Nikolaos I. Ioakimidis† and Georgios T. Anastasselos‡
†DIVISION OF APPLIED MATHEMATICS AND MECHANICS
DEPARTMENT OF ENGINEERING SCIENCES
SCHOOL OF ENGINEERING, UNIVERSITY OF PATRAS
‡DIVISION OF MECHANICS
DEPARTMENT OF ENGINEERING SCIENCE
THE NATIONAL TECHNICAL UNIVERSITY OF ATHENS

Direct Taylor-series solution of singular integral equations with MAPLE

Nikolaos I. Ioakimidis† and Georgios T. Anastasselos‡
†Division of Applied Mathematics and Mechanics, Department of Engineering Sciences,
School of Engineering, University of Patras, GR-265 04 Patras, Greece
‡Division of Mechanics, Department of Engineering Science, The National Technical University of Athens, 5 Heroes of Polytechion Avenue, GR-157 73 Zographou, Athens, Greece

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TECHNICAL NOTE

DIRECT TAYLOR-SERIES SOLUTION
OF SINGULAR INTEGRAL EQUATIONS
WITH MAPLE

Nikolaos I. Ioakimidis† and Georgios T. Anastasselos‡

†Division of Applied Mathematics and Mechanics, School of Engineering,
University of Patras, P. O. Box 1120, GR-261.10 Patras, Greece
‡Division of Mechanics, The National Technical University of Athens,
5 Heroes of Polytechnion Avenue, GR-157.73 Zographou, Athens, Greece

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Abstract—Cauchy-type SIEs (singular integral equations) appear quite frequently in a variety of elasticity problems and, mainly, in crack problems. Here we extend the classical direct numerical methods for the approximate solution of these equations to the SAN (semi-analytical/numerical) environment offered by modern CASes (computer algebra systems). The idea is simply to use Taylor–Maclaurin series in the approximate SAN solution and to reduce the problem to a purely numerical set of systems of linear equations. The approach is illustrated in the classical Lobatto–Chebyshev method with an application to the problem of a periodic array of collinear cracks. Extensive displayed SAN results for the stress intensity factors show the convergence and efficiency of the proposed SAN approach. The cases of other direct methods for SIEs are also directly applicable.

INTRODUCTION

Cauchy-type SIEs (singular integral equations) have become extremely popular in plane and antiplane elasticity problems and, especially, in crack and line inclusion problems in fracture mechanics during the last twenty years because of their natural appearance in several such problems and the development of a variety of numerical algorithms for their approximate solution. For crack and line inclusion problems the Lobatto–Chebyshev method (originally proposed by the senior author in [1]; see also [2]) remains the standard method used in practice. An excellent recent review by Golberg [3] seems to be the ideal source about the work already having been done and the related references. Furthermore, there are several hundreds of application papers in journals devoted to mechanics explicitly using these techniques in practical situations.

Of course, it is obvious that the restriction of the approximate solution to numerical values is not always desirable since no change in the geometric and loading parameters is permissible. Therefore, the SAN (semi-analytical/numerical or symbolic-analytical-numerical) environment seems to
be the ideal one in all cases. This environment is offered by CASes (computer algebra systems) and, especially, by modern CASes, like MAPLE [4,5] and MATHEMATICA [6,7], where numerical computations are equally supported to symbolic ones. Recent related references (on the use of CASes) are the ASME conference proceedings [8,9] with many related applications. An interesting recent review by Beltzer [10] should surely be mentioned as well.

If a SIE possesses a closed-form solution, then the SAN approach can be used directly to the integral in this solution. Taylor–Maclaurin [11], Chebyshev [12] and minimax [13] SAN approaches for the SIFs (stress intensity factors) at the crack tips have already been proposed by the senior author. Unfortunately, this is not always the case and we have to resort to approximate techniques for the SIEs themselves. Up to now we have been able to transfer only iterative numerical methods to the SAN environment [14,15]. These methods make a direct [14] or an indirect [15] transformation of the SIE to a Fredholm integral equation of the second kind. This seems not to be satisfactory for most researchers and it requires sufficient SAN computational effort.

Here we will follow the direct approach on the SIE itself. This approach can be directly used, in principle, only for systems of up to about six linear algebraic equations [16] providing complicated closed-form solutions because of the symbolic inversion of the related matrices. (Analogous is the case in the variational and finite element methods, where SAN techniques were used in the recent pioneering book by Beltzer [17].) Therefore, for larger systems of linear equations (resulting in a SIE after the application of a direct numerical algorithm like the Lobatto–Chebyshev method [1,2]) we have to resort to additional approximations (as has been the case in the iterative methods for SIEs in Refs. [14,15] and quite contrary to what has been the case in Refs. [16,17]).

Our approach will be described in the next section and consists simply in the reduction of the SAN system of linear equations to a set of separate systems of purely numerical linear equations together with the appropriate use of symbolic computations in the SAN environment. To the best of our knowledge the present approach, although elementary in principle and easy for applications, is original and was never used or suggested in the above references for the SAN solution of SIEs or elsewhere. After the description of the method we will apply it to the problem of a periodic array of collinear cracks in plane elasticity and we will display sufficient SAN results for the SIFs at the crack tips showing the power, accuracy and rapid convergence of the method. We will conclude with some comments on the proposed approach. The results were obtained by using the last version of MAPLE (version V, Jan. 1991) recently having become accessible to us. The MAPLE procedure itself will be also displayed below.

THE APPROACH

A SIE of the general form

$$\frac{1}{\pi} \int_{a}^{b} w(t)K(t,x)g(t)\,dt = f(x), \quad a < x < b,$$

probably (in crack problems) supplemented by a condition of the form

$$\frac{1}{\pi} \int_{a}^{b} w(t)g(t)\,dt = 0,$$

where \([a,b]\) is the integration interval (probably the crack), \(K(t,x)\) the known kernel (depending on the geometry of the elastic medium and the crack if any), \(w(t)\) the weight function, \(g(t)\) the unknown function (e.g. \(w(t)g(t)\) denoting the slope of the crack opening displacement) and \(f(x)\)
the right-hand side function (e.g. the loading on the crack) can be directly solved by reduction to a system of linear equations of the form [1–3]

\[ \sum_{i=1}^{n} A_{in} K(t_{in}, x_{kn}) g_{n}(t_{in}) = f(x_{kn}), \quad k = 1, \ldots, n - 1, \]  

(3)
and, further,

\[ \sum_{i=1}^{n} A_{in} g_{n}(t_{in}) = 0, \]  

(4)
where \( g_{n}(t) \) denotes an approximation to \( g(t) \), \( x_{kn} \) are appropriate collocation points [1–3] and an interpolatory numerical integration rule (with \( n \) nodes) of the general form

\[ \frac{1}{\pi} \int_{a}^{b} w(t)g(t)\,dt \simeq \sum_{i=1}^{n} A_{in} g(t_{in}) \]  

(5)
has been used both in (1) and in (2).

For the SAN solution of (3) and (4) we can expand the unknown quantities \( g_{n}(t_{in}) \) into Taylor–Maclaurin series of the form

\[ g_{n}(t_{in}) = \sum_{j=0}^{m} c_{jin} d^{j} + O(d^{m+1}), \]  

(6)
where \( d \) is the selected symbolic (geometric or loading) parameter, \( c_{jin} \) are appropriate coefficients to be determined and \( m \) is the order in the above series. Of course, for increasing values of \( n \) and \( m \), the required computational effort will increase as well.

Now we also perform an analogous Taylor–Maclaurin expansion (again with respect to \( d \), but with coefficients \( p_{j}(t,x) \)) to the kernel \( K(t,x) \) in (1), that is,

\[ K(t,x) = \sum_{j=0}^{m} p_{j}(t,x) d^{j} + O(d^{m+1}) \]  

(7)
and we substitute the values of \( g_{n} \) and \( K \) from (6) and (7) into (3) and (4). Then these equations (\( n \) in number) are written as polynomials in \( d \) of sufficiently high degree. Collecting the coefficients of \( d^{0}, d^{1}, d^{2}, \ldots, d^{m} \) in these equations, we obtain \( m + 1 \) systems of \( n \) purely numerical linear equations. By solving the first of these systems (corresponding to \( d^{0} \)), we determine the unknown coefficients \( c_{0in} \) in the expansion (6) of \( g_{n}(t_{in}) \). Similarly, by solving the second system (corresponding to \( d^{1} \)) and taking also into account the already available numerical results for \( c_{0in} \) (from the first system), we determine the unknown coefficients \( c_{1in} \) in (6) and so on. In general, by solving the \( (j+1) \)th system of linear equations (corresponding to \( d^{j} \)) and taking also into account the numerical values for \( c_{jin} \) with \( l = 0, 1, \ldots, j - 1 \) already having been found from the solutions of the previous \( j \) systems of linear equations (with \( l = 0, 1, \ldots, j - 1 \)), we obtain the numerical values of the coefficients \( c_{jin} \) in (6). The system corresponding to the coefficients of \( d^{m} \) is the last one to be solved although we can stop earlier if we are satisfied from the results already available as far as the convergence of the algorithm is concerned. On the contrary, we can next use higher values of \( m \) to obtain better series approximations to \( g_{n}(t_{in}) \) of even higher values of the number of nodes \( n \) to get better approximations to the integrals in (1) and (2).

All of the above computations (both the symbolic ones and the numerical ones) can be made inside the SAN environment offered by MAPLE or MATHEMATICA by using appropriate commands of these powerful CASes and the corresponding languages.
AN APPLICATION

As an application we consider the classical problem of a periodic array of collinear cracks [1,18] together with the Lobatto–Chebyshev quadrature rule [1,2]. In this case, the singular (Cauchy-type) kernel $K(t,x)$ in (1) is given by

$$K(t,x) = d \cot \left( d (t - x) \right),$$

where the ‘appropriate’ parameter $d$ is a geometric one, defined by

$$d = \frac{\pi a}{b},$$

where $2a$ is the length of each crack and $b$ the period of the array [1,18]. Moreover, the nodes $t_{in}$, the collocation points $x_{kn}$ and the weights $A_{in}$ are given by [1,2]

$$t_{in} = \cos \left( \frac{(i - 1) \pi}{n - 1} \right), \quad i = 1, \ldots, n,$$

$$x_{kn} = \cos \left( \frac{(k - 0.5) \pi}{n - 1} \right), \quad k = 1, \ldots, n - 1,$$

$$A_{in} = \frac{1}{n - 1}, \quad i = 2, \ldots, n - 1, \quad A_{1n} = A_{nn} = \frac{1}{2(n - 1)}.$$

The MAPLE procedure, called sie, with parameters both $m$ and $n$, for the present application is displayed below in its fundamental form and on the basis of the algorithm of the previous section. Some comments are also included in this procedure.

```maple
sie:=proc(m,n)
local i,j,k,mt,kl,a,b,t,x,A,B:
    #definition of the loading distribution f(x)
    F:=(x)->1: c:='c':
    #Taylor-Maclaurin series for the kernel K(t,x)
    readlib(mtaylor): mt:=expand(mtaylor(y*cot(y),y=0,m+1)/y):
    kl:=collect(subs(y=d*(t-x),d*mt),d):
    #definition of some appropriate arrays
    a:=array(1..n): b:=array(1..n): c:=array(0..m,1..n):
    #values of the nodes, collocation points and weights
    for i to n do t[i]:=evalf(cos((i-1)*Pi/(n-1))): od: i:='i':
    for k to n-1 do x[k]:=evalf(cos((k-0.5)*Pi/(n-1))) od:
    for i from 2 to n-1 do A[i]:=1./(n-1) od:
    A[1]:=0.5/(n-1): A[n]:=0.5/(n-1): i:='i': k:='k':
    #numerical values of K(t_{in},x_{kn})
    for i to n do for k to n-1 do
        K[i,k]:=subs(t=t[i],x=x[k],Pi=evalf(Pi),kl)
    od od: i:='i': k:='k':
    #the Taylor-Maclaurin series for g_n(t)
    for i from 1 to n do g[i]:=sum(c[j,i]*d^j,j=0..m) od: i:='i':
    #construction of the main n - 1 SAN equations
    for k to n-1 do
        eq[k]:=collect(sum(A[i]*K[i,k]*g[i],i=1..n),d)-F(x[k])
    od: i:='i': k:='k':
end proc:
```
\textbf{SAN RESULTS}

We display below some SAN results for the SIFs at the crack tips in the application of the previous section by using the above \texttt{sie} procedure. These result show the efficiency of the present algorithm for SIEs in the SAN environment offered by MAPLE V.

At first, we consider the case where the truncation in the Taylor–Maclaurin series \((6)\) and \((7)\) keeps terms of order up to \(m = 12\) and we leave the number of nodes \(n\) in the Lobatto–Chebyshev quadrature rule increasing from \(n = 2\) to \(n = 10\) by 1. The SAN results that we obtained for the approximation \(k_n\) to the SIF \(k = g(1) = -g(-1)\) at the crack tips for a constant loading distribution, \(f(x) = 1\), have as follows:

\[
k_2 = 1.00000000 + 0.3333333333d^2 + 0.1333333333d^4 + 0.05396825397d^6 + 0.02186948854d^8 + 0.00865079365d^{10} + 0.003592128037d^{12},
\]

\[
k_3 = 1.00000000 + 0.1666666667d^2 + 0.05555555556d^4 + 0.02156084656d^6 + 0.008650793651d^8 + 0.00349754155d^{10} + 0.001416655361d^{12},
\]

\[
k_4 = 1.00000000 + 0.1666666667d^2 + 0.05277777778d^4 + 0.01825396825d^6 + 0.006670524691d^8 + 0.002536258752d^{10} + 0.0009896922917d^{12},
\]

\[
k_5 = 1.00000000 + 0.1666666667d^2 + 0.05277777778d^4 + 0.01818783069d^6 + 0.006512345679d^8 + 0.002393211213d^{10} + 0.0008979123677d^{12},
\]

\[
k_6 = 1.00000000 + 0.1666666667d^2 + 0.05277777778d^4 + 0.01818783069d^6 + 0.006510692240d^8 + 0.002386630859d^{10} + 0.0008895307383d^{12},
\]

\[
k_7 = 1.00000000 + 0.1666666667d^2 + 0.05277777778d^4 + 0.01818783069d^6 + 0.006510692240d^8 + 0.002386589105d^{10} + 0.0008892817635d^{12},
\]

\[
k_8 = 1.00000000 + 0.1666666667d^2 + 0.05277777778d^4 + 0.01818783069d^6 + 0.006510692240d^8 + 0.002386589105d^{10} + 0.0008892807066d^{12}.
\]
There has been no change in any coefficient of $k_n$ for $n \geq 8$. Moreover, from the above results we clearly observe the convergence of the method for increasing values of the number of nodes $n$ (as was expected).

Perhaps, it is a better possibility to let both $m$ and $n$ increase simultaneously. This seems to be more reasonable in practical applications. We display below the results for $m = n = 2, 3, \ldots, 11$, but for an exponential loading distribution in the array of collinear cracks, $f(x) = \exp(x)$. Then the SIFs are different at the two crack tips, $t = \pm 1$. (Of course, additional choices of the relation between $m$ and $n$ can be used as well.)

\[
\begin{align*}
k_2(+1) &= 1.00000000 + 0.333333333d^2, \\
k_2(-1) &= 1.00000000 + 0.333333333d^2, \\
k_3(+1) &= 1.803332657 + 0.2100986394d^2, \\
k_3(-1) &= 0.7178710159 + 0.2100986394d^2, \\
k_4(+1) &= 1.830906942 + 0.1888350750d^2 + 0.05582933703d^4, \\
k_4(-1) &= 0.701348588 + 0.1888350750d^2 + 0.06524410439d^4, \\
k_5(+1) &= 1.831223178 + 0.1883900830d^2 + 0.05550670285d^4, \\
k_5(-1) &= 0.7009081793 + 0.1883900830d^2 + 0.0656104514d^4, \\
k_6(+1) &= 1.831224976 + 0.1883863845d^2 + 0.05550028876d^4 + 0.01903700860d^6, \\
k_6(-1) &= 0.7009067787 + 0.1883863845d^2 + 0.06455015997d^4 + 0.02264288793d^6, \\
k_7(+1) &= 1.831224982 + 0.1883863680d^2 + 0.05550023487d^4 + 0.01903687553d^6, \\
k_7(-1) &= 0.7009067738 + 0.1883863680d^2 + 0.06455007961d^4 + 0.02264256694d^6, \\
k_8(+1) &= 1.831224982 + 0.1883863680d^2 + 0.05550023462d^4 + 0.01903687439d^6 + 0.006870381134d^8, \\
k_8(-1) &= 0.7009067738 + 0.1883863680d^2 + 0.06455007927d^4 + 0.02264256470d^6 + 0.008125385416d^8, \\
k_9(+1) &= 1.831224982 + 0.1883863680d^2 + 0.05550023462d^4 + 0.01903687438d^6 + 0.006870381107d^8, \\
k_9(-1) &= 0.7009067738 + 0.1883863680d^2 + 0.06455007927d^4 + 0.02264256469d^6 + 0.08125385350d^8, \\
k_{10}(+1) &= 1.831224982 + 0.1883863680d^2 + 0.05550023462d^4 + 0.01903687438d^6 + 0.006870381107d^8 + 0.002542349898d^{10}, \\
k_{10}(-1) &= 0.7009067738 + 0.1883863680d^2 + 0.06455007927d^4 + 0.02264256469d^6 + 0.008125385350d^8 + 0.002542349898d^{10}, \\
k_{11}(+1) &= 1.831224982 + 0.1883863680d^2 + 0.05550023462d^4 + 0.01903687438d^6 + 0.006870381107d^8 + 0.002542349898d^{10}, \\
k_{11}(-1) &= 0.7009067738 + 0.1883863680d^2 + 0.06455007927d^4 + 0.02264256469d^6 + 0.008125385350d^8 + 0.002542349898d^{10}.
\end{align*}
\]
These results also reveal the convergence of the algorithm for increasing values of \( m = n \). In fact, it cannot be advised that high-order terms in the parameter \( d \) be used in (6) and (7) for small values of the number of nodes \( n \).

Obviously, it is possible to use any loading distribution \( f(x) \) on the cracks of the periodic array (like the constant and the exponential loading distributions in the previous two applications, respectively). But the really challenging possibility is to leave this distribution arbitrary denoted by a set of symbolic values \( f_k \) at the collocation points \( x_{kn} \) used. This is completely permissible in a CAS environment (like MAPLE V here). We display below the obtained SAN results in this case (valid for any loading distribution) including the values \( f_k \). Since these results are somewhat lengthy, we present only those that were obtained for \( m = 1, 2 \) and \( 3 \) and for \( n = 6 \).

\[
\begin{align*}
k_{1,6}(+1) & = 0.3902113033 f_1 + 0.3175570505 f_2 + 0.200000000 f_3 \\
& \quad + 0.08244294954 f_4 + 0.009788696741 f_5, \\
k_{2,6}(+1) & = 0.3902113033 f_1 + 0.3175570505 f_2 + 0.2000000000 f_3 \\
& \quad + 0.08244294954 f_4 + 0.009788696741 f_5 \\
& \quad + (0.006366100188 f_1 + 0.04363389981 f_2 + 0.0666666667 f_3 \\
& \quad + 0.04363389981 f_4 + 0.006366100188 f_5) d^2, \\
k_{3,6}(+1) & = 0.3902113033 f_1 + 0.3175570505 f_2 + 0.2000000000 f_3 \\
& \quad + 0.08244294954 f_4 + 0.009788696741 f_5 \\
& \quad + (0.006366100188 f_1 + 0.04363389981 f_2 + 0.0666666667 f_3 \\
& \quad + 0.04363389981 f_4 + 0.006366100188 f_5) d^2.
\end{align*}
\]

(We observe that the results for \( k_{2,6} \) and \( k_{3,6} \) coincide.)

Beyond the convergence of the above results we can mention that the SIF values for the crack tip \( t = -1 \) are ‘symmetrical’ to the above ones (that is with the subscript \( k \) in \( f_k \) taking the value \( n - k + 1 \) as was expected. Of course, in all cases, we have also available the values of \( g_n(t) \) at the whole set of nodes \( t_{in} \) used and not only at the crack tips. (Interpolation can further be used if required.) In fact, we presented above the crack-tip values of \( g_n(t) \) simply since these are the most important in fracture mechanics because of their relationship to the corresponding SIFs (that is \( k(1) = g(1) \) and \( k(-1) = -g(-1) \)).

**CONCLUSIONS—DISCUSSION**

From the above SAN results we conclude that CASes (like MAPLE [4,5] and MATHEMATICA [6,7]) constitute an ideal environment for the direct SAN solution of SIEs in the form of Taylor–Maclaurin series. The related procedure is a very simple one because of the powerful commands available in CASes. Moreover, the accuracy of the results can increase as much as we like since numerical computations are performed with an accuracy defined in advance by the user (although we used above only 14 digits during the computations and presented our results in 10 digits). The only cost for an increased accuracy is time and computer memory. In our present applications, we used a very small MS-DOS 386SX microcomputer at 16 MHz with only 5 MB of RAM. The CPU times for the calls of the \texttt{solve} procedure varied from 16 s, seconds (for \( n = 2 \)) to 226 s (for \( n = 10 \)) in the first application (with \( m = 12 \)), from 7 s (for \( m = n = 2 \)) to 216 s (for \( m = n = 11 \)) in the second application and from 37 s (for \( m = 1, n = 6 \)) to 93 s (for \( m = 3, n = 6 \)) in the last application. These times could be reduced by a factor of about 5 if a better MS-DOS microcomputer at 33 MHz would...
be available and by a factor of 10 to 20 with a modern RISC workstation. Other possibilities include to use the incorporated evalhf command of MAPLE V, which uses the computer’s hardware for floating-point computations and the analogous possibilities already available as standard ones in MATHEMATICA. In any case, the above results have been very accurate indeed from the practical point of view and the required times were sufficiently reasonable. Therefore, we did not find it necessary to try to reduce these times here. Of course, a final possibility in extremely complicated problems is to use external commands for the solution of the numerical systems of linear equations. This is a rather easy task under the UNIX operating system, but sufficiently less trivial under the MS-DOS 4.01 operating system used here. Of course, it is expected that CASes will be improved with respect to the incorporated numerical algorithms so that the user can get completely rid of any need to resort to external packages (like the IMSL and NAG numerical libraries).

As far as the output of the SAN results is concerned, we directly used the corresponding MAPLE files during the preparation of the present manuscript. We also used the MAPLE \texttt{latex} command for the appropriate (almost final) \TeX formatting of the derived formulae. In this way, practically all reasons for errors from manual intervention are avoided. MAPLE V offers a large number of facilities for the the output and the formatting of its results. (Similar and probably even better is the case with MATHEMATICA, but this strongly depends also on the computer used and, mainly, on the corresponding operating system.)

Although we have restricted above ourselves to SIEs of the first kind by using the Lobatto–Chebyshev method, it is clear that the suggested algorithm for SIEs is quite general and can be applied to the direct SAN solution of any SIE of the first or the second kind and systems of such equations without reduction to Fredholm integral equations directly or indirectly. Moreover, any method for SIEs (like the Gauss–Chebyshev, the Gauss–, Radau– and Lobatto–Jacobi, the Gauss–Laguerre, the Gauss–Hermite, etc.) can be used with very small changes in the procedure displayed above. Similarly, the ‘translation’ of this procedure to MATHEMATICA (or to any other CAS like MACSYMA, REDUCE and SCRATCHPAD II) is also possible, the possibilities and availability of each particular CAS surely taken into serious consideration. Finally, for the automatic algebraic construction of SIEs in crack problems (but without attention to their solution) reference can be made to [19]. Obviously, the results of [19] can be combined with the present ones in a practical fracture mechanics application.

The inclusion of symbolic parameters in the solution of an engineering problem (as has been the case above and in several of the references below) is very important in the sense that the whole problem has not to be solved again if this parameter changes. Moreover, by using this parameter, we are able to get a rather general formula for the quantity of interest, like the SIFs at the crack tips above, which can also be further algebraically manipulated (like differentiated, for maxima and minima, or integrated, etc.). For this reason, we believe that approximate SAN results in engineering problems (which have essentially begun to appear during the last two or three years) have much to gain from the simultaneous tremendous increase of the power and availability of all kinds of computers and CASes used both interactively and in batch mode (like extremely powerful computer languages). References [8], [9] and [17] show clearly this tendency and a lot of further results, probably leading to a ‘boom’ in the area, should be expected (and are really welcome) in the near future.

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