reason to that becomes much more obvious later in the conversion-temperature diagrams. Another interesting observation is that when the initial temperature decreases there is an initial lag where the reaction progresses very slowly until it reaches a specific state at which it is then completed quickly. This is obvious for the case of \( T_{\text{initial}}=0.007T_0 \) that has an initial lag until 2.5ms. Also, this behavior is further supported by the similarity of the 2 curves of 0.007\( T_0 \) and 0.01\( T_0 \). In figure 12 right, the same model of adiabatic batch reactor was constructed in the case of ammonia synthesis, see appendix A. The results for the ammonia synthesis are very similar to the neutrino reaction where the conversion increases until it reaches equilibrium, and for higher initial temperature the conversion at equilibrium is lower. One difference between the 2 is that in the case of ammonia synthesis there is no lag for smaller initial temperatures.

In figure 13 left the temperature-time graphs for neutrino reaction in the adiabatic batch reactor are presented. Here the temperature starts from the given initial temperature and then it increases as the reaction takes place until it reaches equilibrium. Similar to the conversion as the initial temperature decreases the time it takes for the reaction to reach equilibrium increases. Also, as in the conversion diagram for the smaller temperatures there seems to be an initial lag.

Figure 12 Conversion-Time diagram for neutrino reaction left and for ammonia reaction right.

In figure 13 left the temperature-time graphs for neutrino reaction in the adiabatic batch reactor are presented. Here the temperature starts from the given initial temperature and then it increases as the reaction takes place until it reaches equilibrium. Similar to the conversion as the initial temperature decreases the time it takes for the reaction to reach equilibrium increases. Also, as in the conversion diagram for the smaller temperatures there seems to be an initial lag.

Figure 13 Temperature-Time diagram for neutrino reaction left and for ammonia reaction right.
There is another interesting observation about the conversion-temperature diagram, figure 14, is that for every different initial temperature the system reaches equilibrium at a different conversion and temperature. Also, the point of equilibrium must also be on the equilibrium diagram, figure 9, that was constructed earlier. Thus, if we combine the x-T diagram of the Thermodynamic Equilibrium with the Bach reactor as seen in figure 15, the behavior of the reactor becomes much clearer. Here we can start from any point below the equilibrium line and by following the adiabatic rise line until we reach equilibrium, we know all the states the reactor will pass and also the point of equilibrium.

Another interesting question arising by the conversion-time and temperature-time diagrams is what happens when the initial temperature is even lower. In figure 16 the batch reactor system is solved for initial temperature 0.003T₀ and there the time that it takes for the reaction to reach equilibrium increases by about 3 orders of magnitude bigger, from ms to s. But compared with the diagrams from figure 11 the conversion and remains almost 0 for most of the time and when it reaches
a certain temperature and neutrino conversion the reaction is completed very quickly. At the point when the reaction speeds up we have a conversion of 0.019 and a temperature of 1.3 MeV.

![Neutrino Reaction Conversion-Time figure](image1)

![Neutrino Reaction Temperature-Time figure](image2)

![Neutrino Reaction Conversion-Temperature figure](image3)

*Figure 16 Temperature - Time, Neutrino Conversion – Time and Neutrino Conversion – Temperature diagrams for initial temperature 0.003 T_0*

Last, we can examine the dependance on electron concentration. For the previous diagrams, the electron mean value for space was used but this is not always the case. To study this the system of 2 equation (10.12) and (10.13) describing the mass and heat transfer was solved numerically for different electron concentrations and then the time for the completion of the reaction was found in each case. This time was set to be the time when 99% of the neutrinos had reacted. In figure 17 time for 99% for the reaction completion was plotted against the electron concentration. As seen in figure 17 initially the electron concentration does not affect the time of the reaction, but after the electron concentration surpasses 1 e/m^3 the time decreases logarithmically. This is to be expected because the rate of the reaction is a linear function of the electron concentration and thus \( \frac{dx}{dt} \) is also a linear function of the electron concentration.
Figure 17 Time for reaction completion dependance on electron concentration.
12. CONCLUSIONS

First it is again important to note that the Rotating Lepton Model does not contain any adjustable parameters. Starting from the initial idea of the bound state and using Einstein’s special relativity and equivalence principle and De Broglie’s theory for the quantization of angular momentum we can derive the properties of the bound state which for the case of 3 neutrinos match those of the neutron.

Then starting from the equation of the gravitational force holding the bound state we derived analytically the basic thermodynamic properties for the reaction of the neutron synthesis. There we first found the bound energy for the formation of the neutron. Then we calculated the transition temperature, where the Gibbs free energy is 0, and with that we found the expressions for the Gibbs free energy change, the enthalpy change, and the entropy change. Here we can compare this temperature with the phase transition temperature for the quark gluon plasma which is estimated to be around 190 MeV while the RLM predicts a transition temperature for the neutrino reaction at 190.1 MeV which is an excellent approximation. Then the thermodynamic equilibrium diagram was produced for the reaction. As it was also seen from the diagram the reaction for the formation of the neutron behaves as a normal exothermic reaction and thus from the thermodynamic perspective it can be viewed as one. Then the equilibrium diagram of the hadronization reaction was compared with the equilibrium diagram of ammonia synthesis. There, the similarity of the 2 reactions was observed showing the similarities the hadronization reaction and normal chemical reactions with the only difference between them being that the hadronization reaction takes place at temperatures many orders of magnitude bigger.

As for the kinetics of the hadronization reaction first a possible expression for the rate of the reaction was proposed. The main criteria for the rate of the reaction being that it follows the thermodynamics which is crucial to get an accurate description of the reaction.

After this we implement our model in a hypothetical adiabatic batch reactor to see if the description about the suggested kinetics is realistic. Initially we see that as it is the reaction does not take place at a realistic time span and so we need to find a way to increase the speed on the reaction. Here the very important role of electrons and or positrons $e^\pm$ becomes apparent. The $e^\pm$ act as catalysts, lowering the activation energy of the reaction 25 times and thus increasing the rate of the reaction about 50 orders of magnitude for the initial temperature of 2MeV. This is also very similar to exothermic chemical reactions and also in the case of ammonia synthesis because they are favored at lower temperatures where the rate of the reaction is slower. Thus, a catalyst must be present in order for the reaction to have a reasonable rate. When we introduced the catalyst the reaction time dropped to the order of ms.

Again, the behavior of the system in the batch reactor was as we would expect for an exothermic reaction. When we increased the initial temperature, the reaction was less efficient, reaching lower neutrino conversion. Again, we emphasize the similarity with the behavior of the ammonia synthesis in the batch reactor which again suggests that the comparison of baryogenesis with an exothermic reaction at it is viewed according to RLM is a good one.

Another interesting observation of the neutrino reaction is when we have lower initial temperatures. Even though exothermic reactions are favored thermodynamically at lower temperatures the rate of the reaction below a certain temperature approaches 0 and thus the reaction does not take place. Here we see that for an initial temperature of $0.003T_0$ which is approximately 0.57MeV or 6.6 billion K the reaction slows down significantly, requiring a time which is 3 orders of magnitude bigger compared with the other higher temperatures tried. There the rate very small for most of the time until the reaction reached a conversion of 0.019 and a temperature of 1.3 MeV and after that the reaction was completed rapidly.
Last the common graph presented in figure 14 provides a nice picture for the behaviour of the system starting from some arbitrary initial conditions. There we clearly see the path that the system follows in the batch reactor to reach equilibrium.

Now that we have a good idea about the reaction for the formation of the neutron a logical next step is to look at other particles and their formation process. The RLM has a pretty good understanding about the structure of the pion and the kaon [7], thus the thermodynamic properties for those particles can be investigated next. In addition, if we also have a description for the other particles, we can test our assumption about the kinetics of the hadronization reactions. This is because the experiments in particle colliders have as a result the yield of a specific particle compared to other particles. We get for example the yield of neutrons compared to pions. So, if we have the thermodynamic and kinetic description of different particles and the RLM we can compare it with experimental results from particle colliders.
REFERENCES

[3] Bryan R. Webber, Fragmentation and Hadronization
[19] ΑΝΑΛΥΣΗ ΚΑΙ ΣΧΕΔΙΑΣΜΟΣ ΧΗΜΙΚΩΝ ΑΝΤΙΔΡΑΣΤΗΡΩΝ, Κώστας Βαγενάς
14. APPENDIX A THERMODYNAMIC AND KINETICS FOR THE AMONIA SYNTHESIS

The reaction for the synthesis of ammonia from Nitrogen $N_2$ and Hydrogen $H_2$ is

$$N_2 + 3H_2 \rightarrow 2NH_3 \quad (14.1)$$

For reaction (14.1) the enthalpy change, and the Gibbs free energy change are dependent on temperature and are presented in equations (14.2) and (14.3) [19].

$$\Delta H_R = \Delta H_0 + \Delta \alpha T = \frac{1}{2} \Delta \beta T^2 + \frac{1}{3} \Delta \gamma T^3 \quad (14.2)$$

$$\Delta G_R = \Delta H_0 - \Delta \alpha T \ln(T) - \frac{1}{2} \Delta \beta T^2 - \frac{1}{6} \Delta \gamma T^3 - IRT \quad (14.3)$$

In equations (14.2) and (14.3) the constants $\Delta \alpha$, $\Delta \beta$ and $\Delta \gamma$ can be found from [19] for the reaction (14.1) as seen in equations (14.4) – (14.6)

$$\Delta \alpha = -63.56 \text{ J mol}^{-1} \text{ K}^{-1} \quad (14.4)$$

$$\Delta b = 71.04 \times 10^{-3} \text{ J mol}^{-1} \text{ K}^{-2} \quad (14.5)$$

$$\Delta \gamma = -18.66 \times 10^{-6} \text{ J mol}^{-1} \text{ K}^{-3} \quad (14.6)$$

The values for $\Delta H_0$ and $I$ can be calculated by setting the temperature to be 298 K where the values for $\Delta H_{R,298}$ and $\Delta H_{R,298}$ are known to be $\Delta H_{R,298} = -92220 \text{ J mol}^{-1}$ and $\Delta H_{R,298} = -32900 \text{ J mol}^{-1}$.

From this $\Delta H_0$ and $I$ are found to be

$$\Delta H_0 = -76268 \text{ J mol}^{-1} \quad (14.7)$$

$$I = 206 \text{ J mol}^{-1} \text{ K}^{-1} \quad (14.8)$$

Having found an expression for $\Delta G_R$, an expression for the equilibrium constant can be found by starting from equation (14.9) and solving for $K_a$ as seen in equation (14.10)

$$\Delta G_R = -RT \ln(K_a) \quad (14.9)$$
\[ K_a = \exp \left( -\frac{\Delta G_R}{RT} \right) \]  \hfill (14.10)

Supposing that the reactants are in stoichiometric quantities, meaning that \( N_2 : H_2 = 1:3 \), also that there is 1 mol of \( N_2 \) and by denoting \( x \) the nitrogen conversion table 5 that presents the reaction stoichiometry is constructed. From here the molar fractions of \( N_2, H_2 \) and \( NH_3 \) can be found, equations (14.11), (14.12) and (14.13). The molar fraction equilibrium constant can be computed in equation (14.14).

<table>
<thead>
<tr>
<th>( N_2 )</th>
<th>( 3H_2 )</th>
<th>( 2NH_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( [N_{2,0}] )</td>
<td>3([N_{2,0}])</td>
<td></td>
</tr>
<tr>
<td>(-x[N_{2,0}])</td>
<td>(-3x[N_{2,0}])</td>
<td>+2([N_{2,0}])</td>
</tr>
<tr>
<td>((1-x)[N_{2,0}])</td>
<td>((3-3x)[N_{2,0}])</td>
<td>2([N_{2,0}])</td>
</tr>
<tr>
<td>(Total)</td>
<td>((4-2x)[N_{2,0}])</td>
<td></td>
</tr>
</tbody>
</table>

\[ y_{N_2} = \frac{1-x}{4-2x} \]  \hfill (14.11)

\[ y_{H_2} = \frac{3-3x}{4-2x} \]  \hfill (14.12)

\[ y_{NH_3} = \frac{2x}{4-2x} \]  \hfill (14.13)

\[ K_y = \frac{y_{NH_3}^2}{y_{N_2}y_{H_2}^3} = \frac{(2x)^2(4-2x)^2}{(1-x)(3-3x)^3} \]  \hfill (14.14)

Assuming that we have ideal gases, the equilibrium constant is connected with the molar fraction equilibrium constant via equation (14.15)

\[ K_a = K_y p \Sigma n_i \]  \hfill (14.15)

From equation (14.1) we have \( \Sigma n_i = -2 \) and \( p \) is the dimensionless pressure given by equations (14.16), (14.17), (14.18) and (14.19)
Using equations (14.3) to (14.19) the graph of the thermodynamic equilibrium can be constructed, figure 10.

The rate of the reaction is given in equation (14.20)

\[ r = k P_{N_2}^a P_{H_2}^b P_{NH_3}^c \]  (14.20)

In equation (14.20) the partial pressure of N$_2$, H$_2$, and NH$_3$ is given by equations (14.21), (14.22) and (14.23) if again we assume ideal gasses.

\[ P_{N_2} = \frac{[N_2]}{C_{tot}} = \frac{1 - x}{4 - 2x} \]  (14.21)

\[ P_{H_2} = \frac{[H_2]}{C_{tot}} = \frac{3 - 3x}{4 - 2x} \]  (14.22)

\[ P_{NH_3} = \frac{[NH_3]}{C_{tot}} = \frac{2x}{4 - 2x} \]  (14.23)

And for a Ru/C12A7 catalyst the constants a, b and c are a= 0.46 b= 0.97 c=-1 [4]. The kinetic constant k in equation (14.20) is given by equation (14.24) in which the constants $k_0$ and $E_a$ can be found from the Arrhenius diagram for this specific catalyst as found from [ammonia paper]

\[ k = k_0 \exp \left( - \frac{E_a}{RT} \right) \]  (14.24)

\[ E_a = 50 \text{ KJ mol}^{-1} \]  (14.25)

\[ k_0 = 5.9 \times 10^7 \mu \text{mol g}_{\text{cat}}^{-1} \text{ h}^{-1} \]  (14.26)

The equations for mass and heat equilibrium are given in equations (14.27) and (14.28)
\[
\frac{dx}{dt} = \frac{V \rho_{cat}}{N_{tot,0}} r \tag{14.27}
\]

In equation (14.27) and (14.29) \(V \cdot \rho_{cat}\) is the catalyst mass, where for our results is

\[
m_{cat} = 0.025 \text{g} \tag{14.28}
\]

And \(N_0\) are the total initial moles inside the reactor, the enthalpy of the reaction and the specific heat of the reactants are given in (14.30) and (14.31) respectively.

\[
\frac{dT}{dt} = \frac{-(\Delta H_R) V \rho_{cat}}{N_{tot} C_V} r \tag{14.29}
\]

\[
\Delta H_R = 91.72 \text{ KJ mol}^{-1} \tag{14.30}
\]

\[
C_V = 30 \text{ J mol}^{-1} \text{ K}^{-1} \tag{14.31}
\]

Using equations (14.20) to (14.31) the system of the 2 differential equations can be solves numerically with initial conditions:

- \(t_0 = 0\)
- \(x_0 = 0\)
- \(T(0) = T_0\)
- For an initial nitrogen quantity of \(N_{N_2,0} = 1 \text{ mol}\)

The results are presented in figures 14, 13 and 14 where the Nitrogen conversion – Temperature graph, the Nitrogen conversion – Time graph and the Temperature – Time graph are constructed.
15. APPENDIX B MATLAB CODE

15.1 CONVERSION DIAGRAM CALCULATION

clc; clear all

Input

%The maximum temperature that we want the equilibrium diagram to go.
Tmax = 8; %Temperature in MeV divided by T0=190.1MeV

% Do not change beyond this point!

Main Program

T0 = 190.5; %MeV
deltaE0 = -626.4; %MeV
deltaCp = -3/4; %divided by kb
templength = T0*Tmax; %in MeV

%Calculates the equilibrium constant Ka
T = linspace(1,templeNGTH,300); % in MeV
DeltaG = deltaE0*(1-T/T0) + deltaCp*(T-T0) -deltaCp*T.*log(T/T0); %Change of the Gibbs free energy
%for every temperature in MeV
ka = exp(-DeltaG./T); %Equilibrium constant for every temperature

%Finds the x value that solves the thermodynamic equilibrium equation the
%Equation where Equ=0 for every temperature. This is the conversion x at equilib-
rium.
for i=1:length(T);

    %First values to start
    sol= 0.5; %first guess in the midle of the bounds
    lower = 0; %The minimum conversion is 0
    upper = 1; %The maximum conversion is 1
    n = 1; % Counts the number of iterations

    while abs(Equ(sol,ka(i))) > 1e-5 %Finds the conversion within a certain accu-
        if n>300 % if the solution does not converge in a given amount iterations
            the loop is broken
                break
        end

end
if Equ(lower, ka(i))*Equ(upper, ka(i)) < 0 %Makes sure there is a solution inside the bounds
    sol = (lower + upper)/2; % Guesses the solution at the middle of the bounds
    if Equ(lower, ka(i))*Equ(sol, ka(i)) < 0 %Calculates the equation product for the lower bound
        upper = sol; %If that product is negative then the real solution is inside there and the %solution is redefined as the upper bound
    else
        lower = sol; %If that product is positive then the real solution is inside the other bound and the %solution is redefined as the lower bound
    end
end
n = n+1; %Counts the iterations
x(i) = sol;
end

Result Plot

%Plots the equilibrium conversion change over temperature.
plot(T/T0,x);
axis([0, Tmax, 0, 1])
xlabel('T/T_0');
ylabel('Neutrino Conversion (x)');
grid on
title('Thermodynamic Equilibrium');

Equilibrium Function

function f = Equ(x, ka)
% This is the function of the thermodynamic equilibrium after moving all of % the terms in the left side, thus for the equilibrium conversion this % function is 0. The inputs are the conversion and the equilibrium constant

f = ka*(1-x)^3 - x/3;
end

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15.2 BATCH REACTOR SIMULATION

clc; clear all

Input

t0 = 0; %Initial time
tf = 0.01; %Final time
Tin = [0.007, 0.01, 0.1, 1]; %Temperature in MeV divided by T0=190.1MeV

% Do not change beyond this point!

Main Program

T0 = 190.1; %MeV
Cel = 0.25; %e/m3
kb = 8.617e-11; %MeV/K*neu
Cneu0 = 336e6; %neu/m3

% Loop that solves the batch reactor system for every initial temperature
for i = 1:length(Tin)
    x0 = [0; Tin(i),]; %Initial conditions for conversion and temperature
    options = []; %set no options

    % ode45 solves baryonreactor numerically and for the given initial conditions and
gives as output the
    % time, conversion and temperature vector
    [t, ex] = ode45(@baryonreactor,linspace(t0, tf, 1000),x0,options);

    %Combine all conversion vectors for every initial temperature in one table
    conv(:,i) = ex(:,1);

    %Combine all temperature vectors for every initial temperature in one table
    Temp(:,i) = ex(:,2);
end

Result Plot

% Plots how conversion changes over time for every initial temperature
figure('Name','Conversion Time');
plot(t,conv)
grid on
xlabel('t (s)');
ylabel('Neutrino Conversion x');
% Plots how temperature changes over time for every initial temperature
figure('Name','Temperature Time');
plot(t,Temp)
grid on
xlabel('t (s)');
ylabel('T/T0');
legend('T_{\text{inital}}=0.007T0','T_{\text{inital}}=0.01T0','T_{\text{inital}}=0.1T0','T_{\text{inital}}=T0')
title('Neutrino Reaction Temperature-Time figure')

% Plots how conversion changes over temperature for every initial temperature
figure('Name','Conversion Temperature');
plot(Temp,conv)
grid on
xlabel('T/T0');
ylabel('Neutrino Conversion x');
legend('T_{\text{inital}}=0.007T0','T_{\text{inital}}=0.01T0','T_{\text{inital}}=0.1T0','T_{\text{inital}}=T0')
title('Neutrino Reaction Conversion-Temperature figure')

Reaction Rate Function

function eqconstant = k(TM)
%This function given the temperature in MeV divided by T0=190.1MeV as the input
%calculates the rate constant for the hadronization reaction in m6/neu/s

Cneu0 = 336e6; % neu/m3
c = 299792458; % m/s
Ea = 10; % MeV due to the electron/positron catalysis
T0 = 190.1; % MeV
r1 = 1e-3; % m
r2 = 1e-3; % m

saa = 3.14*(r1^2 + r2^2); % cross section for the neutrino reaction
eqconstant = saa*c*exp(-Ea/(TM*T0))/Cneu0; % rate constant in m6/neu*s
end

Equilibrium Function
function equilibrium = K(TM)
%This function calculates the equilibrium constant in neu2/m6 for the neutrino
%hadronization given as input the temperature in MeV divided by T0=190.1MeV

Cneu0 = 336e6; % neu/m3
deltaCp = -3/4; % divided by kb
deltaE0 = -626.4; % MeV
T0 = 190.1; % MeV

% change in the gibbs free energy
deltaG = deltaE0*(1-TM) + deltaCp*(T0*T0-T0) -deltaCp*T0.*log(TM);

% equilibrium constant in neu2/m6
equilibrium = exp(-deltaG ./ (T0*T0))/Cneu0^2;
end

Batch Reactor Function

function xdot = baryonreactor(t, ex)
% This function calculates the conversion and temperature time derivative
% inside the adiabatic batch reactor with inputs the time in s, the conversion-
temperature vector
% with the temperature given in MeV divided by T0=190.1MeV

Cel = 0.25; % e/m3
Cneu0 = 336e6; % neu/m3
kb = 8.617e-11; % MeV/K* neu
Cv = 3/2*kb; % MeV/K* neu
T0 = 190.5; % MeV
deltaHo = -626.4; % MeV/ neu

% Rate of the reaction
r = k(ex(2)) * Cel * ((Cneu0*(1-ex(1)))^2 - (ex(1)/3)/((1-ex(1))*K(ex(2)))

xdot(1) = r/Cneu0; % Conversion time derivative
xdot(2) = -r*deltaHo/((Cneu0*(1-ex(1)))*Cv)*kb/T0; % Temperature time derivative
xdot = xdot(:); % making xdot into a vector (output for ode45 must be a vector)
end