ARISTOTLE’S *METAPHYSICS*, Books M and N: 
Introduction, Translation and Commentary

A PhD Thesis submitted by

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ABSTRACT

The grasp of Aristotle’s *Metaphysics* has proven to be a difficult task for both the ancient and the modern scholars. The reason lies firstly in Aristotle’s complicated argument in most of *Metaphysics’* books and secondly in the many-sided targeting of this treatise. The *Metaphysics* consists of fourteen books, in the majority of which Aristotle discusses the introduction of, and several problems related to, a universal science of being *qua* being. However, in the last two books of the *Metaphysics* -namely in books M and N- Aristotle seems to move the focus away from the primary science of being *qua* being and onto the objects of the mathematical sciences that are numbers and geometrical figures. The subject of these two books is undoubtedly connected to the subject of the other books, because in M and N the discussion concerns the ontological status of mathematical objects and thus their relation to the things-that-are as well as to their principles. Nevertheless, the inclusion of these two books in the *Metaphysics* is highly problematic, since the science of being *qua* being -which is dealt with in the preceding twelve books- has been explicitly distinguished from both physics and mathematics. Therefore, books M and N seem to be a self-contained section of the *Metaphysics*.

This PhD thesis mainly aims at laying the foundation for the following two correlative problems to be more clearly displayed and discussed: a) the relation of books M and N to the other books of the *Metaphysics* and, hence, b) the relation of the mathematical sciences to the universal science of being *qua* being. For this goal to be achieved, the ancient text of *Metaphysics’* last two books has been accurately translated and thoroughly commented upon. Aristotle’s philosophy of mathematics can be detected in the questions that are raised by him and the reasoning through which he answers them in M and N. Aristotle’s reasoning, the problems that arise and the scholars’ and commentators’ views on the right way of understanding the Aristotelian claims are the principal topics that are densely dealt with in the introduction and extensively analyzed in the detailed interpretative account of the main part of the thesis. The originality of this study can be found in the constant and
systematic effort to elucidate the obscure points of books M and N firstly through the method of close-reading analysis and secondly through individual observations and comments that are put forward within the framework of a fruitful dialogue with the most recent hermeneutic approaches.
INTRODUCTION

A. THE PLACE OF BOOKS M & N IN ARISTOTLE’S METAPHYSICS AS A WHOLE

The books M and N of *Metaphysics* have caused a lot of embarrassment to scholars. There is an undeniable difficulty in understanding Aristotle’s argument in these two books. The reason for this difficulty lies firstly in the fact that the topics that are discussed in M and N are extremely complicated and secondly in the fact that M and N constitute the only written account of a debate that took place within the Platonic Academy and is not saved until today.

Apart from the hermeneutic difficulties, it is the inclusion of M and N in the *Metaphysics* that seems problematic, since Aristotle explicitly regards mathematical objects as falling outside the domain of first philosophy. Moreover, the endpoint of book M and the starting point of book N are a matter of discussion. Therefore, it is necessary that we try to give a satisfactory answer to the following two questions: firstly, how M and N are related to the other books of the *Metaphysics* and, secondly, how M and N are related to each other.

In *Metaphysics* E.1, 1026a18-19, Aristotle divides the theoretical sciences into mathematics, physics and theology. Theology is said to deal with objects which (i) exist separately from other things -namely they do not exist inside other things and thus they are not ontologically dependent on any other entity- and (ii) are immutable. As regards physics, it deals with things which (i) have a separate existence, but (ii) are not immutable. As far as mathematics is concerned, in Γ.1, 1003a23-25, Aristotle claims that the mathematical sciences study the attributes of some certain portion of the being. Moreover, in E.1, 1026a6-10, he says that the mathematical sciences belong to the theoretical sciences, but it is “not clear at present” (νῦν ἄδηλον)\(^1\) whether mathematical objects are immutable and separately existing.

\(^1\) At this point Aristotle seems to announce his intention of discussing the mathematical objects in a later section of the same treatise.
It is noteworthy that Aristotle has devoted a specific treatise (i.e. *Physics*) to the discussion of physical objects (which exist separately and are not immutable), of their properties and of the problems related to them (such as the ontological status of time, place, infinity etc.). Concerning the objects that exist separately and are immutable, they are mainly discussed in another treatise, which is the *Metaphysics.* However, it is noteworthy that the final two books of *Metaphysics*, i.e. books M and N, deal with the objects of mathematics. Furthermore, there seems to be no Aristotelian treatise that especially focuses on the science of mathematics. This fact could lead us to the conclusion that M and N should be treated as a separate treatise, given that they deal with one of the three sciences which are clearly distinguished from each other in E.

According to Aristotle, it is clear that some branches of mathematics study their objects *qua* immutable and *qua* separate from matter. Therefore, it could be Aristotle’s purpose that these mathematical sciences are one of the subjects discussed in *Metaphysics*, given that their objects are thought of as being of a similar nature to the objects of the first philosophy.

It should be noted that the term “separate/separable” has two meanings: (i) separable/separate from matter, i.e. immaterial potentially or actually, and (ii) self-subsisting as substance. In both senses, only the objects of theology are actually separate. The objects of mathematics are mentally separate or else abstracted from matter. This means that the mathematical objects are not separated in reality. Their separation from matter is the result of a mental process of abstraction that is performed by the mathematician. The objects of mathematics are never self-subsisting, unlike the objects of physics, which are self-subsisting -and in this sense separate- but not separate from matter in the former sense.

Moreover, in E.1, 1026a14-15, Aristotle talks about the existence of “some branches of mathematics” (τῆς δὲ μαθηματικῆς ēνα) that deal with things which are immutable, but not separately existing, for they are present in matter. The inclusion
of these branches of mathematics among the subjects discussed in *Metaphysics* causes complications.

There is an obvious relation among books M and N and the other books of *Metaphysics*. These two books have a close relation to books A and B. For instance, in B.1, 996a1-9, Aristotle formulates the *aporia* whether the One and the being are not distinct, but are the substance of things. This question, which is described by Aristotle as “the hardest and most perplexing of all” is dealt with mostly in book N. Furthermore, according to Jaeger⁴, chapters 1-9 (to 1086a20) of book M discuss the *aporia* formulated in B.2, 997a34-997b3 (“...and is there really only one kind of substance, or more than one, as they who speak of the Forms and the Intermediates hold, which they maintain to be the objects of the mathematical sciences?”). Moreover, it is likely that chapters 1-3 of book M and chapters 1-3, 5 and 6 of book N answer the *aporia* put forward in B.5, 1001b26-1002b11 (“Out of this arises the question whether numbers, bodies, planes and points are substances or not...”). It is also noteworthy that chapters 4-5 and chapters 6-9 of book M practically repeat the Aristotelian arguments against the Platonic theory of Forms and the Form numbers put forward in A.9. Finally, chapters 9 (from 1086a21) and 10 of book M have many references to books A and B. For example, M.9, 1086a34 (“That this is impossible has been already shown by a consideration of the difficulties involved.”) probably refers back to 1003a6-17 of book B. Similarly, M.9, 1086b2 (“as we have said in an earlier passage”), seems to refer back to 987b1-6 of Book A.

Apart from books A and B, there seems to be also a relation between books M and N and books Ζ, Η and Θ of the *Metaphysics*. In particular, 1088b24 of book N (“as we have had occasion to say elsewhere”) may refer back to 1050b7-17 of book Θ. Furthermore, both Ζ and Η refer to the “Forms and the objects of mathematics”, that are discussed in M and N, as something that “must be considered later” (1037a12 and 1042a22, respectively).

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⁴ Jaeger (1923), 186-199
Thus, it could be assumed that books M and N are integrated into the *Metaphysics* as an appendix dealing with mathematical objects. Indeed, it is noteworthy that M.1 begins with a μὲν οὖν...δὲ transition, which indicates “not the beginning of a new treatise, but a new stage within a developing argument”\(^5\).

The position of these two books in the end of the whole treatise seems right for two reasons: firstly, it has already been said that the objects of “pure” mathematics are studied *qua* immutable and *qua* separate from matter. Therefore, they are somehow related to the objects of theology (or first philosophy) that are discussed in the *Metaphysics*. Secondly, according to A.5, 985b23-26, and also according to A.6, 987b18-21, the mathematical objects play an important role both in the Pythagorean and in the Platonic ontology. Therefore, in his *Metaphysics*, where the principles of things are discussed, Aristotle reasonably devotes an explicit study to the mathematical objects, given that they have already been treated as principles of the universe by previous thinkers. If nobody had claimed that numbers, points, lines, planes and solids were principles of things -and possibly also higher substances- Aristotle probably would not bother to deal with them in an extensive manner.

In conclusion, the immutability of mathematical objects and the belief that numbers are principles of things make mathematics a suitable subject for Aristotle’s *Metaphysics*.

**B. THE RELATION OF BOOKS M & N TO EACH OTHER**

It is a matter of question how books M and N are related to each other and, also, where does book M really stop and what is the point from which book N really starts. To give a satisfying answer to these two questions, we should take into account the structure as well as the content of these two books. Books M and N could be divided into the following sections:

\(^5\) Menn (2022), 4-5
Book M:

A. **1076a8-37** (Chapter 1): Announcement of the three inquiries that Aristotle intends to discuss: (a) way of mathematical objects’ existence, (b) way of Forms’ existence and (c) whether numbers and Forms are the principles of things.

B. **1076a38-1077b17** (Chapter 2): Explanation why mathematical objects (a) cannot exist in sensible things and (b) cannot exist apart from sensible things. Conclusion that either mathematical objects do not exist at all or they exist in some qualified sense.

C. **1077b17-1078b6** (Chapter 3): Claim that mathematical objects exist and the best way to think of them is that of arithmetic and geometry.

D. **1078b7-1080a11** (Chapters 4-5): Criticism of the Theory of Forms.

E. **1080a12-1086a20** (Chapters 6-9): Distinction between Form numbers and mathematical numbers. Criticism of Plato and other thinkers’ theories about the way numbers exist and also about their belief that the elements of numbers are the first principles of things.

F. **1086a21-1087a25** (Chapters 9-10): Summary of the criticism formulated by Aristotle in the previous section. Discussion of a problem about principles and of a solution to that.

Book N:

A. **1087a29-1088b13** (Chapter 1): Objection to the theory that treats contraries as first principles and to the belief that there are principles of eternal entities.

B. **1088b14-1090a2** (Chapter 2): Argument against considering the two principles as elements of being and not-being.

C. **1090a2-1091a29** (Chapters 2-4): Criticism of the theory that numbers exist separately and that they are generated.

D. **1091a29-1092a21** (Chapters 4-5): Examination of the difficulties resulting from a relation between the first principles and the good.
E. 1092a21-1092b8 (Chapter 5): Discussion of the relation between number and its first principles.

F. 1092b8-1093b29 (Chapters 5-6): Argument against numbers as causes of other things.

Books M and N discuss the topics that are announced in the first chapter of book M. In particular, book M deals with the first two inquiries, i.e. about the existence of mathematical objects and about the Forms, while book N deals with the third inquiry, i.e. about whether numbers and Forms are the principles of things.

However, there seem to be two problems undermining the theory that M and N form a unity: firstly, in M.6-M.9 (1086a21), Aristotle argues against Platonist theories concerning number. Nevertheless, he does not appear to do so only with the aim of answering the specific questions formulated in M.1. On the contrary, he treats the subject in more general terms. Therefore, the section M.6-9 seems to be a digression.

Secondly, although at M.9, 1086a21, Aristotle returns to the specific topics of M1, he seems to make an introduction to the discussion of the third inquiry, which basically takes place in book N. In particular, in M.9 (1086a21)-M.10, Aristotle sums up his analysis of the first two topics introduced in M.1 and immediately afterwards he seems to start a new discussion concerning the principles of things. This prologue has led both Syrianus in antiquity and Jaeger in modern times to the conclusion that the section M.9-M.10 is more strongly connected to N and that the real beginning of book N should be placed at 1086a21 of book M.

In my opinion, the introduction to the discussion of principles made by M.9-M.10 does not make the inclusion of this section in book N necessary. We should take into account that there are often digressions and untidiness in Aristotle’s writings. Moreover, there is so little distinction in subject matter between the two books, that sometimes the point of the real transition from the one book to the other is not very clear. In fact, the opening sentence of book N at 1087a29 (“With regard to this kind of substance, then, let the foregoing account suffice”) seems to be the real start of the discussion about principles. For, this sentence firmly declares that the Platonic Forms

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6 Syr., 160.6-9
7 Jaeger (1923), 176-193, 205-208, 223-227
and numbers have sufficiently been discussed and now it is time to move on to the next topic, i.e. that of principles.

As for the section M.9 (1086a21)-M.10, it begins with the claim that the views of the thinkers who assert that there are other substances besides the sensible ones should be investigated (1086a24-25) and then lays emphasis on the necessity of discussing the theory that the elements of Forms and numbers are the principles of reality (1086a26-29). Although M.9-M.10 discusses the subject of principles, it does not do so in the systematic and detailed manner of book N and, also, it does not make the crucial distinction between the discussion of principles and that of Forms and numbers as clearly as it happens in the beginning of book N.

C. ARISTOTLE’S AIMS IN BOOKS M & N

i. The intended audience

Aristotle’s main purpose in books M and N of *Metaphysics* is to express his theory about the ontological role of mathematical objects through a constant negation and refutation of the Academic views. In particular, Aristotle seems to have two goals. Firstly, he aims at displaying the relation between the mathematical objects and the physical world. Secondly, unlike Plato, who considers that the principles of mathematical objects are identical to those of the physical world, Aristotle intends to show that they differ from each other.

The content of M and N as well as the arguments which are formulated in them are highly sophisticated. So, it is a matter of question what the audience to which Aristotle addresses is. For it seems that, contrary to the rest of *Metaphysics’* books, which-though demanding- may still be read and comprehended by philosophically minded persons quite generally, the audience of the last two books is limited to some people who have focused their interest especially on the complicated subjects of the sense in which mathematical objects exist and of the way in which they are involved in the generation of the universe.
Furthermore, in M and N Aristotle adopts a polemic tone while arguing against rival theories and principally against the Platonists. He does not adopt the positive tone of arguing in favor of his own theory. Thus, the Aristotelian views arise out of Aristotle’s attempt to disprove the Academic assertions. It seems that his primary intention is to oppose and finally deconstruct the Platonic theory of numbers and first principles. He could do this by means of unveiling the difficulties and inadequacies of the Platonic theory while teaching his pupils a lesson; or by means of beginning from the Academic claims and arriving at new assumptions either in the form of questions or in the form of conclusions while giving a lecture to a very special audience fascinated about the history—or better the philosophy—of mathematics; or, finally, he could address to his fellow students of Academy, trying to convince them that the abandonment of the doctrine of “separateness” is indispensable for the preservation of the essence of Plato’s philosophy.

ii. The polemical attitude

As it has already been mentioned, in books M and N Aristotle formulates his views in the form of arguments that are addressed against the Academic views. His method focuses mainly on pointing out the absurdities to which the Platonic theory of Forms and numbers leads. According to Aristotle, all these absurdities follow, (a) because they make every principle an element, (b) because they make the contraries principles, (c) because they make the one a principle and (d) because they treat the numbers as Forms, i.e. as primary substances that are capable of existing separately (N.4, 1092a5-8).

It is noteworthy that, although Aristotle addresses his criticism to Plato and the Academic thinkers in a quite harsh way (which sometimes seems to be a little unfair), he does not hesitate to admit that “all those thinkers” speak correctly in some way, though not correctly over all (M.9, 1086a12-13). This could mean that Aristotle’s real intention in books M and N is not just to attack and deconstruct the Platonic

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8 Jaeger (1934), 184
ontology by arguing against its absurdities. It is also to discuss with a critical eye what has already been said by the Platonists in order to contribute to the comprehension of the real nature and ontological role of mathematical objects. However, there is a serious difficulty that causes a great deal of embarrassment to all the scholars who comment upon M and N. This is the difficulty of reconstructing with certainty the Academic views due to the insufficient sources of information we have at our disposal.

One further difficulty is Aristotle's constant tendency to talk about “some people” who hold a specific opinion on an issue and “some others” who have a different belief about the same subject. For example, Aristotle arrives at the conclusion that it is impossible for number and magnitudes to be separate from perceptible things. In M.8, 1083a21-25, he refers to “those” who believe that only mathematical objects exist besides sensible beings, because they have realized the absurdity that is caused by the theory of Forms. These thinkers are said by Aristotle to have moved away from Form number and set up mathematical number. The aforementioned opinion probably belonged to Speusippus, but it is difficult to say precisely who else shared the same belief. We face the same difficulty with the identity of those by whom “it is said” that both Form number and mathematical number exists and also that Form number is identical to mathematical number (M.8, 1083b1-8). It is likely that Aristotle has Xenocrates in his mind here, but we cannot say for sure who else had embraced the same ideas. On the contrary, it is quite clear where exactly Aristotle refers to the Pythagorean theory of numbers. The Pythagorean theory is presented as the most reasonable of all, on account of the fact that the Pythagoreans do not treat the number as existing separately from the beings. Nevertheless, what causes a great deal of difficulty, according to Aristotle, is the Pythagorean claim that the bodies are composed of numbers, given that it is impossible that the number of which the bodies are made up is the mathematical one (M.8, 1083b8-13).

In general, the task of matching the views that are negated by Aristotle with the thinkers who have adopted and supported them is undeniably demanding. The difficulty of this task increases due to the deficiency of other sources displaying the
rival theories and also due to the need of assuming the exact content of these theories as well as the identity of their (Platonic, Academic and Pythagorean) supporters exclusively through the Aristotelian text.

iii. The endoxic method

It is noteworthy that throughout books M and N, Aristotle usually resorts to the use of *endoxa*, so as to present the views about the mathematical objects which he wishes to discuss. The *endoxa* that are used in M and N differ from the *endoxa* used in the other books of *Metaphysics* - and especially in book A - because in the last two books of the treatise Aristotle refers almost exclusively to the Platonic and generally the Academic views which he plans to disprove and only occasionally to other thinkers or schools of thought (such as the Pythagoreans).

The *endoxic* method is common in Aristotelian treatises. Aristotle's purpose of using *endoxa* seems to be both pointing out the theoretical difficulties of the complex subject matter he deals with and proving the truth of some reliable *endoxa* in order to better convince his audience through achieving consistency with the most authoritative opinions. This double purpose becomes evident also in *Metaphysics*' M and N. In M.1, 1076a33-35, two *endoxa* are presented, that are also refuted in M.2, 1076a38-1077a14: firstly that the mathematical objects, “as some people say” (καθ’απ’κὶ λέγουσι τινὲς), are substances that exist inside perceptible things and secondly that the mathematical objects, “for some (other) people say so, too” (λέγονσι δὲ καὶ οὔτω τινὲς), are substances that exist separately from perceptible things⁹. In the aforementioned *endoxa*, it seems that Aristotle’s aim is to present two refutable opinions that complicate the question of the status of mathematical objects and thus give rise to discussion about the way in which mathematical objects exist.

Further down in the same chapter (1077a14-16), Aristotle says that “in general, conclusions opposing truth and received opinion will be reached, should somebody assume that the mathematical objects exist this way, namely as some sort of

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⁹ Katz (2014), 343-368
separately-existent natures” (ὅλως δὲ τοιναντίων συμβαίνει καὶ τοῦ ἀληθοῦς καὶ τοῦ εἰσθώτος ύπολαμβάνεσθαι, εἰ τις θήσει ὑστερ χαὶ τὰ μαθηματικὰ ὡς κεχωρισμένας τινὰς φύσες). In this case, Aristotle suggests that there are acceptable endoxa which are opposed to his rivals’ refutable endoxa that have just been negated.

In general, it could be observed that Aristotle does not hesitate to make use of the human everyday experience for the sake of a better treatment of the mathematical objects.

D. ARISTOTLE’S ARGUMENT IN BOOKS M & N

i. Aristotle’s argument against Platonic Forms. The relation between A.9 and M.4-5 of Metaphysics.

Aristotle’s criticism against the Platonic theory of Forms becomes evident mainly in M.4 and M.5, where he deals with the Platonic Forms, without any reference to numbers. The argumentation against the theory of Forms is integrated in Aristotle’s plan of considering whether there is anything imperishable that exists apart from perceptible things and, if there is, what it is (M.9, 1086a24-29). According to Aristotle, Socrates was the first who tried to formulate universal definitions of perceptible things, but he did not claim that the objects of those definitions exist in separation from the perceptible things they define. It was Plato and his followers, who ascribed a separate existence to the universal entities that belong to definitions, and they called them “Forms”. In seeking the causes of existing things, Platonists claimed that there are Forms of all of them (M.4, 1078b23-32). So, they are said by Aristotle to believe in the existence of Forms as universal substances that exist separately from the particular ones.

Aristotle truly thought that there must necessarily be something universal apart from the perceptible things. However, he definitely did not claim that the universals exist separately from perceptible things (M.9, 1086b5-7). The separate existence of
Forms is the Platonic claim that mainly caused Aristotle’s attack. For, the role played by Platonic Forms could be assigned to any Aristotelian genus or species of individual things and any Aristotelian universal for that matter. However, according to Aristotle, it cannot be assumed that genera, species and universals quite generally exist separately from the things that they qualify.

It is noteworthy that many points of Aristotle’s criticism against the Platonic Forms in A.9 are duplicated in M.4-5. In M.4-5, 1078b31-1080a11, Aristotle’s argumentation against Forms is in fact a repetition of his arguments that are formulated in A.9, 990b2-991b9. The similarities between A.9 and M.4-5 become evident in the following table of correspondences:

<table>
<thead>
<tr>
<th>A.9</th>
<th>M.4-5</th>
<th>Resemblances in Aristotelian criticism</th>
</tr>
</thead>
<tbody>
<tr>
<td>990b2-27</td>
<td>1078b32-1079a24</td>
<td>In their attempt to make a transition from the variety of particular beings to the universal elementary principles of the world, the Platonists introduced a number of new entities, which they named Forms. However, Forms will turn out to be much more in number than particular beings, given that there are Forms not only of substances, but also of many other things that are objects of sciences.</td>
</tr>
<tr>
<td>990b27-991a8</td>
<td>1079a24-1079b3</td>
<td>Nevertheless, if Forms are to be participated in by beings of our experience, then they must be participated in only by substances. And this leads to the conclusion that the Forms are substances, too. This is reasonable, since, if we do not accept that the Forms and the things which participate in them are of the same sort, i.e. substances, then they will simply be homonyms without being linked to each other by any other sort of relationship or community.</td>
</tr>
<tr>
<td>991a9-991b9</td>
<td>1079b12-1080a11</td>
<td>Moreover, Forms contribute nothing to sensible things, for they are not causes of their generation or of any sort of change in</td>
</tr>
</tbody>
</table>

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10 Düring (2015), 444
them. On the contrary, it is possible for anything that is similar to something else, which is a Form, both to be and to come into being without imitating that specific Form. Furthermore, there will be several patterns and therefore several Forms of the same thing. For example, Man, Animal and Two-Footed will be patterns and thus Forms of one particular being, i.e. an individual man. Finally, there must be Forms also of many artificial things (e.g. a house or a ring) that are generated, but of which the Platonists do not claim there are any Forms.

It is a matter of discussion what is the reason for this similarity and also whether it is the text of A.9 or the text of M.4-5 that was written first. It is usually believed that Aristotle firstly wrote the text of A.9, because he uses the pronoun “we” in order to refer to the Platonists, while in M.4-5, instead of “we” he uses the pronoun “they” for the same purpose. In other words, it seems that in A.9 Aristotle uses the first person plural because he includes himself in the group of Platonists who have adopted the aforementioned beliefs concerning the Forms, while in M.4-5 he uses the third person plural so as to keep a distance from them showing that he treats Platonists as a rival school.

Jaeger\(^\text{11}\) considered this to be evidence that Aristotle wrote the text of A.9 at an earlier stage -when he defined himself as a member of the Academy- and later he incorporated almost the same text in M.4-5, which belonged to a separate treatise. In my opinion, the fact that Aristotle uses the first person plural in A.9 and the third person plural in M.4-5 does not necessarily indicate that these two almost identical passages from *Metaphysics* have been created in different periods of Aristotle’s thought. It rather seems to me that the two almost identical texts have been written with a different intention: in A.9 Aristotle wishes to present the Platonic theory of first principles and to point out its difficult points, while in M.4-5 he mainly wishes to address his intense polemic against the Platonic views and disprove them. Hence,

\(^{11}\) Jaeger (1923), 171-172
it seems that the same text has been used twice in order to unveil two different intentions of the same author. Aristotle, who starts the *Metaphysics* with a questioning attitude, ends his treatise in an aggressive mood.

ii. **Aristotle’s argument against Platonic Form Numbers**

In books M and N of *Metaphysics*, Aristotle deals with the role of mathematical objects within the framework of his ontology and criticizes the Academic views about mathematical objects. As it has already been said, the section consisting of books M and N is not the only part of *Metaphysics* in which Aristotle tries to raise objections against the ontology of Plato and his successors. The same intention becomes evident in A.9, 991b9-992b18, where he explains how the turn of the Platonic theory of Forms to a mathematical direction led to even more absurdities and difficulties. Moreover, in the same chapter, Aristotle complains that, although philosophy should be concerned with the causes of visible things, Plato and his successors try to explain the existence of other substances that are not perceptible. Thus, Aristotle attacks systematically the mathematized version of the Platonic theory of Forms, which undermines the understanding of the natural reality.

In M.1, 1076a22-32, Aristotle formulates the three questions which he intends to answer in the course of books M and N. The first question that Aristotle poses is about mathematical objects, the second question concerns Forms, while the third one is whether the principles of beings are numbers and Forms.

Aristotle’s question about mathematical objects is divided into three sub-questions: 

a) Are or aren’t the numbers Forms? b) Are or aren’t the numbers principles? c) Do or don’t numbers exist? And, if they exist, in what way do they do so? It is necessary to summarize Aristotle’s argumentation concerning each of these questions.

As far as the first question about the existence of Form numbers is concerned, Aristotle’s argument can be derived from M.6-7 and briefly be presented in the following points:
According to Aristotle, if we accept the existence of Form numbers, for instance the existence of the original number Three, number Four etc., then we also have to accept that each number differs in kind from any other number, given that each original number is a different Form. Therefore, the units of the Form number that is primary in succession, for example the units of number Three, and the units of the Form number that follows, for example the units of number Four, are also of a different species and they cannot be added to each other. Thus, it is impossible for a Form number that is primary in succession to produce the Form number that comes next. For instance, number Four cannot be derived from number Three. For, the units of different Form numbers are non-combinable with each other (M.6, 1080a15-20 & 1080a33-37).

However, if we accept that Form number Ten can be said to consist either of ten units or of two Fives and number Four can be said to consist either of four units or of two Twos, then it necessarily follows that the units which exist in each Form number should be different in kind from each other, too. The reason is that, as it has already been said, the units that comprise each Form number are different in kind from the units of any other Form number. For instance, in the case of Form number Ten, the ten units that comprise number Ten are different in kind from the units that comprise the two Fives of which number Ten can also be said to consist. Nevertheless, if the units are different in kind, then they are non-combinable with each other in the same number. And, if the units are non-combinable in the same number, then the existence of Form numbers is impossible (M.7, 1081b35-1082a14).

In case units of different numbers were combinable with each other, then the number in discussion would not be the Form number, but the mathematical number, which is the number that is dealt with by the mathematicians. For, the units of different mathematical numbers do not differ in nature and thus they can be added to each other. Therefore, the mathematical number that comes after one is counted as number two and consists of another unit that is added to the former one. Similarly, mathematical number three consists of one more unit that is added to the former two units that comprise mathematical number two (M.6, 1080a20-33). However, if it is
accepted that the units of different numbers are combinable with each other, a great deal of difficulty will be caused to Platonists, because it proves impossible for Form number to exist, while the only number that can exist is the mathematical one.

Aristotle also mentions one third possibility, i.e. that the units are all non-combinable with each other. Nevertheless, in such a case the sort of number that is generated is neither Form number nor mathematical number (M.7, 1081a17-25). For, it is necessary that both Form numbers and mathematical numbers come from a process of adding units (M.7, 1081b10-19).

In conclusion, it seems more reasonable to Aristotle that the units are combinable with each other and that the only sort of number that really exists is the mathematical one, while the Forms cannot be numbers. Moreover, no mathematical theorem applies to Form numbers. Therefore, the postulation of Form numbers and magnitudes posits a realm of mathematical objects in which no mathematical relations apply. For, in the case of Form numbers, mathematical operations -such as addition and subtraction- cannot be performed; and similarly with geometrical objects that are conceived in the same fashion as Forms. The mathematical truths are not true of them or at least they are not true of them in the same way in which they are derived in actual mathematics (N.3, 1090b20-33). But, if the Forms are not numbers, then they cannot exist at all, given that the first principles, according to Plato, are the One and the Indefinite Dyad, which are principles of number (M.7, 1081a5-17).

iii. Aristotle’s argument about numbers in relation to first principles

a. Are or aren’t the numbers principles?

Aristotle attacks the view that numbers are the first principles of reality, but he also argues against the derivation of all the existing things from the two Platonic first principles that are the One and the Dyad.
According to Aristotle, numbers neither act obviously as a cause of existence for the beings nor are in general needed for the explanation of the being of substances (N.2, 1090a3-15). So, the question remains in what sense numbers can be responsible for the substances and the being.

Firstly, Aristotle denies that numbers could act as boundaries of the existing things or as ratios that express the internal structure of them (N.5, 1092b8-25). Moreover, it does not seem to be due to the kind of a specific number that there are seven vowels in Greek language, that the musical scale has seven strings, that there are seven stars in the constellation of the Pleiads, that the animals lose their teeth at the age of seven, and that according to tradition there were seven warriors against Thebes. For example, the heroes against Thebes were seven because of the number of gates or due to some other reason and, while we count the Pleiads as seven stars, other people may count more or less stars in them, depending on the geographical point from which they see the constellation. In general, there is no difficulty in finding and stating such analogies, because they often occur coincidentally among the things (N.6, 1093a13-1093b11).

Furthermore, if everything could be expressed by number, then many difficulties would follow, since the same number would be used to measure many different things. And, if two things were measured by the same number, then it would turn out that they necessarily are the same, which is absurd (N.6, 1092b30-1093a13).

Therefore, numbers do not seem to act either as causes of generation or as means of explanation or as the outline of the form or as the inner organization of the matter of beings.

b. Aristotle’s argument against the Platonic First Principles

Aristotle also addresses his criticism against the two Platonic first principles, i.e. the One (or else the Equal) and the Dyad (or else the Unequal or the Great and Small). First of all, according to Aristotle, Platonists, although they seem to speak of two elements, in fact speak of more than two elements, given that the Dyad can be
thought of as one thing in formula but as more than one thing in number (N.1, 1087b4-12). Thus, the dualistic system of Platonic first principles is disproved.

In addition, if the principles are the same for all beings, then different objects, such as the line, the plane and the solid will turn out to be the same. For, what comes from the same principles is one and the same thing. Nevertheless, if the material principles are more than one – i.e. one of the line, another of the plane and another of the solid – then the aforementioned objects must necessarily consist of different types of matter and, as a result, the plane will not be able to contain the line and the solid will not be able contain the plane (M.9, 1085a35-1085b4). In general, it seems totally absurd that one principle, which is a being of one single nature, is responsible for the wide diversity of existing things and their qualities (Ν.2, 1089a6-19).

In any case, the principles cannot be thought of as substances, given that they are of elementary nature. But, again, not all the principles should be thought of as incomplete and undeveloped. For, especially in the case of animals, the undeveloped seed comes from a fully developed parent. Therefore, not all the principles can be treated in the same way, for they are not all complete or incomplete to the same extent (N.5, 1092a9-21).

Now, as far as the One is concerned, it is in fact a measure of some plurality. Given that any number is a measured plurality, i.e. a plurality of measures, the One is not a number; it is more of a starting point. The measure must always be one and the same thing applying to all alike; for instance, if the measure is human being, it will apply to human beings. And, if the things to be measured are a human being and a horse and a god, the measure will perhaps be living being. If the things to be measured are a human being and a white thing and a walking thing, there is nothing they have in common. However, human, walking and white are different kinds or ways of being, i.e. different categories/predications (N.1, 1087b33-1088a14). What they have in common is that they are one thing in the sense that this particular human being is both walking and white. Therefore, the One seems to be the first principle that acts as a universal measure imitated by all the individual measures of beings which act analogously to the One.
As regards the Indefinite Dyad (or the Unequal or the Great and Small), it cannot easily be thought of as a material substrate of the existing things. For, the Unequal and the Great and Small are features and properties of the mathematical objects. On top of this mistake, the Great and Small and all the similar things must be relative to something; and the relative is of all the categories the most remote from substance, and also it is posterior to both quality and quantity. Therefore, it cannot be an element and starting point of substance. It is absurd, or rather impossible, to make non-substance an element of, and prior to, substance; for all the categories are posterior to substance (N.1, 1088a15-1088b4). Aristotle’s criticism here is founded on his theory about the categories of being, the most primary of which is substance.

Moreover, the Indefinite Dyad cannot be thought of as a principle of qualities, such as colors, flavors and shapes, given that it can exclusively produce numbers and units. However, in the case of qualities too, there should be some generative element that is analogous to that of numbers and units. Some thinkers introduce the Relative, which is another term for the Unequal, in their attempt to explain the generation of qualities and substances apart from numbers. Nevertheless, they do not ask how there are many kinds of unequal things besides the Unequal, while they make use of them and talk about great and small, many and few (from which the number comes), long and short (from which the length comes), broad and narrow (from which the plane comes) and deep and shallow (from which the masses come) (N.2, 1089a31-1089b15).

Furthermore, while some Platonists describe the material principle as the Great and Small, others call it Many and Few and still others speak of the universal term covering these, i.e. the Exceeding and the Exceeded. However, none of these variations makes any significant difference for Aristotle. And also, if the first principles are the Exceeding and the Exceeded, which are more universal notions than the Great and the Small, then on the same principle number, which is more universal, should be derived from the first principles before the Dyad, which is more particular (N.1, 1087b12-27).
On the other hand, if the elements of substances were not universal, but unique individual things, then substances would be limited in their number which would not surpass the number of the elements. But, if this were true, then there would not be other existing things besides the elements. Moreover, the elements would not be knowable; for they would not be universal, but individual things, while the sciences deal with universal objects (M.10, 1086b20-37).

It is noteworthy that Aristotle, while formulating his views about the inadequacy of the Platonic first principles, in many cases seems to conceive of the One and the Indefinite Dyad in the same way he conceives of the common mathematical numbers one and two. Probably, his intention here is again to lay emphasis on the difficulties lying in obscure and controversial points of the Platonic theory that seem to be opposed to common sense. However, the Aristotelian understanding of the Indefinite Dyad as two elements instead of only one could also be the result of a misconception of the Platonic ontology. For, it seems that Aristotle sometimes overlooks the aspect of indefiniteness of the Dyad and gives prominence to its arithmetic character.12

c. How is number derived from the First Principles?

One further question is in which sense the number is derived from the first principles. Those thinkers who posited the One as first principle and generated the numbers out of it would answer the question about how the numbers can be generated out of the One with the reply “the one is the seed of numbers”. However, the seed is not indivisible like the One; hence the One cannot act like a seed. Moreover, nothing can come out of something indivisible. So, it was assumed that the number was not generated only from the One but it was also generated from another principle that is contrary to the One. One thinker (probably Speusippus) posited the One as contrary to the Plurality, while another (probably Xenocrates)

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12 Κάλφας (2009), 125
posited it as contrary to the Unequal—for he treated the One as Equal. Therefore, number was treated as coming from contraries (N.5, 1092a29-1092b3).

However, the first principles cannot be contraries, given that in such a case two serious problems would arise: not only would the first principles generate substances that are contrary to one another, but there would also be contrasts within the same substance, which is impossible (N.1, 1087a29-1087b4). Further, even if the thinkers who claim that the first principles are contraries were right to say so, their theory would prove to be inadequate; for, it is not the One but the fewness that is opposite to the plurality (N.1, 1087b27-33).

Also, if the elements of the beings were contraries, that would cause the destruction of the existing things. For, the one contrary would fight against the other and, as a result, there could not be any cohesion among the constituent parts of the being. Finally, if there are no other elements apart from the One and the Indefinite Dyad or the Great and Small, then it turns out that both the Form number and the mathematical number have to be composed out of the same elements, i.e. the One and the Indefinite Dyad, which is absurd, because in such a case Form numbers would be identified with mathematical numbers. In conclusion, we cannot think of contraries as elements of number (N.3, 1090b31-1091a12 & N.5, 1092b3-8).

d. What is the relation of the First Principles to the good and the beautiful?

According to Aristotle, the first principles can be related to the good and the beautiful in two ways: either the good and the best are of the same sort as the first principles or the good and the beautiful appear at a later stage as a result of the first principles’ interaction.

Firstly, there are those who identify the original One with the original Good and treat the unity as the essence of the goodness. These people identify the One with the Good on the assumption that the essence of good is the one, namely that whenever we say that something is good what we mean is that it is unified (N.4, 1091b13-25).
If the One is to be identified with the Good, then every unit becomes, on this view, something that is good in itself, so that there is rather a great profusion of good things. Further, if the Forms are numbers, then all the Forms are identical with sorts of good. And, if there are Forms of living substances, then all the animals and the plants will be treated as good things, which is not reasonable (N.4, 1091b25-30).

Moreover, provided that the first principles are contraries, if the one principle is good, then the other principle must necessarily be bad. This means that the One will be good and the Indefinite Dyad (or the Unequal or the Great and Small) will be bad. Therefore, the principle which is treated as matter will be bad. However, if this is true, then the good, i.e. the formal principle, will interact with the bad, i.e. the material principle, which is also absurd (N.4, 1091b30-1092a5).

e. Do or do not numbers exist? And, if they exist, in what way do they do so?

In his attempt to deal with the third and final question concerning numbers, which has been announced in M.1, Aristotle formulates an extensive argument about whether numbers and, in general, mathematical objects exist or not and also about the way in which they exist.

Firstly, he claims that mathematical objects cannot exist inside perceptible things. As it has been said previously in the Metaphysics (B.2, 998a13-17), it is not possible for more than one object, e.g. a mathematical and a physical object, to occupy the same place. Also, Platonists thought of mathematical objects -such as the point as well as the line and the plane- as substances that exist independently without the one being divided into the other. So, in case mathematical objects are indivisible, then the physical objects have to be indivisible too, if it is accepted that mathematical objects exist inside them. However, any physical object can be cut into pieces. Therefore, claiming that mathematical objects exist inside perceptible things and thus perceptible things are indivisible is absolutely absurd (M.2, 1076a37-1076b11).
Nevertheless, the objects of mathematics cannot exist separately from perceptible beings. If we accept the separate existence of mathematical objects, there will turn out to be too many such objects without it being clear which of them is to be studied by mathematical sciences. We will also have to accept that the objects of astronomy, geometry, optics and harmonics exist separately from perceptible things. And, similarly, the mathematical theorems will have to exist separately from physical objects too (M.2, 1076b11-1077a17).

The only way mathematical objects can be prior to and thus exist separately from physical bodies is by definition. Solid bodies are prior to planes, lines and points in terms of being, due to solid bodies' being more complete as substances than the mathematical objects. Nevertheless, planes, lines and points are prior to solid bodies in terms of definition, due to their being necessary for the definition of physical bodies to be formulated. But, even if we accept this sort of priority of mathematical objects, they can in no way be located in a specific place and exist separately from physical objects (M.2, 1077a24-1077b11).

It is noteworthy that the mathematical theorems are concerned with physical objects, not \textit{qua} being perceptible things, but \textit{qua} having certain mathematical properties. In particular, the mathematicians study physical objects \textit{qua} solids, planes, lines or \textit{qua} divisible and indivisible things. Therefore, the mathematical sciences deal with the aforementioned mathematical aspects of physical beings. These aspects are the objects of mathematical sciences, or else the mathematical objects. What the arithmetician and the geometer do is separately consider and study properties of things that cannot be thought of as separately existing entities. Therefore, it proves necessary to release the physical object from its material properties for the sake of quick, easy and right calculation. This means that the mathematical objects do exist, but they can be studied only by abstraction from physical objects. Physical objects are a sort of subject matter for mathematical sciences. Hence, mathematical sciences are neither about physical objects \textit{qua} physical nor about other objects that exist separately from physical ones (M.3, 1077b17-1078a30).
In conclusion, books M and N should be considered as integral part of *Metaphysics*, firstly because the mathematical objects are of the same nature as the objects of the first philosophy and secondly because the mathematical objects have been considered to be principles of the universe by previous thinkers. Therefore, the subjects of books M and N fit in the *Metaphysics* and there is no serious reason why we should treat these two books as a separate Aristotelian treatise.

Moreover, books M and N are indissolubly linked to each other in a coherent whole. In book M, Aristotle deals with the first and the second questions that are formulated in M.1, while book N is devoted to the discussion of the third inquiry. The transition to the discussion concerning numbers in relation to first principles is made in M.9-10, though not as clearly as it is made in book N. Hence, M.9-10 does not need to be treated as the starting point of book N, as it seems to end book M by announcing the topics that Aristotle is about to discuss in detail in book N.

Finally, the unique character of books M and N is owed mainly to Aristotle’s polemical attitude against the theories of Plato and his successors. It is likely that Aristotle’s criticism has a double purpose: firstly, Aristotle intends to demarcate his own theory concerning mathematical objects by highlighting all the disputable points of Platonic claims about them and, secondly, he wishes to specify the ontological role played by mathematical objects through the discussion of their relation to the first principles of the universe.

E. BIBLIOGRAPHICAL REVIEW

There has already been a great deal of discussion among the scholars about the books M and N of the *Metaphysics*. As far as the ancient scholars are concerned, I have chosen to use the ancient commentaries of Syrianus and Alexander, knowing that the first writer often wishes to defend Plato against the Aristotelian objections and that the text which has been created by the second writer is in fact (from book E
et seq.) the work of the Byzantine scholar Michael of Ephesus who lived in the 12th century A.D.¹³

Now, it is worth mentioning and evaluating the contribution of modern scholars to the study of different aspects of Metaphysics' M and N and to the comprehension of Aristotle's argument in these two books.

a. The relation of M and N to the rest books of Metaphysics

One of the most important, thorough and detailed modern commentaries of M and N has been done by W. D. Ross (1924). Ross¹⁴ treats M and N as part of the original version of Metaphysics, especially due to the way in which these two books frequently answer some of the problems that are formulated in the book B. W. Jaeger (1934)¹⁵, who has especially dealt with the problem of the relation of M and N with the rest books of the Metaphysics, also accepts that there is a close relation between M and N on the one hand and A and B on the other that is founded on the criticism of Plato's doctrine of Forms. Actually, the inclusion of some almost identical chapters in A and in M led Jaeger to the conclusion that Aristotle's intention was to use parts of A as material for the subsequent reconstruction of his argument in M. However, as regards the place of books M and N in the Metaphysics, he remarks that these two concluding books have no obvious relation to the immediately preceding books of the same treatise. S. Menn (2022)¹⁶, who considers the books M and N to be part of the Metaphysics, argues that, in particular, M and N address aporiai from B, take up topics deferred from Z and H and raise difficulties against the Academic accounts of the ἀρχαί that will be resolved in Aristotle's positive account in Λ. According to Menn, the Metaphysics would be defective without M and N, firstly because the critical discussion of Academic unmoved φύσει and their ἀρχαί would never be

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¹³ Taking into account that Alexander's commentary has been written by Michael of Ephesus, I have chosen to put Alexander's name into brackets ([Alex.]) every time I use it in footnotes, with the aim of making clear that I am referring to this particular ancient source.
¹⁴ Ross (1924), xiii-xxiv, xxxii-xlvi, li-lxvi
¹⁵ Jaeger (1934), 167-227
¹⁶ Menn (2022), 1-71
delivered and secondly because certain Aristotelian *aporiae* would not receive answers. In general, it seems that the recent studies on M and N tend to recognize that these two books fit in the project of the *Metaphysics*.

b. The ontological status of geometrical objects

The Aristotelian geometry has prompted scholars’ interest to a greater extent than the Aristotelian arithmetic. I. Mueller (1970)17 systematically deals in his article with the way in which Aristotle “separates” or “abstracts” what is “not separate or separable”. In particular, he states that Aristotle denies the Platonic mathematical ontology, but he does accept the Platonic mathematical epistemology (i.e. the assumption that there must be a correlation between the mathematical and the physical objects) and reaches the conclusion that Aristotle could hardly have conceived the objects of mathematics as being totally separated from any sort of underlying matter. It is highly probable, according to Mueller, that Aristotle considered the “purely dimensional” (i.e. the line, the plane and the solid) as an “intelligible matter” upon which other geometrical properties are imposed. Nevertheless, as E. M. Cadavid (2009)18 points out, it is not made clear through Mueller’s interpretation what is the ontological status of these “other” or else “secondary” geometrical objects.

E. Hussey (1991)19 emphasizes that Aristotle’s mathematical objects are “representative objects”, or else objects having generic properties which belong to the individuals of a class *qua* members of that specific class. The Aristotelian representative (i.e. mathematical) objects are reached through a process of “abstraction”. Hussey seems to be in agreement with Mueller in that the mathematical objects are considered to serve as a kind of mathematical “matter”, analogous to the matter of the physical world. On the basis of this hermeneutic

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17 Mueller (1970), 156-171
18 Cadavid (2009), 81-99
19 Hussey (1991), 105-133
approach, mathematics may be seen as a science that explores the possibilities inherent in that fictitious matter.

J. Annas (1976)\(^{20}\), who has translated and thoroughly commented upon M and N, partly disagrees with Mueller and states that the treatment of all geometrical objects as properties suits better Aristotle’s conception. As regards the Aristotelian concept of “intelligible matter”, Annas believes that it was probably formulated so that Aristotle could avoid the difficulties of the concept of “abstraction” from the matter of physical objects that brought him dangerously close to Plato.

c. The Aristotelian “abstraction”

J. Lear (1982)\(^{21}\) admits that Aristotle’s philosophy of mathematics has its obvious limitations, but he also recognizes that it has its certain virtues. According to Lear, even though there may be no purely geometrical objects for Aristotle, they are postulated as separated objects for heuristic purposes. To make a link between geometry and the physical world, Aristotle claims that the elements of a geometrical construction are abstractions from the physical world. In order that the Aristotelian abstraction of mathematics from the physical world becomes clearer, Lear stresses the necessity of a bridge between the physical world and the world of mathematical objects enabling us to see how we can cross the world of mathematical objects and return to the physical world or else how the world of mathematical objects and the physical world overlap. Provided that the Aristotelian “abstraction” should be treated as a process of knowing and understanding the objects of mathematics (i.e. as an epistemological process), in order that this process is accomplished, it is necessary that the physical world is seen from a “mathematical” point of view. The “mathematical” conception of the universe requires the existence of a mathematical structure in the physical world or the applicability of mathematical laws and theorems in it. This concept of mathematical organization could be the “bridge”

\(^{20}\) Annas (1976), 26-41
\(^{21}\) Lear (1982), 161-192
between the world of physical objects and the world of mathematical objects that is asked for by Lear.

J. Cleary (1995)\textsuperscript{22} states that, unlike Platonists, Aristotle did not consider either Forms or mathematical objects to exist separately, since they are unable to provide principles (such as the Aristotelian Unmoved Mover) for the cosmos. Abstraction is proposed by Aristotle as an epistemological process for understanding the objects of mathematics. This process is totally different from the epistemological process of induction from experience, which makes us familiar with the principles of other sciences like ethics and physics that are not concerned with universals, but only with particulars. Y. K. Cho (2009)\textsuperscript{23} claims that the Aristotelian abstraction is in fact a process of linguistic analysis and not an epistemological process. A. Younan (2019)\textsuperscript{24} says that the mathematical realities are neither independent Platonic Forms nor simply nominalist patterns that exist only in the mind, but rather mental abstractions of the symmetries that are truly there in nature.

d. The Aristotelian “fictionalism”

It is admitted that there is a difficulty in reconciling Aristotle’s mathematical fictionalism with his realism. According to Aristotle’s realism, mathematics studies aspects and properties of sensible things. Cho\textsuperscript{25} criticizes Lear for complicating things through the introduction of the idea of geometrical figures that are mentally conceived and not physically instantiated, while they are constructed out of basic geometrical elements that require physical instantiation. Cho’s criticism is founded on the fact that the Aristotelian fictionalism which is proposed by Lear undermines Aristotle’s realism.

In his article, P. Corkum (2012)\textsuperscript{26} draws a distinction between two interpretative approaches to the Aristotelian philosophy of mathematics. Firstly, he refers to

\begin{itemize}
\item Cleary (1995), 248-343
\item Cho (2009), 36-78
\item Younan (2019), 1-20
\item Cho (2009), 79-203
\item Corkum (2012), 1057-1075
\end{itemize}
“literalism”, which is founded on the idea that Aristotle believed in the existence of distinctly mathematical matter that gives mathematical objects the status of being perceptible things. Secondly, Corkum refers to “fictionalism”, which is based on Aristotle’s saying that mathematicians draw the harmless and useful fictitious assumption that mathematical objects exist. According to Corkum, both theories have their weaknesses. Moreover, he says, Aristotle probably had thought of the mathematical objects as inseparable from the physical beings, though only in sense that they have their ontological status in virtue of standing in relation to sensible substances.

Corkum is definitely right in pointing out that Aristotle believed in the existence of a relation between mathematical and physical objects. Given that the conception of mathematical objects stems from the empirical perception of the physical world, a relation between the mathematical and the physical realm seems indispensable for the grasp of mathematical objects. However, we should keep in mind that, though this relation could be described as “inseparability” - in the sense that the conception of mathematical objects is inseparably linked to the perception of the physical objects- it can equally be described as “separability”, in the sense that the process of “abstraction” requires the mental separation of the mathematical objects from the physical beings.

As far as Aristotle’s mathematical fictionalism is concerned, it can be generally assumed that the mathematical “fiction” which is described by Aristotle does not amount to a mere inexistence of mathematical objects, but it is equal to the ontological dependence of the mathematical objects on substances. Therefore, it seems that Aristotle’s fictionalism does not necessarily negate his realism.

e. The Aristotelian theory of numbers

L. Tarán (1991)\textsuperscript{27} focuses on Aristotle’s exhaustive classification of numbers with reference to their units in *Metaphysics’* M.6. Moreover, he emphasizes that Aristotle

\textsuperscript{27} Tarán (1991), 199-231
does not ascribe to Plato the treatment of Form numbers as themselves simple and single unitary things. On the contrary, Aristotle seems to claim (incorrectly, according to Tarán) that Plato had thought of Form numbers as congeries of units that are “incomparable”, i.e. as numbers comprising of units that are non-combinable with one another. Tarán considers this Aristotelian claim to be the reason why this part of Aristotle’s testimony should be rejected.

M. Crubellier (1994)\textsuperscript{28}, who has most recently translated and extensively commented upon M and N, embraces the idea that the assumptions formulated in books M and N are not of solely mathematical interest, but they can equally find application in other sciences that are founded on the principles of rational thought. According to Crubellier, the argument of M and N is of paramount significance, since the questions and problems discussed in these two books concern mathematicians only insofar as they are also preoccupied with philosophy. Therefore, the “mathematical” or “non-mathematical” status of numbers is a primary issue of M and N. In this context, it is totally unacceptable that the Platonic Form numbers have no relation to mathematics. On the contrary, it is indispensable that we admit their usefulness for the comprehension of the mathematical numbers, especially on account of the fact that the principles of Form numbers, i.e. the One and the Indefinite Dyad, are of arithmetic nature.

It is noteworthy that the modern scholars seem to discuss the Aristotelian geometry to a greater extent than the Aristotelian arithmetic so far. The reason why such prominence is given to the geometrical over the arithmetical aspect of Aristotle’s mathematics may be that Aristotle himself seems to lay emphasis on the objects of geometry while expressing his views about the way in which mathematical objects exist. Aristotle’s philosophical reflection upon number is elaborated mainly in his argument against Form number and in favor of mathematical number and also in his polemic against the Platonic first principles that are of numerical nature. Besides, the status of the Aristotelian geometrical objects constitutes a greater challenge for discussion among the scholars than the status of the Aristotelian numbers, because

\textsuperscript{28} Crubellier (1994), 11-26
the dimensional character of geometry complicates the understanding of its objects, for it tempts the mind to conceive the geometrical objects as material entities that resemble in a way the physical objects.

Now that we have roughly referred to the characteristics that feature the special and nature of books M and N as well as to the main topics that have challenged the scholarly interest, let us see the great hermeneutic difficulties that arise from closely reading each one of these two books’ chapters.
### BOOK M

**M.1 (1076a8-37)**

“Announcement of enquiries and objectives”

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<th>SECTION</th>
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<tr>
<td>1076a8-10</td>
<td>Περὶ μὲν οὖν τῆς τῶν αἰσθητῶν οὐσίας εἰρηται τις ἐστιν, ἐν μὲν τῇ μεθόδῳ τῇ τῶν φυσικῶν περὶ τῆς ὑλῆς, ὑστερον δὲ περὶ τῆς κατ᾽ ἐνέργειαν.</td>
<td>As far as the substance of perceptible things is concerned, we have already explained what it is, firstly in the investigation about matter in <em>Physics</em> and secondly in the investigation about substance in terms of actuality.</td>
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<tr>
<td>1076a10-16</td>
<td>Ἐπεὶ δ’ ἡ σκέψις ἐστι πότερον ἐστι τις παρὰ τὰς αἰσθητὰς οὐσίας ἄκινητος καὶ ἄδιος ἢ οὐκ ἐστι, καὶ εἰ ἐστὶ τίς ἐστι, πρῶτον τὰ παρὰ τῶν ἀλλῶν λεγόμενα θεωρητέον, ὡς εἴτε οἱ καλῶς λέγουσι, καὶ εἰ τὶ δόγμα κοινὸν ἡμῖν κάκεινος, τούτ’ ἰδία μὴ καθ’ ἡμῖν δυσχεραίνωμεν· ἀγαπητόν γὰρ εἰ τις τὰ μὲν κάλλιον λέγοι τὰ δὲ μὴ χείρον.</td>
<td>However, since our inquiry is about whether, apart from the perceptible substances, there is any other substance that is unchangeable and eternal or not, and, if there is, what it is, we should first examine what has been stated by the other thinkers on this subject, so that, if they have said anything incorrectly, we are not bound by the same mistakes, and, in case there is any doctrine common to us and them, we may not on that account feel any private self-irritation; for one should be content if they put some things better and some others not worse than the others.</td>
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<td>1076a16-19</td>
<td>Δύο δ’ εἰσὶ δόξαι περὶ τούτων: τὰ τε γὰρ μαθηματικὰ φασιν οὐσίας εἶναι τινες, οίον ἀρίθμοὺς καὶ γραμμὰς καὶ τὰ συγγενῆ τούτοις, καὶ πάλιν τὰς ἰδέας.</td>
<td>So, there are two opinions on this subject; some thinkers say that the mathematical objects, for instance numbers and lines as well as anything of the same kind, are substances, but also that the Forms are substances.</td>
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<tr>
<td>1076a19-26</td>
<td>Ἐπεὶ δὲ οἱ μὲν δύο ταῦτα γένη ποιοῦσι, τὰς τε ἰδέας καὶ τοὺς μαθηματικοὺς ἀρίθμους, οἱ δὲ μίαν φύσιν ἀμφοτέρων, ἢτεροι δὲ τινες τὰς μαθηματικὰς μόνον οὐσίας εἶναι φασι, σκεπτέον πρῶτον μὲν περὶ τῶν μαθηματικῶν, μηδεμίαν προστίθεντας φύσιν ἄλλην αὐτοῖς, οίον πότερον ἰδέα τυγχάνουσιν οὐσαί ἢ οὔ, καὶ πότερον ἀρχαί καὶ οὐσίαι τῶν ὄντων ἢ οὔ, ἀλλ᾽ ἡς περὶ μαθηματικῶν μόνον εἰτε ἰσιν εἰτε μὴ ἰσιν, καὶ εἰ εἰσὶ πάς εἰσίν.</td>
<td>However, since some thinkers claim that they are two distinct kinds of things -the Forms and the mathematical numbers-, while some others assume that they are of one and the same nature, and others state that only the mathematical substances are substances, we had better first examine the objects of mathematics without attaching any other kind to them, namely without wondering whether the mathematical objects happen to be Forms, or not, and whether they happen to be principles and natures of beings, or not, but wondering if they, regarded exclusively as mathematical objects, exist or not and, in case they exist, in what way they do so.</td>
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<tr>
<td>1076a27-31</td>
<td>Ἐπειτὰ μετὰ ταῦτα χωρὶς περὶ τῶν ἰδεῶν αὐτῶν ἀπλῶς καὶ ὅσον νόμον χάριν· τεθρύληται γὰρ τὰ πολλὰ καὶ</td>
<td>Then after this we should separately consider Forms themselves, but only in general and as much as our</td>
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<tr>
<td>1076a31-37</td>
<td>Ἀνάγκη δ’, εἰπερ ἐστι τὰ μαθηματικὰ, ἢ ἐν τοῖς αἰσθητοῖς εἶναι αὕτα καθάπερ λέγουσί τινες, ἢ κεχωρισμένα τῶν αἰσθητῶν (λέγουσι δὲ καὶ οὕτω τινὲς) ἢ εἰ μηδετέρως, ἢ οὐκ εἰσίν ἢ ἄλλον τρόπον εἰσίν. Ωσπέρ ἡ ἀμφισβήτησις ἡμῖν ἔσται οὐ περὶ τοῦ εἶναι ἢ ἄλλα περὶ τοῦ τρόπου.</td>
<td>Nevertheless, if the mathematical objects exist, it is necessary that they exist either inside perceptible objects, as some thinkers say, or separately from them (for some thinkers maintain this too); otherwise, if they exist in neither of these two ways, then either they do not exist at all or they exist in some other way. As a result, our dispute will be not whether they exist, but in what way they exist.</td>
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OVERVIEW:

In the first introductory chapter of Book M, Aristotle raises the questions with which he intends to deal in books M and N of Metaphysics. These questions reveal that Aristotle's interest is now oriented towards a discussion about Forms and mathematical numbers and magnitudes.

- It is necessary to discuss whether an eternal and immutable substance exists or not.
- There are many conflicting opinions on this subject:
  
  (a) some people claim that the mathematical objects and also the Forms are substances,
  
  (b) some others hold that the Forms and the objects of mathematics are substances of a different kind,
  
  (c) some others identify them and
  
  (d) finally, some believe only in the existence of mathematical objects.

- Therefore, the following topics deserve to be investigated:
  
  (a) The first question that should be dealt with is whether mathematical objects exist and, if they exist, in what way they do so.
  
  (b) Secondly, a separate and brief discussion concerning Forms should be made.
  
  (c) Thirdly, it should be considered whether the natures and the principles of beings are numbers and Forms.
COMMENTARY:

1076a8-10: In the beginning of *Metaphysics*’ book M, Aristotle states that he has already investigated the substance of perceptible things. His investigation about perceptible things is said to have focused firstly on the material substance of perceptible things and secondly on their substance “in terms of actuality”. By referring to substance “in terms of actuality”, Aristotle may mean the actualization of the matter as substance having a particular outline, or else as substance having a particular form. As far as the Aristotelian views about matter are concerned, they are certainly discussed in book A from *Physics*.  

As regards the Aristotelian views about form, the commentators’ opinions are in a conflict: according to Alexander, substance in terms of actuality, i.e. formal substance (*εἰδικὴ ὄνσα*), is discussed in book B from *Physics*. However, Ross points out that the adverb ὑστερον (: later, afterwards) which is used in M.1, 1076a9, does not refer to a next part from *Physics*, but most probably to books Ζ, Η and Θ from *Metaphysics*. It is noteworthy that, in case Ross is right -which is highly probable-, this would mean that books M and N presuppose the central part of the *Metaphysics*.

1076a10-16: Aristotle argues that, since the subject of perceptible substances has been adequately treated previously, it is now time to focus the discussion on the existence of eternal substances. Stressing the need of examining other thinkers’ opinions before going on to unfold his own thought, Aristotle aims at the following two goals:  

(a) He wants to avoid the mistakes committed by other thinkers and  

(b) even if he has already fallen into the same mistakes, he wants to avoid feeling displeased about it.  

If he manages to avoid the same mistakes, he will subsequently be able to tackle the problem in a better way than his predecessors did, not bearing the same guilt and attracting the same accusations. And, even if he commits the same mistakes, he will be aware that he did not treat the subject in a worse way, but just as bad as the others.

29 [Alex.], 698.15  
30 Ross (1924), v. 2, 407
did. So, in any case, Aristotle’s treatment will not prove subordinate than the other thinkers’, which is a prerequisite for the success of his work.

1076a16-19: There are two different translating options here: Annas\textsuperscript{31} translates καὶ πάλιν τὰς ἰδέας as if there was one group of thinkers holding that the mathematical objects are substances and another one holding that the Forms are substances. On the contrary, Ross\textsuperscript{32} translates as if it was the same group of people regarding both mathematical objects and Forms as substances. Ross’s option seems more reasonable, since Aristotle does not use any additional word or phrase that indicates a reference to two different categories of thinkers. Furthermore, the use of the adverb πάλιν (: again, also) encourages us to conclude that Aristotle has in mind only one and the same group of thinkers, i.e. the thinkers he has already referred to with τινες, whom he is going to divide into different classes afterwards. So, what Aristotle does here is to expose two already expressed and generally admitted opinions (endoxa) on the existence of unchangeable and eternal substances, without clarifying who has expressed each one of them.

1076a19-22: Straight afterwards, Aristotle exposes three particular ways of conceiving the things generally admitted by some thinkers as unchangeable and eternal, i.e. mathematical objects and Forms:

(a) Ὅι μὲν -whom Aristotle in all likelihood considers to be Plato and the Platonists whose convictions were consistent with Plato’s teachings- think of Forms and mathematical objects as two distinct kinds of things.

(b) Ὅι δὲ, i.e. probably Xenocrates and the thinkers holding the same beliefs with him, consider Forms and mathematical objects to be one and the same kind of things.

\textsuperscript{31} Annas (1976), 91
\textsuperscript{32} Barnes (1991), v. 2, 184
(c) Ἑτεροι δε τινες, whom Aristotle maybe considers to be Speusippus and his followers, do not treat Forms as substances at all, but they accept only numbers.  

It is interesting that in this account Aristotle seems to move from the most admissive of Forms opinion to the least (or not in the least) admissive one. This could be a sign of Aristotle’s intention to insist on the investigation of mathematical objects and not lay the emphasis on Forms.

1076a22-26: Aristotle’s intention to lay emphasis on mathematical objects becomes evident in the first question he poses: do mathematical objects exist or not? And, if they exist, in what way they do so? It is worth wondering what would be the possible answers to the question about the way in which mathematical objects exist. The way in which mathematical objects exist could be either in actuality, i.e. as perceptible objects, or in abstraction, i.e. as intelligible objects. So, Aristotle intends to examine whether mathematical objects should be thought of as perceptible or as intelligible things.

1076a27-28: The second question Aristotle poses concerns Forms. However, he refers to the investigation about Forms as being secondary in comparison to the investigation about mathematical objects. This is confirmed by the fact that Aristotle confines himself to examining Forms only to the extent dictated by the “law”, i.e. the rules or limitations, constituting the frame of his current investigation.

As far as Forms are concerned, there are some boundaries that Aristotle does not want to overcome. But in what exactly could these limits consist? According to Alexander, when anyone treats a subject, they should abide by the rule dictating that the opinions of others are also taken into account so that people who have

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33 Ross (v. 2, 408) thinks that in this third category Aristotle would also include Pythagoreans. However, Annas (p. 136) notes that “the Pythagoreans do not fit into this classification, because they are not Platonists”. Annas is right in that Pythagoreans, not believing in the existence of abstract objects, would not easily have been classified by Aristotle in the same category with thinkers like Speusippus as sharing common beliefs about numbers.

34 [Alex.], 700.12-18
already read or will read these opinions are not troubled. If this is the νόμος that Aristotle obeys here, then he probably intends to examine Forms only to the extent that his treatment of mathematical objects is not incomplete, given that mathematical objects and Forms are both candidates for the same position, i.e. that of eternal substances. Furthermore, mathematical objects and Forms are identified by some thinkers. As a result, Aristotle cannot treat mathematical objects adequately without also taking Forms into account. Nevertheless, given that Aristotle’s current purpose is to deal mainly with the mathematical objects -while the discussion concerning Forms generally belongs to the past-, the treatment of Forms will be restricted to summarizing already formulated arguments about them\(^{35}\).

1076a28-29: By mentioning the “treatises for wide audience” (ἐξωτερικῶν λόγων), Aristotle hints at the prominent distinction between his acroamatic and exoteric treatises, namely between the treatises aimed at a small group of pupils and the treatises aimed at a large group of people who do not come exclusively from the Peripatetic School. Aristotle is said to express his criticism for Forms in lost published works such as the dialogue De Philosophia and the treatises De Ideis and De Bono.\(^{36}\)

It should, further, be noted that Aristotle, by saying here not ἐν τοῖς λόγοις (= in the treatises) but ὑπὸ τῶν λόγων (= by the treatises), could treat the noun λόγος as a personified notion. Λόγος can also be translated as “argument”, which means that here we could understand λόγος as argument acting like a person. This is not peculiar, given that “argument” was frequently treated in Greek as if it were a person (cf. Δίκαιος Λόγος and Ἀδίκος Λόγος).

1076a29-32: After proving if and in what way mathematical objects and Forms exist, Aristotle intends to give an answer to a third question: are numbers\(^{37}\) and Forms the

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\(^{35}\) Crubellier (1994), 87

\(^{36}\) Ross (1924), v. 2, 409

\(^{37}\) Having stated that mathematical objects are “numbers and lines as well as anything of the same kind” (1076a18), Aristotle clearly takes mathematical objects to be numbers and
principles of beings or not? This final question presupposes the previous two questions, since it has to be proven firstly whether mathematical objects and Forms exist in general and afterwards if they exist as principles of beings in particular.

It is noteworthy that Aristotle, by asking constantly about the way in which mathematical objects exist, does not seem to doubt that they exist in general but only that they exist in the particular way described by Platonists and Academics.

1076a32-37: Starting from mathematical objects Aristotle says that, if they exist, they do so:

(a) either inside physical beings
(b) or separately from them.

The members of the second group of thinkers, i.e. those thinking of mathematical objects as existing separately from physical beings, are probably Platonists who treated mathematical objects as separate entities. What should be discussed is the identity of the thinkers comprising the first group, i.e. those considering mathematical objects as existing inside physical beings. According to Annas these thinkers could be called “partial Platonists”, for they agree with Plato in holding that the mathematical objects are substances, but they disagree with him in supposing that mathematical objects exist inside physical objects. It seems that, for Aristotle, these thinkers are still confused, since they treat mathematical objects as distinct substances, but with right instinct, since they have taken an important step towards the truth, not treating them as independently existing substances.

Aristotle goes on to state that, if mathematical objects exist in neither of these two ways, then they either do not exist at all or they exist in some other way and finally arrives at the conclusion that mathematical objects certainly exist, but the way in which they do so is still a matter of dispute. So, it becomes clear that Aristotle does not reject the existence of mathematical objects, but aims at attaining the truth about the way in which they exist with the greatest possible accuracy.

geometrical objects, namely points, lines, planes and solids. So, talking about numbers, he talks about a group of mathematical objects.

38 Annas (1976), 137
### M.2 (1076a37-1077b16)

“Mathematical objects can exist neither inside nor outside physical objects”

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<tr>
<td>1076a38 - 1076b3</td>
<td>Ὅτι μὲν τοίνυν ἐν γε τοῖς αἰσθητοῖς ἀδύνατον εἶναι καὶ ἁμα πλασματίας ὁ λόγος, εἴρηται μὲν καὶ ἐν τοῖς διαπορήμασιν ὅτι δύο ἁμα στερεὰ εἶναι ἀδύνατον, ἕτι δὲ καὶ ὅτι τοῦ αὐτοῦ λόγου καὶ τὰς ἄλλας δυνάμεις καὶ φύσεις ἐν τοῖς αἰσθητοῖς εἶναι καὶ μηδὲμίαν κεχωρισμένην.</td>
<td>It has already been pointed out also in our Discussion of Problems that it is impossible for the objects of mathematics to exist inside perceptible objects and that this is a fictitious statement, since it is impossible for two solids to occupy at the same time the same place, as well as since, by the same argument, the rest capacities and natures exist inside perceptible objects and none of them exists in separation.</td>
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| 1076b3-11 | Ταῦτα μὲν οὖν εἰρήται πρῶτοιν, ἀλλά πρὸς τούτως φανερόν ὅτι ἀδύνατον διαιρεθήσεται, καὶ τοῦτο κατὰ γραμμὴν καὶ αὐτὴ κατὰ στιγμήν, ἦσται εἰ τὴν στιγμὴν διελεῖν ἀδύνατον, καὶ τὴν γραμμὴν, εἰ δὲ ταύτην, καὶ τάλλα. Τί οὖν διαφέρει ἢ ταῦτας εἶναι τοιαύτας φύσεις, ἢ αὐτὰς μὲν μή, εἰναι δ᾽ ἐν αὐταῖς τοιαύτας φύσεις; Τὸ αὐτό γὰρ συμβήσεται διαιρουμένων γὰρ τῶν αἰσθητῶν | These things, then, have been stated beforehand. However it is also clear that, by the same argument, it is impossible for any body to be divided. For the body will be divided by plane and the plane by line and the line by point, so that, if the division of the point is impossible, the division of the line will be impossible too and, if the division of the line is impossible, so will be the division of the rest. Therefore, what difference does it make whether perceptible natures...
| 1076b11-16 | Ἀλλὰ μὴν οὐδὲ κεχωρισμένας γῇ εἶναι φύσεις τοιαύταις δυνατόν. Εἰ γὰρ ἔσται στερεά παρὰ τὰ αἰσθήτα κεχωρισμένα τούτων ἑτερα καὶ πρότερα τῶν αἰσθητῶν, δήλω ὅτι καὶ παρὰ τὰ ἑπίπεδα ἑτερὰ ἀναγκαῖον εἶναι ἑπίπεδα κεχωρισμένα καὶ στιγμὰς καὶ γραμμάς τοῦ γὰρ αὐτοῦ λόγου. | Yet truly natures of this kind cannot exist in separation either; for, if besides perceptible solids there are to be some other solids which are separate from these and prior to the perceptible ones, then it is clear that besides perceptible planes there have to be some other separate planes as well as points and lines, by the same argument. |
| 1076b16-21 | Εἰ, δὲ ταύτα, πάλιν παρὰ τὰ τοῦ στερεοῦ τοῦ μαθηματικοῦ ἑπίπεδα καὶ γραμμὰς καὶ στιγμὰς ἑτερὰ κεχωρισμένα. Πρότερα γὰρ τῶν συγκειμένων ἔστι τὰ ἀσύνθετα καὶ ἐπεὶ τῶν αἰσθητῶν πρότερα σώματα μὴ αἰσθητά, τῷ αὐτῷ λόγῳ καὶ τῶν ἑπίπεδων τῶν ἐν τοῖς ἀκινήτοις στερεοῖς τὰ αὐτά καθ’ αὐτά. | But, if these exist, then besides planes and lines and points of the mathematical solids there must again be others which are separate. For incomposite things are prior to composite ones; and, if there are non-perceptible bodies that are prior to the perceptible ones, then by the same argument the planes by themselves must be prior to the planes existing in the unchangeable solids. |
| 1076b21-26 | Ὄστε ἐτερα ταύτα ἐπίπεδα καὶ γραμμαὶ τῶν ἀμα τοῖς στερεοῖς τοῖς κεχωρισμένοις· τὰ μὲν γὰρ ἀμα τοῖς μαθηματικοῖς στερεοῖς τὰ ὑπὸ πρότερα τῶν μαθηματικῶν στερεῶν. Πάλιν τοίνυν τούτων τῶν ἐπιπέδων ἔσονται γραμμαί, ἂν πρότερον δεήσει ἕτερας γραμμᾶς καὶ στιγμᾶς εἶναι διὰ τὸν αὐτὸν λόγον· | And, as a result, these planes and lines will be different from those belonging together with the separate solids; for the latter coexist with the mathematical solids, while the former are prior to them. Then, again, there will be lines belonging to these planes and by the same argument there must be other lines and points prior to these; |
| 1076b27-29 | καὶ τούτων τῶν ἐν39 ταῖς προτέραις γραμμαῖς ἕτερας προτέρας στιγμάς, ἂν οὐκέτι πρότερα ἐτεραί. Ατοπὸς τε δὴ γίγνεται ἡ σώρευσις· | and other points must be prior to these points that belong to the prior lines, to which no other points will be prior. Then, this accumulation becomes absurd; |
| 1076b29-36 | συμβαίνει γὰρ στερεὰ μὲν μοναχὰ παρὰ τὰ αἰσθητά, ἐπίπεδα δὲ τριττὰ παρὰ τὰ αἰσθητα -τὰ τε παρὰ τὰ αἰσθητά καὶ τὰ ἐν τοῖς μαθηματικώις στερεοίς καὶ τὰ παρὰ τὰ ἐν τούτοις- γραμμαί δὲ τετραξαί, στιγμαί δὲ πενταξαί ὡστε περὶ ποὺ μὲν ἐπιστήμη δέχονται αἱ μαθηματικαὶ τούτων; οὐ γὰρ δὴ περὶ τὰ ἐν τῷ στερεῷ τῷ ακινήτῳ ἐπίπεδα καὶ γραμμαίς καὶ στιγμαίς· ἂεὶ γὰρ περὶ τὰ πρότερα ἡ ἐπιστήμη· | because on the one hand only one class of solids happens to exist besides the perceptible ones, while on the other hand three classes of planes happen to exist besides the perceptible ones (namely those which exist separately from the perceptible planes, those which exist in the mathematical solids and those which exist separately from those in mathematical solids), as well as four classes of lines and five classes of points. So, which of these groups are to be objects |

39 1076b27: ἐκ Ross, ἐν codd. Given that the preposition is constructed with dative (ταῖς προτέραις γραμμαῖς), ἐν is preferable.
of the mathematical sciences? For, the mathematical sciences are surely not to deal with the planes and lines and points that exist in the unchangeable solid; for, the science is always concerned with what is prior.

| 1076b36-39 | ὁ δ’ αὐτὸς λόγος καὶ περὶ τῶν ἁριθμῶν· παρ’ ἐκάστας γὰρ τὰς στιγμὰς ἐτεραὶ ἔσονται μονάδες, καὶ παρ’ ἐκαστα τὰ ὅντα αἰσθητά\(^{40}\), εἴτα τὰ νοητά, ὃστ’ ἔσται γένη ἀπειρά τῶν μαθηματικῶν ἁριθμῶν. | And the same argument is valid in the case of numbers; for there will be other units besides each class of points, and besides each class of things that are perceptible, and then besides each class of things that are intelligible, so that there will be countless kinds of mathematical numbers. |

| 1076b39-1077a5 | Ἐτι ἀπερ καὶ ἐν τοῖς ἀπορήμασιν ἐπήλθομεν πῶς ἐνδέχεται λύειν; Περὶ ἅ γὰρ ἡ ἀστρολογία ἐστίν, ὁμοίως ἔσται παρὰ τὰ αἰσθητά καὶ περὶ ἅ ἡ γεωμετρία· εἴναι δ’ ὑφανόν καὶ τὰ μόρια αὐτοῦ πώς δυνατόν, ἢ ἄλλο ὦτοιν ἔχον κίνησιν; Ὁμοίως δὲ καὶ τὰ ὀπτικά καὶ τὰ ἄρμονικά· | Moreover, how is it possible that we solve the difficulties which we went through in the Discussion of Problems? For the objects of astronomy will similarly be distinct from the perceptible objects and the same will happen with the objects of geometry; how could the sky and its parts or anything else undergoing change be objects of astronomy? And the same will happen with the objects of optics and |

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\(^{40}\) 1076b38: τὰ ὅντα, <τὰ> αἰσθητά Ross, τὰ ὅντα αἰσθητά codd. On account that the formulation of εἴτα τὰ νοητά is obviously modelled on the formulation of τὰ ὅντα αἰσθητά, there seems to be no serious reason why τὰ should be added before αἰσθητά.
| 1077a5-9 | ἔσται γὰρ φωνὴ τε καὶ ὄψις παρὰ τὰ αἰσθητὰ καὶ τὰ καθ’ ἐκαστα, ὥστε δήλον ὅτι καὶ αἱ ἄλλαι αἰσθήσεις καὶ τὰ ἄλλα αἰσθητα: τὶ γὰρ μᾶλλον τάδε ἢ τάδε; εἰ δὲ ταῦτα, καὶ ζῶα ἔσονται, εἰπεῖ καὶ αἰσθήσεις. | for there will be both voice and sight besides the perceptible and particular ones. Hence, it is clear that the same will happen with the rest senses as well as the rest objects of sense; for why should this happen more to one class of objects than to another? But, if these are true, there will also be animals distinct from the perceptible ones, should the same apply to the senses. |
| 1077a9-14 | Ἑτὶ γράφεται ἐνὶ καθόλου ὑπὸ τῶν μαθηματικῶν παρὰ ταῦτας τὰς ὑσίαις. Ἐσται οὖν καὶ αὕτη τις ἄλλη ὑσία μεταξὺ κεχωρισμένη τῶν τ᾽ ἱδεών καὶ τῶν μεταξὺ, ἢ οὔτε ἀριθμός ἐστιν οὔτε στιγμαί οὔτε μέγεθος οὔτε χρόνος. Εἰ δὲ τούτῳ ἀδύνατον, δήλον ὅτι κάκεινα ἀδύνατον εἶναι κεχωρισμένα τῶν αἰσθητῶν. | In addition, there are some general theorems which mathematicians represent by lines and which exist apart from these substances. So, they will constitute some other kind of substances intermediate and separate from Forms and Intermediates, which is neither number nor points nor magnitude nor time. But, if this is impossible, then it is clear that it is also impossible for the aforesaid objects too to exist separately from the perceptible ones. |
| 1077a14-20 | Ὑλῶς δὲ τούναντιον συμβαίνει καὶ τοῦ ἀληθοῦς καὶ τοῦ εἰσθότος ὑπολαμβάνεσθαι, εἰ τις θήσει οὕτως εἶναι | And, in general, conclusions opposing truth and received opinion will be reached, should somebody assume that the |
τὰ μαθηματικὰ ἂς κεχωρισμένας τινὰς φύσεις. Ανάγκη γὰρ διὰ τὸ μὲν οὕτως εἶναι αὐτὰς προτέρας εἶναι τῶν αἰσθητῶν μεγεθῶν, κατὰ τὸ ἀληθὲς δὲ ύστερας τὸ γὰρ ἀπελεξίς μέγεθος γενέσθαι μὲν πρῶτον ἐστὶ, τῇ οὐσίᾳ δ᾽ ύστερον, οἷον ἄψυχον ἐμψύχου.

mathematical objects exist this way, namely as some sort of separately existing natures. For these natures, due to their existing this way, will necessarily be prior to perceptible magnitudes on the one hand, while, on the basis of truth on the other hand, they will necessarily be subsequent; for the incomplete magnitude is on the one hand prior in terms of generation, but on the other hand it is posterior in terms of substance, just as the living precedes the lifeless.

Ἐτὶ ἔτι τίνι καὶ πότ᾽ ἐσται ἐν τὰ μαθηματικὰ μεγέθη; Τὰ μὲν γὰρ ἐνταῦθα ψυχὴ ἢ μέρει ψυχῆς ἢ ἄλλω τινί, εὐλόγως, εἰ δὲ μῆ, πολλὰ, καὶ διώκεται, ἐκείνοις δὲ διαφερότοις καὶ ποσοῖς οὕσι τί αἰτίον τοῦ ἐν εἶναι καὶ συμμένειν; Ἐτὶ αἱ γενέσεις δηλοῦσιν. πρῶτον μὲν γὰρ ἐπὶ μῆκος γίγνεται, εἰτὰ ἐπὶ πλάτος, τελευταίον δὲ εἰς βάθος, καὶ τέλος ἐσχέν.

Furthermore, in virtue of what and when will mathematical magnitudes be one? For the things around us are one in virtue of soul or a part of soul or something else, and it is reasonable; otherwise, the thing is not one but many and it breaks up. But what is the reason for those, while being divisible and of a certain quantity, to be one and keep together? Moreover, their way of generation makes the point clear. For they are generated firstly across the dimension of length, secondly across the dimension of width, finally across the dimension of depth and the whole
| 1077a26-30 | Εἰ εἰ οὖν τὸ τῇ γενέσει ύστερον τῇ οὐσίᾳ πρῶτερον, τὸ σῶμα πρῶτερον ἢν εἰ ἐπιπέδου καὶ μήκους καὶ ταύτη καὶ τέλειον καὶ ὅλον μᾶλλον, ὅτι ἐμψυχον γίγνεται· γραμμὴ δὲ ἐμψυχος ἢ ἐπίπεδον πῶς ἢ ἐν εἴ. | So, if the posterior in terms of generation is prior in terms of being, then the body should be prior to plane and length; and, in this sense, it must be more complete and whole, because it comes into being as animate; but how could the existence of animate lines or animate planes be possible? |
| 1077a30-35 | Ὑπὲρ γὰρ τὰς αἰσθήσεις τὰς ἡμετέρας ἢν εἰ ἡ τὸ ἀξίωμα. Ἐτι τὸ μὲν σῶμα οὐσία τε· ἢδη γὰρ ἐχει πῶς τὸ τέλειον· αἱ δὲ γραμμαί πῶς οὐσίας· Ὑπὲρ γὰρ ἢς εἴδους καὶ μορφής τις, οἴον εἰ ἄρα ἢ ψυχὴ τοιούτων, ὢτε ἢς ἢ ἥλι, οἴον τὸ σῶμα· οὔθεν γὰρ ἐκ γραμμῶν ὑπὸ ἐπιπέδων οὔθε στιγμῶν φαίνεται συνύστασθαι δυνάμενον· | This supposition would be beyond the limits of our senses. Moreover, the body is substance of some sort, as it already has completeness in a way; but how could lines be thought of as substances? For they could exist neither as form or shape of some sort, as maybe soul is, nor as matter, as body is; and there is nothing that seems capable of being put together out of lines or planes or points; |
| 1077a35 - 1077b4 | εἰ δ᾽ ἦν οὐσία τις ύλική, τοῦτ᾽ ἢν ἐφαίνετο δυνάμενα πάσχειν. Τῷ μὲν οὖν λόγῳ ἢστω πρότερα, ἀλλ᾽ οὖ πάντα ὅσα τῷ λόγῳ πρότερα καὶ τῇ οὐσίᾳ πρότερα. Τῇ μὲν γὰρ οὐσίᾳ πρότερα ὅσα χωροῖσθεν τῷ εἶναι ύπερβάλλει, τῷ λόγῳ δὲ ὅσαν οἱ λόγοι ἢ τῶν λόγων· | However, they would seem capable of undergoing this, provided that they were material substances of some sort. So, let them be prior in definition. Nevertheless, not everything that is prior in definition is also prior in terms of being. For the objects which, when separated, surpass the |
| 1077b4-11 | ῥαῦτα δὲ οὐχ ἀμα ὑπάρχει. Εἰ γὰρ μὴ ἔστι τὰ πάθη
παρὰ τὰς οὐσίας, οἴον κινούμενόν τι ἢ λευκόν, τοῦ
λευκοῦ ἀνθρώπου τὸ λευκὸν πρότερον κατὰ τὸν λόγον
ἀλλ᾽ οὐ κατὰ τὴν οὐσίαν· οὔ γὰρ ἐνδέχεται εἶναι
κεχωρισμένον ἀλλ᾽ ἀεὶ ἁμα τῷ συνόλῳ ἐστίν· σύνολον
dὲ λέγω τὸν ἀνθρώπον τὸν λευκόν. Ὡσεὶ φανερὸν ὅτι
οὔτε τὸ ἔξ ἀφαίρέσεως πρότερον οὔτε τὸ ἐκ
προσθέσεως ὑποτερον· ἐκ προσθέσεως γὰρ τῷ λευκῷ ὁ
λευκὸς ἀνθρώπος λέγεται.

1077b12-14 | Ὅτι μὲν οὖν οὔτε οὐσίαι μᾶλλον τῶν σωμάτων εἰσιν
οὔτε πρότερα τῷ εἶναι τῶν ἁπάθειαν ἀλλὰ τῷ λόγῳ
μόνον, οὔτε κεχωρισμένα που εἶναι δυνατόν, εἰρηται
ἰκανός.

So, it has been sufficiently shown that the objects of
mathematics are neither more substantial than bodies nor
prior to perceptible objects in existence -but only in
definition- and also that it is not in any way possible for

however, these cannot apply simultaneously. Since, if the
attributes do not exist besides substances, for example,
something “moving” or “white”, then “white” is prior to
“white man” in definition but not in terms of being, as it
cannot exist separately, but always together with the
compound; by “compound” I mean the “white man”. As a
consequence, it becomes clear that neither what results from
abstraction is prior nor what results from addition is
posterior; for it is by adding to “white” that “white man” is
talked about.

So, it has been sufficiently shown that the objects of
mathematics are neither more substantial than bodies nor
prior to perceptible objects in existence -but only in
definition- and also that it is not in any way possible for
| 1077b14-17 | Έπει δ’ οὐδ’ ἐν τοῖς οἰσθητοῖς ἐνεδέχετο αὐτὰ εἶναι, φανερὸν ὅτι ἡ ὅλας οὐκ ἔστιν ἡ τρόπον τινὰ ἔστι καὶ διὰ τούτο οὐκ ἀπλῶς ἔστιν· πολλαχῶς γὰρ τὸ εἶναι λέγομεν. | However, since we have also seen that they cannot exist in perceptible things, it is clear that either they do not exist at all or they exist in a way and therefore not without qualification; for we say “exist” in several senses. |
OVERVIEW:

In M.2, Aristotle explains why mathematical objects cannot exist either inside sensible things or separately from them.

- If it is accepted that mathematical objects do not exist separately from but inside sensible things, then many difficulties would follow:
  
  (a) Firstly, two solids would occupy the same place.
  
  (b) Secondly, all the other properties (apart from the mathematical ones) should also exist inside sensible things.
  
  (c) Thirdly, no physical object would be divisible. For the body would be divided by plane and the plane by line and the line by point. However, the point is indivisible. Therefore, the line, the plane and the solid as well as any perceptible thing would be indivisible, too.

- However, it cannot be accepted that mathematical objects are separate from and prior to sensible things. For, then:
  
  (a) besides perceptible solids, planes, lines and points there will be an accumulation of other separate solids, planes, lines and points.
  
  (b) Moreover, in the case of numbers, there will similarly be many sorts of units.
  
  (c) Also, the objects of astronomy as well as the objects of optics and of harmonics will exist separately from sensible things.
  
  (d) Finally, there will be one distinct class of mathematical theorems that should be treated as separately existing objects.

- This way of thinking is contrary to common sense. For, mathematical objects are treated as prior to sensible things, while they are in fact posterior to them due to their being incomplete. Given that what is prior in generation is posterior in substance and what is posterior in generation is prior in substance, then body is prior to plane and line, since it is more complete and has a soul.
The attributes cannot exist separately from sensible things. So, the attributes can be described as prior in definition to the things that are composite of substance and attributes. For, the definitions of the attributes are a prerequisite for the definitions of composite things. Hence, mathematical objects are not prior in substance—but only in definition—to sensible bodies.

In conclusion, mathematical objects can exist neither separately from perceptible objects nor inside them. Therefore, they exist either not at all or in some other sense.
COMMENTARY:

1076a38-1076b3: Aristotle denies that the objects of mathematics exist inside perceptible things, because:

(a) it is impossible for two solids to occupy the same place and
(b) it would follow that the other properties (apart from the mathematical ones) must also exist inside perceptible objects.

The rejection of the existence of mathematical objects inside perceptible things seems to refer back to a doctrine which has already been discussed in B.2, 998a13-17 from *Metaphysics*. There, Aristotle states that it is impossible for two solids to occupy the same space at the same time. Furthermore, he says, this would be impossible because an object which is unchangeable, i.e. a mathematical object, cannot exist inside another object which is changeable, i.e. a physical body. In general, if it is impossible for any two different mathematical figures to occupy the same place (for then they would be identical to each other), it is equally -or even more- impossible for an immaterial geometrical solid and a physical body to occupy the same space.⁴¹

The Aristotelian line of thought divulges the absurdity resulting from the belief that the objects of mathematics exist inside physical objects and it could be briefly presented as follows:

(a) Some thinkers take Intermediates, i.e. mathematical objects, to be substances that exist inside other substances, i.e. physical objects.

↓

So, two different objects have to be in the same place at the same time, without the one being part of the other, which is absurd.

(b) Consistency requires that the same thinkers should think of the “rest capacities and natures” in the same way too, namely as existing inside physical beings. The “rest capacities and natures” are, in general,

⁴¹ Crubellier (1994), 98
characteristics of beings which Platonists considered to be separate Forms. However, a Platonist would never think of Forms as existing inside physical beings. What is more, even if this could be accepted, it would then turn out that unchangeable things like Forms exist inside changeable beings, which is also absurd.

1076b4-11: Aristotle adds here a third reason why the objects of mathematics cannot exist inside perceptible things:

(a) Any perceptible thing is a solid body that can be divided at planes.
(b) The plane can be divided at lines.
(c) The line can be divided at points.
(d) However, the point cannot be divided.
(e) Therefore, the solid body will finally prove to be indivisible.

A question effortlessly arising here is why the division of point is precondition for the division of line and, subsequently, for the division of plane. Couldn’t the point be conceived as the ultimate -and thus indivisible- component of solid? Alexander gives the following reasonable explanation on this point: the plane is constituent part of the solid. However, the plane is continuous. It is not many distinct planes with intervals between them, i.e. planes that can be separately seen and touched. To divide the solid by plane presupposes the ability of plane itself to be divided. The plane is divided by line. To divide the plane by line presupposes the divisibility of line; since only by somehow “cutting” the continuous line that constitutes the plane can the plane be divided. The line is divided by point. To divide the line by point means to “cut” the point that constitutes the line and extends with it. Let us try to think of line as a series of many points without intervals between them. Due to interval missing, it would be wrong to think of them as different points. In fact, it would be righter to think of them as one. But, what name could be given to this “one”? This “one” would be point. And this “one”, or else point, should be divisible.

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42 Ross (1924), v. 2, 412
43 [Alex.], 702.12-21
for the line to be divided. Nevertheless, it would be absolutely absurd to claim that
the point is divisible. But, if the point is indivisible, so will be the rest.

Platonists thought of the point as well as of line and plane as substances existing
independently without being divided the one into the other. However, if it is true
that they exist inside perceptible physical beings, then they would have to be
divisible. Any physical object can be cut into pieces. Hence, provided that any
physical object, as solid, is (or involves) plane, line and point, it would be necessary
for plane, line and point to be also divisible. In case they were indivisible, the beings
would have to be indivisible too, which is an absolutely absurd assumption.

1076b11-36: Aristotle’s argument in this section could be summarized as follows:

(a) We have already explained why it is impossible for the objects of
mathematics to exist inside physical beings.
   - It is firstly because of the inability of two things to occupy the same place
     at the same time
   - and secondly because such a supposition would result in the indivisibility
     of physical beings (for mathematical objects are indivisible, due to the
     undeniable indivisibility of the point).

(b) Nevertheless, the objects of mathematics cannot exist separately from beings.
   - If there are to be mathematical solids separate from and prior to the
     perceptible solids, then there will turn out to be two different types of
     solids.
   - There must also be three different types of planes: (1) planes in the
     perceptible solids, (2) planes in the mathematical solids and (3) planes
     separate from both perceptible and mathematical solids.
   - Furthermore, there must be four different types of lines: (1) lines in the
     planes of perceptible solids, (2) lines in the planes of mathematical solids,
     (3) lines in the planes existing separately from both perceptible and
     mathematical solids and (4) lines separate from any type of planes.
- Finally, there must be five different types of points: (1) points in lines of perceptible solids, (2) points in lines of mathematical solids, (3) points in lines of separate planes, (4) points in separate lines and (5) points separate from any type of lines.

The accumulation of mathematical objects becomes absurd. The reason for the absurdity of this accumulation lies first of all in the confusion resulting from such an “uneconomical” outcome and secondly in the difficulty in selecting among all these objects the ones that the mathematical sciences deal with. It seems, in fact, that the mathematical sciences would not deal with planes, lines and points of the mathematical solid, but with planes separate from any type of solids, lines separate from any type of planes and points separate from any type of lines.

As Annas\(^{44}\) notes, it is likely that, according to Aristotle, Platonists’ ideal objects are nothing more than reduplication of the physical ones. This means that the ideal objects have been conceived by Platonists through a process of doubling the physical objects. According to Syrianus\(^{45}\), a Platonist could retort that this way of reaching the ideal mathematical object is non-profitable when compared to other ways, i.e. deriving the ideal plane straight from the ideal solid. Therefore, this “non-profitable” accumulation may have been done on purpose here by Aristotle to prove that the platonic thought is faulty. A difficulty that arises from Syrianus’ claim is that in all likelihood a Platonist would not derive the ideal line straight from the ideal plane or the ideal plane straight from the ideal solid, given that, according to the theory of Forms, a Form should not be treated as a composite entity that consists of other Forms.

1076b36-1076b39: Aristotle stresses the need of making a correlation between five different types of point and five different types of number. It is essential that five different types of unit exist, so that there is one type of unit for counting each type of point. By the same argument, according to Alexander\(^{46}\), each and every different

\(^{44}\) Annas (1976), 140  
\(^{45}\) Syr., 88.32-35  
\(^{46}\) [Alex.], 704.25-705.1
physical being seen as one entity should be counted by a different ἕν. And, similarly, each and every intelligible thing seen as one entity should be counted by a different ἕν. However, such a conception would result in introducing number in numerous different ways, which contradicts the Aristotelian conception of number as one in kind, i.e. mathematical.

1076b39-1077a9: In this section, Aristotle examines the difficulties that arise from the application of the thesis that has already been applied to the objects of geometry and to numbers, also to the objects of astronomy, optics and harmonics:

a) Firstly, there cannot be moving but non-sensible heavens.

b) Secondly, there cannot be non-sensible vision and voice.

c) Thirdly, there cannot be non-sensible animals.

A Platonist would agree with the idea that applied mathematical sciences study ideal objects analogously to pure mathematical sciences, namely geometry and arithmetic. However, there can be no other place outside heavens. Moreover, movement and, in general, change is a characteristic property of heavens. Nevertheless, how could there be change in anything that is non-sensible? In the case of sciences that are relevant to human senses, such as optics and harmonics, this idea also leads to absurdity. For, how anyone could become expert in harmonics without studying perceptible instances of physical voice? Aristotle may exaggerate a little here, since a teacher of harmonics would admit that they deal with people’s actual sounds aiming at grasping the ideal sound. As far as the existence of ideal animals is concerned, Syrianus defends the Platonic doctrine by offering the following interesting example: given that human in the proper sense (κυρίως) is called νοῦς (= intelligence) in Nicomachean Ethics and that the κυρίως ἄνθρωπος is the one who exists inside us, then this, i.e. νοῦς or κυρίως ἄνθρωπος, is the ἔτερον ζώον (= the “other (ideal) animal).

47 Syr., 89.20-31
Aristotle has already stated that, if mathematical objects exist separately from perceptible things, the logical consequence would be that for the objects of other sciences as well as for the animate beings and the perceptible things in general there have to be some intelligible objects that exist separately from the sensible things.

Then, Aristotle talks about mathematical theorems and proofs, as another type of substance which is “neither number nor points nor magnitude nor time”. This type of substance is placed by Aristotle in between Forms and mathematical objects. So, it seems that the classes of substances are four: (a) physical objects, (b) mathematical objects, (c) mathematical theorems and (d) Forms.

Probably it is not clear enough why mathematical theorems should constitute a distinct class of substances. Wouldn’t it be more correct to state that they are in fact propositions including numbers and geometrical magnitudes that are the objects of mathematical sciences? What is the point of treating them as a class of beings that is totally different from the class of mathematical objects? It is true that mathematical theorems and proofs involve mathematical notions such as numbers or figures. However, these numbers and figures cannot be identified with the mathematical objects referred to previously. The mathematical objects talked about beforehand are those particular objects which the mathematician studies, before formulating a general mathematical theorem or proof. On the contrary, the numbers and figures used during the formulation of mathematical theorems and proofs are general terms embracing all the particular numbers and figures of the same sort. For instance, a mathematical theorem about right-angle triangles uses the general term “right-angle triangle” in a way that is true of all the right-angle triangles regardless of properties such as the area of triangle or the length of its sides. Nevertheless, these general mathematical terms are not Forms. Consequently, according to Aristotle, it would be better that they are classified in between Forms and mathematical objects.
According to Aristotle, a complete object is temporally posterior to an incomplete one, since it is generated at a later time. However, it is irrefutably prior in reality, given that it is substance.

"ἵν" should be understood as meaning “one in number/unitary”. In this sense, concrete objects seem favored over abstract ones. For, any concrete object is unitary due to its having a soul. But, Aristotle asks, given that mathematical objects are divisible entities, is there any principle, analogous to soul which could hold them together being abstracts?

It can be noticed that the Aristotelian question: “What causes the unity of mathematical magnitudes?” is between two relevant to genesis arguments: (1) perceptible magnitudes are prior in essence but later in generation to mathematical ones and (2) solids are prior in essence but later in generation to planes, lines and points.\(^{48}\) Aristotle could be suggesting here that the unity of a body is acquired at the latest stage of its development.

It is noteworthy that Aristotle likens form to soul and matter to body. Just as any living being cannot be fully created, unless it is composed of soul and body, in the same way any physical object cannot exist, unless it consists of form and matter. Lines, planes and points cannot act either as form or as matter in order to contribute to the creation of a physical object. Therefore, they cannot in the least be conceived as substances.

Given that soul is the factor which unifies physical substances, it is indispensable for any physical being to have soul in order that it is thought of as substance. Our experience also confirms the fact that the animate body attains perfection to the greatest extent than anything else in the universe\(^{49}\). However, soul is acquired by the physical being after its three-dimensional generation, i.e. its generation across the dimensions of length, width and depth. In other words, to acquire soul, the physical being must necessarily be a solid composed of planes

\(^{48}\) Ross (1924), v. 2, 413-414

\(^{49}\) Crubellier (1994), 117
which are formed by lines. Only a physical solid can be animate, for it is more complete than a plane, a line etc.

On the contrary, mathematical objects such as lines and planes cannot be animate, since they lack completeness. Furthermore, lines and planes cannot be said to function either as form or as matter. If they functioned as form, then they would determine the characteristics that qualify the physical object and, if they functioned as matter, then planes, lines and points would be the material out of which the physical being is made. However, this does not happen. Therefore, solid bodies are ontologically prior to planes, lines and points, while planes, lines and points are prior to solid bodies in terms of definition.

This is reasonable, as the solid body is a being in a fuller sense than the plane is, but the plane is presupposed for defining the solid. Similarly, mathematical number is posterior in being, but prior in definition, since it is present in the definition of any measurable physical object.

1077b4-11: This is the most important and interesting point of the simple example given by Aristotle. The outcome of “addition” is made out of components which, in this case, may be words like “white”, that are unable to stand for an independent item. The addition of “man” to “white” leads to the creation of an independent whole. This whole is later in generation, but prior in substance.

The reverse process is the “abstraction”, namely the extraction of something from the whole. Annas\(^5\) notes that numbers could be considered as products of abstraction in the sense that they are words which are unable to stand for independently existing items. However, Aristotle does not use this as an example of abstraction. What is given by abstraction is a part of the whole and thus prior in generation, but later in substance. So, it cannot be said either that what is received by abstraction is unqualifiedly prior or that what is received by addition is unqualifiedly subsequent.

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\(^5\) Annas (1976), 147
This section provides the conclusion which Aristotle draws after his previous analysis (cf. 1076a38-1077b11). Aristotle assumes that mathematical objects cannot exist in the following three ways:

a) as a class of substances having the nature of substance to a greater extent than bodies do,

b) as a class of things having been in existence before bodies did so and
c) as a class of things existing separately from perceptible things.

It has already been clearly pointed out by Aristotle that mathematical objects do exist and that the only thing we could argue about is the way in which they do so (1076a36-37: "ὥσθ᾽ ἡ ἀμφισβήτησις ἡμῖν ἔσται οὐ περὶ τοῦ εἶναι, ἀλλὰ περὶ τοῦ τρόπου"). Therefore, Aristotle does not aim at disproving the existence of mathematical objects, but only at arguing in favor of their existence in a special sense -given that there are many senses in which we can talk about existence.

It is not the first time Aristotle expresses the idea that "εἶναι" ("be" or "exist"), can be used in several senses. The same idea is formulated in H.2, 1042b8-1043a28 of Metaphysics. There, Aristotle states that it has already been said in Physics what is the difference between "coming into existence in an unqualified way" and "coming into existence in a qualified way".

He then goes on to say that there are two types of substance: the first type of substance is agreed to be identified with the material substrate of things and it can also be called "potential substance". The second type of substance is associated with qualities of things, such as shape, position, arrangement, place, time and excess or deficiency in properties like hardness, softness, density, rarity, dryness and humidity. This second type of substance is agreed to be identified with the form of things and it can also be called "actual substance". Aristotle admits that "εἶναι" has as many senses as substance has. So, there are numerous different senses in which we can talk about "being" (or "existing").
### M.3 (1077b17-1078b6)

“The way mathematical objects really exist”

<table>
<thead>
<tr>
<th>SECTION</th>
<th>ANCIENT GREEK TEXT</th>
<th>TRANSLATION</th>
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<tbody>
<tr>
<td>1077b17-22</td>
<td>Ὡσπερ γὰρ καὶ τὰ καθόλου ἐν τοῖς μαθήμασιν οὐ περὶ κεχωρισμένων ἐστὶ παρὰ τὰ μεγέθη καὶ τοὺς ἁριθμοὺς ἀλλὰ περὶ τούτων μὲν, οὐχ ἢ δὲ τοιαῦτα οία ἔχειν μέγεθος ἢ εἶναι διαιρετά, δήλον ὅτι ἐνδέχεται καὶ περὶ τῶν αἰσθητῶν μεγεθῶν εἶναι καὶ λόγους καὶ ἀποδείξεις, μή ἢ δὲ αἰσθητὰ ἅλλ᾽ ἢ τοιαδ.</td>
<td>For just as the universal propositions in mathematics are not concerned with objects existing in separation apart from magnitudes and numbers, but they are concerned with these (i.e. magnitudes and numbers), however not <em>qua</em> such as having magnitude or being divisible, clearly it is also possible that there may be both definitions and demonstrations applicable to perceptible magnitudes, however not <em>qua</em> perceptible, but <em>qua</em> having certain qualities.</td>
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<tr>
<td>1077b22-30</td>
<td>Ὡσπερ γὰρ καὶ ἡ κινούμενα μόνον πολλοὶ λόγοι εἰσί, χωρὶς τοῦ τί ἔκαστον ἐστὶ τῶν τοιούτων καὶ τῶν συμβεβηκότων αὐτοῖς, καὶ οὐκ ἄναγκη διὰ ταύτα ἢ κεχωρισμένον τι εἶναι κινούμενον τῶν αἰσθητῶν ἢ ἐν τούτως τινὰ φύσιν εἶναι ἁριθμημένην, οὕτω καὶ ἐπὶ τῶν κινούμενων ἐσονται λόγοι καὶ</td>
<td>For just as there are also many definitions of things merely <em>qua</em> moving -without any reference to the essence of each of such things or to their properties- and it is not therefore necessary either that there is something moving which exists in separation from perceptible things or that there is some moving nature marked off in them, in the same way too in the case of moving things there will be definitions and branches of knowledge which</td>
</tr>
</tbody>
</table>
ἐπιστήμα, ὡς ἢ κινούμενα δὲ ἀλλ' ἢ σώματα μόνον, καὶ πάλιν ἢ ἐπίπεδα μόνον καὶ ἢ μήκε μόνον, καὶ ἢ διαφετὰ καὶ ἢ ἀδιαφετὰ ἔχοντα δὲ θέσιν καὶ ἢ ἀδιαφετὰ μόνον.

1077b30-34 Ωστ" ἐπεὶ ἀπλῶς λέγειν ἀληθὲς μὴ μόνον τὰ χωρίστα εἶναι ἀλλὰ καὶ τὰ μὴ χωρίστα, οἷον κινούμενα εἶναι, καὶ τὰ μαθηματικὰ ὅτι ἔστιν ἀπλῶς ἀληθὲς ἐπείν, καὶ τοιαῦτα γε οία λέγουσιν.

1077b34-1078a2 Καὶ ὡσπερ καὶ τὰς ἄλλας ἐπιστήμας ἀπλῶς ἀληθὲς ἐπείν τούτου εἶναι, οὕς τού συμβεβηκότος, οἷον ὑπὸ τοῦ λευκοῦ, εἰ τὸ ὑγιεινὸν λευκόν, ἢ δ' ἔστιν ὑγιεινοῦ, ἀλλ' ἔκεινον οὐ ἔστιν ἐκάστη, εἰ ἢ ὑγιεινὸν

Thus, due to the fact that it is true to say in a general sense that not only separate but also non-separate things exist, for instance, that moving things exist, it is also true to say in a general sense that the objects of mathematics exist, and that they are such as they are said to be.

And just as it is true to say generally of the other sciences that they deal with a certain object -not with its property, for instance not with the white, if the healthy thing is white, and the science has the healthy as its object, but with that which is the object of the particular science; for example, with the healthy if the science

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51 1077b36-1078a1: Codices E and A and also Bonitz read “ἡ ἔστιν ἐκάστου”, while Alexander and Bekker read “οὐ ἔστιν ἐκάστῃ”. I would agree with the second reading, since, looking for the object of scientific knowledge, it is more reasonable to say “the branches of knowledge are not about this (namely about properties) but they are about that which is the object of each branch of knowledge”. If we say “the branches of knowledge are not about this (namely about
ći τὴν γεωμετρίαν.

If it has happened that the things with which mathematics deals are perceptible but it deals with them not as perceptible, it will follow that the mathematical sciences do not deal with perceptible things, nor for that matter with other things separated from them and besides them. Many attributes are essential properties of things *qua* being, each of such things, of a certain type, since even in the case of animals there are proper attributes that belong to the animal *qua* female and *qua* male, but there is neither something female nor something male in separation from the animals. Therefore, the same is also true insofar as the animals are treated only *qua* lines or *qua* planes.

And, indeed, the more a science deals with things which are prior in definition and simpler, so much the more it is accurate. And this is the simple. So, a science which does not take the properties, but they are about that *qua* which the science is about each thing”, we mean that scientific knowledge practically deals with everything - but only with respect to a specific property of it - and therefore we do not successfully define the object of scientific knowledge.
magnitude of its objects into account is more accurate than that which takes magnitude into account and it is most precise if it does not take movement into account. However, if it involves movement, it most of all deals with the primary one; for this is the simplest of all, and again uniform movement is the simplest form of this. The same account also applies to harmonics and optics; for neither considers (its object) qua sight or qua voice, but qua lines and numbers; however the latter (i.e. lines and numbers) are attributes proper to the former (i.e. sight and voice). And the same is true of mechanics too.

So, if somebody, who has put things independently of their properties, studies any aspect of them qua so regarded, he shall not on this account be guilty of any error, just as he shall not be guilty of any error when he draws a diagram on the ground and says that a line is one foot long, while the line is not one foot long; for the error does not lie in the premises.

And every object would be best investigated in this way, i.e. if somebody supposed that which does not exist in separation and
άριθμητικός ποιεί καὶ ὁ γεωμέτρης. Ἐν μὲν γὰρ καὶ ἀδιαίρετον ὁ ἄνθρωπος ἡ ἀνθρώπως: ὁ δὲ ἐδέστο ἐν ἀδιαίρετον, εἰτ᾽ ἐλεώρησεν εἰ τὶ τῶ ἄνθρωπω ἑμβέβηκεν ἡ ἀδιαίρετος. Ὁ δὲ γεωμέτρης οὐθ᾽ ἡ ἀνθρώπως οὐθ᾽ ἡ ἀδιαίρετος ἀλλ᾽ ἡ στερεόν. Ἄ γὰρ κἂν εἰ μὴ που ἦν ἀδιαίρετος ὑπήρξεν αὐτῶ, δῆλον ὅτι καὶ ἀνευ τούτων ἐνδέχεται αὐτῷ ὑπάρχειν [τὸ δυνατὸν]52, ὥστε διὰ τούτο ὀρθῶς οἱ γεωμέτραι λέγουσι, καὶ περὶ ὀντῶν διαλέγονται, καὶ ὑπάρχει ἣ ἐστὶν· διττὸν γὰρ τὸ ὅν, τὸ μὲν ἐντελεχεία τὸ δ᾽ ὑλικῶς.

regarded it, having separated it, as separate, which is what the arithmetician and the geometrician does. For human qua human is one and indivisible thing; and the arithmetician assumes human to be one and indivisible thing, and then considers if there is any property of human qua indivisible. However, the geometer considers human neither qua human nor qua indivisible, but qua something solid. For, concerning the attributes that would belong to human, even if he was somehow not indivisible, it is clear that such attributes can belong to him irrespectively of these things (i.e. of his humanity or indivisibility). Therefore, the geometricians speak rightly for this reason, i.e. because they discuss about existing things, and the objects of their science really exist; for the being can exist in two ways, either in actuality or as matter.

1078a31-1078b6 Επεὶ δὲ τὸ ἄγαθον καὶ τὸ καλὸν ἔτερον (τὸ μὲν γὰρ ἄει ἐν πράξει, τὸ δὲ καλὸν καὶ ἐν

Now, since the good and the beautiful are different (for the former is always present in action, while the beautiful is also

52 1078a28: "τὸ δυνατὸν" is adopted by Ross, but it is suspectum according to Christ and it is omitted by codex Γ. I think that "τὸ δυνατὸν" seems to be - at least semantically and hermeneutically - superfluous here, given that it is semantically relevant to "ἐνδέχεται" in the same line.
τοῖς ἀκανήτοις), οἱ φάσκοντες οὐδὲν λέγειν τὰς μαθηματικὰς ἐπιστήμας περὶ καλοῦ ἢ ἀγαθοῦ ψεύδονται. Λέγουσι γὰρ καὶ δεικνύουσι μάλιστα οὐ γὰρ εἰ μὴ ὄνομάζουσι τὰ δ’ ἔργα καὶ τοὺς λόγους δεικνύουσιν, οὐ λέγουσι περὶ αὐτῶν. Τοῦ δὲ καλοῦ μέγιστα εἰδὴ τάξεις καὶ συμμετρία καὶ τὸ ἄρισμένον, ἃ μάλιστα δεικνύουσιν αἱ μαθηματικαὶ ἐπιστήμαι. Καὶ ἐπεὶ γε πολλῶν αἰτία φαίνεται ταύτα (λέγω δ’ οἴον ἢ τάξεις καὶ τὸ ἄρισμένον), δήλον ὅτι λέγοιεν ἀν καὶ τὴν τοιαύτην αἰτίαν τὴν ἄρα τὸ καλὸν αἰτίον τρόπον τινά. Μᾶλλον δὲ γνωρίμως ἐν ἄλλοις περὶ αὐτῶν ἐρούμεν.

found in immovable things), those who assert that the mathematical sciences say nothing about the beautiful or the good are in error. For, they (i.e. mathematical sciences) speak about and prove them to the greatest extent; since, if they do not mention them by name, but demonstrate their effects or definitions, it is not true to say that they tell us nothing about them. Order and symmetry and definiteness, which mathematical sciences demonstrate in the greatest degree, are the main forms of the beautiful. And due to the fact that these things (such as, I mean, order and definiteness) obviously are causes of many things, it is clear that the mathematical sciences must treat this sort of cause too, which is the beautiful as cause in some sense. But we shall speak about these matters more explicitly elsewhere.
OVERVIEW:

M.3 presents Aristotle’s account of how mathematical objects are to be considered and also how their existence is to be explained. It seems that Aristotle puts forward two different but related points so as to explain the nature of mathematical objects. One further point discussed is the non-separate existence of mathematical objects. It is finally deduced that mathematical objects are more primary, simpler and thus more accurate than the objects of the other branches of scientific knowledge. Moreover, the objects of mathematics are said to display the beautiful.

- Just as universal propositions in mathematics are not about numbers and magnitudes \textit{qua} divisible things, in the same way the formulae and proofs about sensible magnitudes are not about them \textit{qua} sensible, but \textit{qua} having certain qualities.
- As there are definitions and demonstrations about things \textit{qua} movable, there can also be about things \textit{qua} planes, \textit{qua} bodies etc.
- So, it can be said without qualification that mathematical objects exist and that they are such as they are considered by the mathematicians.
- Each science deals with its object \textit{qua} having particular qualities. In the same way, mathematics deals with perceptible things \textit{qua} lines, planes etc.
- The simpler the object which is studied by a science, the more accurate the knowledge of it. Hence, the sciences of harmonics and optics deal with their objects \textit{qua} numbers and lines and not \textit{qua} voice and sight.
- A science may treat its object as being separate from its attributes, in the same way in which a geometer supposes that a line is a foot long, when it is not.
- Therefore, the way in which the arithmetician and the geometrician investigate their objects, i.e. treating them as separately existing, is the best method.
- Finally, there is a mistake in holding that there is no relation between mathematics and good or beautiful. For, mathematics proves attributes that
are principal forms of the beautiful, such as order, symmetry and definiteness.
COMMENTARY:

1077b17-22: Aristotle goes on making reference to universal statements and demonstrations of mathematics and finally reaches a conclusion about universal formulae and demonstrations of any branch of knowledge.

As it is also stated by Aristotle in M.2 (cf. 1077a9-14), universal propositions in mathematics do not deal with magnitudes and numbers \textit{qua} having magnitude and \textit{qua} being divisible things.

In order to understand Aristotle’s point here, we need first to draw a distinction between magnitudes and numbers conceived as physical objects and magnitudes and numbers conceived as non-physical objects. As a physical object, “magnitude” should be thought of as a perceptible body having size and “number” should be thought of as a numerable plurality of perceptible things. Any physical object has a size and can be seen as a structure divisible in its components, or else as a plurality of countable constituent parts. Such properties could be possessed by a cube made of marble or by a dozen of roses. As non-physical objects, “magnitude” and “number” should be considered to be properties or aspects of physical objects. The objects of mathematical sciences are not the physical objects without qualification, but the physical objects \textit{qua} having some specific properties, such as shape, size and the ability to be counted.

Mathematical definitions, theorems and demonstrations are propositions which are not about magnitudes and numbers insofar as they are measurable and divisible objects studied by geometry and arithmetic respectively\textsuperscript{53}. Mueller\textsuperscript{54} talks about the distinction that is drawn by Aristotle between “ordinary geometric reasoning” and universal “conceptual syllogistic reasoning” and he assumes that universal knowledge, for instance the knowledge that any triangle has interior angles equal to two rights, arises out of ordinary reasoning. Therefore, Mueller claims, universal reasoning does not have any object over and above the objects of ordinary reasoning; for, universal reasoning is only a formulation of the ordinary reasoning.

\textsuperscript{53}Crubellier (1994), 131
\textsuperscript{54}Mueller (1970), 171
Nevertheless, Aristotle seems to treat mathematical theorems as propositions, for the formulation of which we need “objects” that differ from the magnitudes and numbers studied by mathematical sciences. In fact, mathematical theorems deal with mathematical objects insofar as they have certain properties. For example, the mathematical theorem that any triangle has interior angles equal to two rights deals with “the triangle” as a universal object and not with the numerous particular geometrical triangles. Thus, it could be inferred that mathematical demonstrations and theorems concern magnitude and number in the most universal sense. This means that they do not refer to individual geometrical objects which have size and can be divided. They refer to magnitude and number as universal aspects, whose values vary from thing to thing.

It could be a matter of discussion whether the mathematical objects should be conceived as specific parts or aspects of physical things, which the mathematicians study, or as perspectives from which mathematicians regard physical things. The former interpretation treats mathematical objects as beings—though not in the sense of substance—, while the latter interpretation treats them not as beings, but as points of view in the study of physical objects. It seems to me that the “/qu/” (qua, as) locution could be understood in both ways, since they are related to each other. Firstly, “qua” could be conceived as indicating the mathematical study of certain aspects that belong to physical things, for example their shape and, secondly, it could be conceived as indicating the particular approach that is adopted by mathematical sciences when studying physical things. The choice of one certain feature of a physical thing as an object of study presupposes that this physical thing is approached in a particular way, so that we can study a specific feature of it. Therefore, the two possible aforementioned conceptions of mathematical objects seem to be in fact two sides of the same coin.

1077b22-30: As previously said, mathematicians—though dealing with perceptible things—do not study them as physical objects, but as things having the mathematical

55 Annas (1976), 148
properties of magnitude, number etc. Aristotle gives here another similar example concerning moving things.

(a) Physicists study physical objects only insofar as they move. So, they formulate definitions and other universal propositions concerning their property of moving and not their physical essence or any other property of them.

(b) So, “universals” both in the case of mathematics and in the case of physics do not just deal with the nature of objects \textit{qua} physical objects, but they deal with particular properties of objects, which are different in each scientific field.

Although the branches of scientific knowledge differ, it turns out that they all deal with physical beings. But, even though they all deal with the same physical beings, the approach of each science is not the same with any other science’s approach, since each science treats the same objects from a different perspective. For instance, both physics and geometry study physical objects. However, physics studies their motion, while geometry studies other aspects of them, for example their shape. So, physics studies physical objects as moving things, while geometry studies them as triangular or square things. Therefore, the objects of mathematics are the same physical moving objects which are studied by physics, but mathematical proofs are about them not insofar as they are sensible or moving, but only insofar as they are dimensional and quantifiable.

Definitions and demonstrations about perceptible things can be formulated in any discipline. However, each discipline treats its object not insofar as it is a physical object, but insofar as it has the properties that are relevant to this specific field of knowledge. So, all branches of knowledge study the same things, but they treat it in different ways.

Furthermore, Aristotle points out that we should not conceive qualities that are studied by sciences as natures having separate existence from the objects to which they belong. This point reinforces our previous assumption that all branches of knowledge deal with the same objects, but each of them treats it focusing on different aspects of them.
Aristotle here clarifies that:

(a) not only separate but also non-separate things exist.

(b) The objects of scientific knowledge have been said to be non-separate from physical objects. However, they do have existence.

(c) Besides, it sounds absurd to say that something which is object of science does not exist. Anything that is able to be studied and explained by science must exist.

Nevertheless, it is wrong to conceive the existence of non-separate things as identical with the existence of separate things, i.e. of individual substances. This is why Aristotle adds that we can say only in a general sense (“ἅπλῶς”) that non-separate things, such as mathematical objects, exist. So, we should be careful here to take “ἅπλῶς” as modifying “εἰπεῖν” and not “ἔστιν”, for Aristotle has already said that mathematical objects do not exist in an unqualified (i.e. general) way (cf. 1077b16).

This section raises two quite significant points:

(a) The first one is that every branch of knowledge deals with a different object. However, we should be careful not to identify the scientific object with any incidental quality it has. It is necessary here to explain the distinction between a thing’s essential and incidental properties. For Aristotle, essential properties are those which are indispensable in order that a being is the kind of thing it is. For instance, to be two-legged and rational are essential properties of man. Incidental properties are those which are not necessary for something’s being the kind of thing it is. For instance, to be thin or healthy are incidental properties of man. From the viewpoint of a science, essential properties of physical things are those which constitute the object studied by the science, while incidental properties are the rest of properties possessed by physical things, which are not studied by this specific science. The object

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Ross (1924), v. 2, 417
of medicine, for example, is health. Even though the color which better suits
the healthy condition is white, we cannot say that medicine deals with white,
since white is only an incidental property of the physical thing when it is
treated as healthy.

(b) The second point is that all branches of knowledge in fact deal with sensible
things, however from different perspectives. This is the reason why Aristotle
insists on using here the modifier “ἣ” (qua, as). Every branch of knowledge
studies the same beings qua certain things. For example medicine studies
beings qua healthy, physics studies them qua moving, geometry studies them
qua solids etc.

1078a2-9: The point that every different branch of knowledge deals with the same
physical objects from a different perspective is better explained in this section,
through the example of mathematical sciences.

Geometry does not deal with the beings it studies qua perceptible things. One
further point that is introduced here is that geometry does not deal with any object
existing separately from perceptible beings. Therefore, it can be concluded that
geometry deals with perceptible beings, but not insofar as they are perceptible
things. The perceptibility, in other words, is not the aspect of physical beings that is
studied by geometry.

Afterwards, it is said that the objects studied by geometry are lengths and planes.
However, Aristotle emphasizes that lengths and planes are not separate beings. On
the contrary, he presents them as properties of beings, just like masculinity and
femininity. For, just as we cannot think of masculinity and femininity as separate
natures, in the same way we should not think of length and plane as separate
entities. However, we should also keep in mind that length and plane are
independent of the other properties of the same being. Therefore, the right way of
conceiving mathematical objects is as properties of physical objects that can be
studied separately from the other properties of the same objects57.

57 Crubellier (1994), 139
Summarizing the three points which are mentioned in the previous section (1077b34-1078a2) and in this one, it can be assumed that each branch of scientific knowledge (and therefore mathematics, too):

(a) studies physical objects focusing on certain qualities of them, which are thought of as essential and not incidental properties of beings,
(b) treats physical objects *qua* certain things, namely from a certain perspective that is different from all the other sciences’ perspectives and
(c) focuses on properties of physical objects which cannot be thought of as separately existing entities.

1078a9-17: In this section, Aristotle expresses the view that the more primary and the simpler something is, it is more accurate. This position could be explained as follows:

(a) First of all, we say that a thing is prior in definition to others in the sense that its definition does not involve the definitions of those other things.
(b) The fact that its definition does not entail other definitions means that the thing which is prior in definition is also prior in composition. In other words the thing, of which the definition does not require many other definitions, is simpler in structure and does not involve a great number of different components. Otherwise, the definitions of its components would constitute parts of its own definition. So, the simplicity of something’s definition reflects its simplicity in structure.
(c) It is therefore evident that something which has magnitude and moves, or else something which occupies place and changes with time, is not as simple as something unmoving which has no magnitude. Moreover, the science of something lacking magnitude and motion—and therefore not bound by place and time restrictions—attains accuracy to the greatest extent, since it can under all circumstances precisely describe the thing it investigates in the same way.
(d) The objects of mathematical sciences are such things. The fact that they are more primary, simpler and more accurate than the objects of any other
science is confirmed by the fact that any other science analyses its object in lines and numbers so that it can make measurements and thus study it with the best possible accuracy. The sciences of optics and harmonics aim at analyzing the phenomena of sight and voice respectively in such a way. Hence, approaching the objects of scientific knowledge in a mathematical way is required for both of these sciences.

(e) It is true that an optician’s or harmonics teacher’s study of sight and voice can be systematized on the basis of measurements. And the measurements can be performed by means of geometrical objects, such as lines, and arithmetical objects, such as numbers, since lines and numbers are qualities of sight and voice. However, the sciences of harmonics and optics are posterior to the simpler mathematical sciences, given that they do not study directly mathematical objects, but they study voice and sight by means of performing mathematical measurements.

(f) Being prior, mathematical objects are characterized by great accuracy, since accuracy is typical of what is prior, essential, without magnitude and generally unchangeable. It becomes even clearer here that the objects of mathematical sciences cannot be the physical beings of our experience.

1078a17-1078a21: In this section, Aristotle talks about any object of scientific study, saying that we are allowed to think of it independently of its properties.

(a) In the case of mathematics, it seems that Aristotle by “properties” means qualities that characterize physical beings which are treated by mathematicians qua mathematical objects.

(b) This fact is displayed in the example used by Aristotle here. He says that, if somebody draws a line on the ground and assumes it is one foot long, while it is not, he does not lie. What Aristotle really means is that, when mathematical measurements are about great dimensions, it is often non-practical or even impossible to make diagrams having such dimensions. So, it is necessary to release the mathematical object from the material properties
possessed by the physical object, such as the real length or width or height, and make a diagram which -though it may in fact be half a foot long- can be assumed to be one foot long, for the sake of quick, easy and right calculation.

**1078a21-31:** Aristotle claims that arithmetics and geometry have adopted the best way to treat their objects of study:

(a) It has already been said that mathematical objects, as properties and thus non-separate things, cannot be physically separated from the things of which they are properties.

(b) However, this physically impossible separation can be performed by the mathematician’s mind, i.e. the geometrician’s and the arithmetician’s, who considers and study mathematical objects thinking of them as separate entities. Mathematical objects are, then, separate only in abstraction (ἐξ ἀφαίρεσεως).

(c) Aristotle says that human *qua* human is one and indivisible, meaning that the definition of human cannot be valid in the case of a human being which is divided into parts. In other words, the definition of human can apply only to human thought of as a plurality of features which though is an indivisible entity. This could remind us of the procedure followed by arithmetic, which studies numbers defined as a plurality of monads. So, human *qua* human is one and indivisible and the arithmetician studies human *qua* indivisible unit.

(d) Hence, any property studied by the arithmetician has to be property of human *qua* indivisible unit and not property of human *qua* human. For instance, if human is studied *qua* indivisible unit, the quality of being rational is a property that is not studied. On the contrary, if human is studied *qua* human, it is the property of being indivisible that is not studied. Similarly, the geometer studies human *qua* solid body and not human *qua* human. So, both the arithmetician and the geometer focus on and abstract from human being the aspect which is object of their study, thinking of all the other features as properties that are not studied by them.
The objects of geometry are said to “really exist”. Saying that, Aristotle may mean that geometry focuses on qualities of beings by dint of which it can be felt that these beings do exist. In other words, geometry deals with aspects of things which are closely linked with their perceptibility. An example of these aspects is the solidity of physical objects.

Since ὑλικῶς is opposed to ἐντελεχεία it could be interpreted as δυνάμει. What Aristotle may have in mind here is that the being can be treated in two ways, first as composed of material elements and second as able to be analyzed into mathematical parts. Mathematical objects exist potentially in the sense that they are potentially present in physical objects and receive actual existence by the geometer’s act of separation. So, the mathematical parts into which a physical object can be analyzed are to be conceived as its ὑλή νοητή, i.e. its matter that is thought of, which is different from its ὑλή αἰσθητή, i.e. its matter that is perceived. This distinction is made in Book Z of Metaphysics (1035a12, 1036a9-12 & 1036b32-1037a5). Conceiving the objects of mathematics as matter, we attribute the material aspect to them. It seems quite risky to think of mathematical objects as matter. However this could maybe be possible provided that we think of mathematical objects as some sort of “matter” that mathematicians work on.

1078a31-1078b6: In this section, Aristotle expresses the idea that mathematical sciences display the beautiful.

(a) Leveling criticism probably against thinkers, like Aristippus, Aristotle begins his account with reference both to good and to beautiful as regards their link to mathematics. According to Ross, this happens because the thinkers against whom Aristotle addresses his criticism are presented as attacking mathematics because it never uses ἀγαθόν and καλόν as a term.

(b) Aristotle clarifies that ἀγαθόν (i.e. good) can always be found in action, while καλόν (i.e. beautiful) can also be found in things lacking motion and generally change. The use of the conjunction “καί” (meaning “also”) before

58 Ross (1924), v. 2, 418
“ἐν τοῖς ἀκινήτοις” (i.e. “in immovable things”) (cf. 1078a32) indicates that the beautiful can be found in action as well.

(c) Mathematicians never use “good” and “beautiful” as names. Aristotle says that the good and the beautiful are displayed in mathematics by means of their “ἐργα” (i.e. their effects or actions) and their “λόγοι” (i.e. their formulae/definitions).

After this general reference to the relation of good and beautiful with mathematics, Aristotle focuses on beautiful, which could be conceived as the genus which involves order, symmetry and definiteness as its species. Mathematics display order (i.e. spatial arrangement of the parts), symmetry (i.e. proportional size of the parts) and definiteness (i.e. limitation in size of the whole). Since mathematical sciences display forms of beautiful, even without naming them, they cannot be thought of as saying nothing about beautiful.

It is remarkable that although at first Aristotle makes reference to both good and beautiful, he then goes on to show how mathematics uses the beautiful, but not how it uses good. The reason why Aristotle speaks exclusively about mathematics may be that “καλόν” is primarily applicable to the physical realm, while “ἀγαθόν” is primarily applicable to the moral realm.

Aristotle also adds that, due to the fact that order, symmetry and definition seem to be causes of things, it can be assumed that beautiful, as genus of order, symmetry and definition, is some sort of cause too. However, his promise to deal with this matter more explicitly elsewhere seems to remain unfulfilled.59

59 Ross (1924), v. 2, 419
### M.4 (1078b7-1079b11)

**“Discussion of the Theory of Forms”**

<table>
<thead>
<tr>
<th>SECTION</th>
<th>ANCIENT GREEK TEXT</th>
<th>TRANSLATION</th>
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<tbody>
<tr>
<td>1078b7-12</td>
<td>Περὶ μὲν οὐν τῶν μαθηματικῶν, ὅτι τε ὅντα ( \text{ἔστι} ) καὶ πῶς ( \text{ὸντα} ), καὶ πῶς πρότερα καὶ πῶς ( \text{οὐ \ πρότερα} ), τοσάντα εἰρήσθω· περὶ δὲ τῶν ἰδεῶν πρῶτον αὐτὴν τὴν κατὰ τὴν ἰδέαν δόξαν ἐπισκεπτέον, μηδὲν συνάπτοντας πρὸς τὴν τῶν ἀριθμῶν φύσιν, ἀλλ᾽ ὡς ὑπέλαβον ἐξ ἀρχῆς οἱ πρῶτοι τὰς ἰδέας φήσαντες εἶναι.</td>
<td>So, as regards the mathematical objects -namely as regards the fact that they exist and in what sense they exist and in what sense they are prior and in what sense they are not prior- let so much suffice. Now, as far as Forms are concerned, we must first of all examine the opinion itself about Form, not connecting it at all with the nature of numbers, but treating it in the way in which it was originally thought of by the people who first maintained that Forms exist.</td>
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<tr>
<td>1078b12-23</td>
<td>Συνέβη δ᾽ ἡ περὶ τῶν εἰδῶν δόξα τοῖς εἰπούσι διὰ τὸ πεισθῆναι περὶ τῆς ἀληθείας τοῖς Ἡρακλείτειοις λόγοις ὡς πάντων τῶν αἰσθητῶν ἀεὶ ἰδεῶν, ἀστερ᾽ ἐπερ ἐπιστήμη τινός ἦστα καὶ φρόνησις, ἐτέρας δεῖν τινὰς φύσεις εἶναι παρὰ τὰς αἰσθητὰς μενοῦσας· οὐ γὰρ εἶναι τῶν ἱερῶν ἐπιστήμην.</td>
<td>The theory of Forms occurred to the people who, due to having been convinced about the true nature of reality by Heraclitus’ doctrine that all the perceptible things are eternally in a state of flux, they said that, consequently, if there is to be any knowledge or thought about anything, there must be some other natures besides the perceptible ones, which stand still; for there can be no knowledge of things that are in flux. But Socrates was the first to</td>
</tr>
</tbody>
</table>
Σωκράτους δὲ περὶ τὰς ἡθικὰς ἀρετὰς προαγματευομένου καὶ περὶ τούτων ὀρίζοντα καθὸλου ζητοῦντος πρώτου (τῶν μὲν γὰρ φυσικῶν ἐπὶ μικρὸν Δημόκριτος ἦφαι σῶν καὶ ἀρίστατό πώς τὸ θερμὸν καὶ τὸ ψυχρὸν· οἱ δὲ Πυθαγόρειοι πρῶτερον περὶ τινῶν ὀλίγων, ὡς τοὺς λόγους ἐις τοὺς ἀριθμοὺς ἀνήπτονος, οἶον τί ἐστι καὶ ὁ ἀρίθμος ή τὸ δῦκαιον ή γάμος.

elaborate the moral virtues and seek to formulate general definitions about these (for, among the natural scientists, Democritus touched only superficially this subject and defined in a way the “hot” and the “cold”; while the Pythagoreans had beforehand done so about a few certain things, whose definitions they connected to numbers -for instance what “opportunity” is or “justice” or “marriage”;

But he (i.e. Socrates) was reasonably inquiring into «what (a thing) is». For he was seeking to make syllogisms/arguments, but the starting point of syllogisms is «what (a thing) is». At that time there was not yet the dialectical power so that men be able to speculate contraries independently of «what (a thing) is», and

1078b23-31
Εκείνος δ᾽ εὐλόγως ἐξῆτε τὸ τί ἐστιν. συλλογίζεσθαι γὰρ ἐξῆτε, ἄρχῃ δὲ τῶν συλλογισμῶν τὸ τί ἐστιν. διαλεκτικὴ γὰρ ἴσχὺς οὕτω τότε ἢν ἤστε δύνασθαι καὶ χορίς τοῦ τί ἐστι τάναντια ἐπισκοπεῖν, καὶ τῶν

But he (i.e. Socrates) was reasonably inquiring into »what (a thing) is». For he was seeking to make syllogisms/arguments, but the starting point of syllogisms is »what (a thing) is». At that time there was not yet the dialectical power so that men be able to speculate contraries independently of »what (a thing) is», and

60 1078b22: Codices A and E adopt the reading "ἀνήπτον", while Alexander proposes the reading "ἀνῆγον". I would prefer the reading "ἀνήπτον" as more appropriate, since its meaning ("connected/ascribed to") suits the context better than the meaning of "ἀνῆγον" ("referred/reduced to"), which is mainly used in cases of referring something to its principles. Aristotle makes here reference to the way in which Pythagoreans attached the formula of each thing to a number, in order to indicate that they were among the thinkers who formulated definitions after a fashion. However, it is still too early to talk about reducing arguments to syllogisms.
<table>
<thead>
<tr>
<th>Greek</th>
<th>Latin</th>
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<tbody>
<tr>
<td>ἐναντίων εἰ ἢ αὐτὴ ἐπιστήμη; δύο γὰρ ἔστιν ἂ τις ἢ ἀποδοθῆ Σωκράτει δικαίως, τούς τ᾽ ἐπακτικούς λόγους καὶ τὸ ὀρίζεσθαι καθὸλου· ταῦτα γὰρ ἔστιν ἄμφοι περὶ ἄρχην ἐπιστήμης· ἀλλ᾽ ὁ μὲν Σωκράτης τὰ καθὸλου ὁ πόσιτα ἐποίει οὐδὲ τοὺς ὀρισμοὺς;</td>
<td>inquire if it is the same science that deals with contraries. So, there are two things which somebody would fairly ascribe to Socrates: inductive arguments and formulation of general definitions; for both of these are concerned with the starting-point of scientific knowledge). However, Socrates regarded neither universals nor definitions as existing in separation; but they gave them separate existence and called such existing things Forms. Thus, almost by the same argument, it followed for them that there are Forms of all things which are spoken of in universal terms and it was nearly the same as if somebody, who wished to count a number of things and, since the things were a few, thought that he could not count them, would finally be able to count them after making them more in number; for, one may say, the Forms are more in number than particular sensible things, in seeking the causes of which these thinkers proceeded from these to the Forms. for, to each thing corresponds an entity having the same name.</td>
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<tr>
<td>Line</td>
<td>Greek Text</td>
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<tr>
<td>91</td>
<td>παρὰ τὰς οὐσίας, τῶν τε ἄλλων ἐν ἑστιν ἐπὶ πολλῶν, καὶ ἐπὶ τοίοθε καὶ ἐπὶ τοῖς ἁίδιοις.</td>
</tr>
<tr>
<td>1072a4-7</td>
<td>Ἐτι ἐτι καθ’ ὀς τρόπους δείκνυται ὁτι ἑστὶ τὰ εἰδή, κατ’ οὐθένα φαίνεται τούτων· ἔς ἐνίων μὲν γὰρ οὐκ ἀνάγκη γίγνεσθαι συλλογισμόν, ἔς ἐνίων δὲ καὶ οὐχ ἃν οἶνται τούτων εἰδὴ γίγνεται.</td>
</tr>
<tr>
<td>1079a7-11</td>
<td>Κατὰ τὲ γὰρ τοὺς λόγους τοὺς ἐκ τῶν ἐπιστήμων ἐσται εἰδή πάντων ὅσιων ἐπιστήμηα εἰσίν, καὶ κατὰ τὸ ἐν ἑπὶ πολλῶν καὶ τῶν ἀποφάσεων, κατὰ δὲ τὸ νοεῖν τι φθαρόντων τῶν φθαρτῶν· φάντασμα γάρ τι τοῦτον ἐστιν.</td>
</tr>
<tr>
<td>1079a11-13</td>
<td>Ἐτι δὲ ὅι ἀκριβεστάτατοι τῶν λόγων οἱ μὲν τῶν πρὸς τὶ ποιούσιν ιδέαις, ὡς οὐ φασιν εἶναι</td>
</tr>
</tbody>
</table>

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61 1079a11: Codex Ab reads “τοῦτων”, while codex E reads “τοῦ”. I agree with the first reading, since the Forms of the things which have perished function as a mental image of these things.
καθ᾽ αὐτό γένος, οἱ δὲ τῶν τρίτων ἀνθρώπων λέγουσιν.

| 1079a14-19 | Ὅλως τε ἀναφοροῦσιν οἱ περὶ τῶν εἰδῶν λόγοι ἑαυτοῦ καὶ ἀρκετῶν, εἰσὶ οἱ λέγοντες εἰδή τού τάς ἱδέας εἶναι συμβαίνει γὰρ μή εἶναι πρῶτον τὴν δύναμα ἄλλα τὸν ἀριθμόν, καὶ τὸ πρῶτον τοῦ καθ᾽ αὐτὸ, καὶ τὰ τῶν ἱδεῶν δόξας ἠναντιώθησαν ταῖς ἀρχαῖς. | And in general the arguments for the Forms totally do away with the things whose existence the exponents of Forms want more to be than the existence of the Forms; for it follows that what is first is not the dyad but number, which means that the relative is prior to the thing “itself”; and all the other points in respect of which some people, who have followed up the opinions held about the Forms, have contradicted the principles. |
| 1079a19-23 | ἔτι ἐτι κατὰ μὲν τὴν ὑπόληψιν καθ᾽ ἢν φασιν εἶναι τὰς ἱδέας οὐ μόνον τῶν οὐσιῶν ἐσονται εἰδή ἄλλα καὶ ἄλλων πολλών (τὸ γάρ νόμημα ἐν οὐ μόνον περὶ τὰς οὐσίας ἄλλα καὶ κατὰ μὴ οὐσίων ἐστὶ, καὶ ἐπιστήμη αἱ οὐ μόνον τῆς οὐσίας εἰσὶ. | In addition, according to the assumption based on which they state that Forms exist, there will be Forms not only of the substances but also of many other things (for the concept is single, not only in the case of the substances, but also in that of non-substantial things, and there will be sciences not only of substances but also of other things; and a thousand other similar conclusions will also result). |

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62 1079a17: Codex E reads “καὶ τὸν πρῶτον τοῦ καθ᾽ αὐτό”, while codex A reads “καὶ τῷ καθ᾽ αὐτῷ” and Ross proposes “καὶ τούτον τῷ πρῶτῳ καὶ τοῦτο τοῦ καθ᾽ αὐτῷ”. I think that Ross’s additions should be rejected, since in Α.9, 990b20-21, the reading “μή εἶναι πρῶτον τὴν δύναμα ἄλλα τὸν ἀριθμόν, καὶ τῷ πρῶτῳ τοῦ καθ᾽ αὐτῷ” is adopted without serious disagreements.
| 1079a31-1079b3 | δὲ τὸ ἀναγκαῖον καὶ τὰς δόξας τὰς περὶ αὐτῶν, εἰ ἦστι μεθεκτὰ τὰ εἰδῆ, τῶν οὐσιῶν ἀναγκαῖον ἰδέας εἶναι μόνον· οὐ γὰρ κατὰ συμβεβηκὸς μετέχοντα, ἀλλὰ δεὶ ταύτῃ ἐκάστου μετέχειν ἢ μὴ καθ᾽ ὑποκειμένου λέγονται (λέγω δ᾽ οίον εἰ τὶ αὐτοῦ διπλασίον μετέχει, τούτῳ καὶ αὐτίκῳ μετέχει, ἀλλὰ κατὰ συμβεβηκὸς· συμβέβηκε γὰρ τῷ διπλασίῳ αὐτίκῳ εἶναι), ὡστε ἦσται οὐσία τὰ εἰδῆ. | However, by necessity and according to the opinions about them, if the Forms can be participated in, there must be Forms of substances only; for they are not participated in incidentally, but a thing must participate in each Form in so far as it is not predicated of a subject. I mean, for instance, if a thing participates in the double “itself”, then the same thing participates also in something eternal, but incidentally; for the double incidentally happens to be eternal. Therefore, Forms will be substance. |
| 1079a31-1079b3 | Ταύτα δ᾽ ἐν τὰ ὁμοῦν σημαίνει κάκει· ἢ τί ἦσται τὸ εἶναι φάναι τὶ παρὰ ταύτα, τὸ ἐν ἐπὶ πολλῶν; καὶ εἰ μὲν ταύτῳ εἴδος τῶν ἰδεῶν καὶ τῶν μεταχόντων, ἦσται τὶ κοινὸν τί γὰρ μᾶλλον ἐπὶ τῶν φθορῶν δυάδων, καὶ τῶν δυάδων τῶν πολλῶν μὲν ἀυτίκῳ ἐν, τὸ δυάς ἐν καὶ ταύτῳ, ἢ ἐπ᾽ αὐτῆς καὶ τῆς τινὸς;)· εἰ δὲ μὴ τὸ αὐτὸ εἴδος, ὀμώνυμα ἄν εἴη, καὶ ὀμοιὸν ὡσπερ ἄν εἰ τις καλῶς ἀνθρωπον τὸν And the same names denote substance here and there; or else, what will be the meaning of stating that something exists apart from these, i.e. the One-over-Many argument? And, if the Forms and the things which participate in them have the same form, there will be something they have in common; for, why should 2 mean one and the same thing in the case of perishable 2’s and the 2’s that are many on the one hand but eternal on the other hand, but not in the case of 2 “itself” and a particular 2? But, if they have not the same form, they will simply be homonyms, and it is as if |
| 1079b3-11 | Εἰ δὲ τὰ μὲν ἄλλα τοὺς κοινοὺς λόγους ἐφαρμόττειν θήσομεν τοῖς εἰδεσιν, οἷον ἐπὶ αὐτὸν τὸν κύκλον σχῆμα ἐπίπεδον καὶ τὰ λοιπὰ μέρη τοῦ λόγου, τὸ δὲ ἐστιν προστεθήσεται, σκοπεῖν δὲί μὴ κενὸν ἢ τούτο παντελῶς. Τίνι τε γὰρ προστεθήσεται; Τῶ μέσω ἢ τῷ ἐπίπεδῳ ἢ πάσιν; Πάντα γὰρ τὰ ἐν τῇ οὐσίᾳ ἱδέα, οἷον τὸ ἔριον καὶ τὸ δύσιον. ΄Ετί δὴλον ὅτι ἀνάγκη αὐτὸ εἶναι τι, ὧσπερ τὸ ἐπίπεδον, φύσιν τινὰ ἢ πάσιν ἐνυπάρξει τοῖς εἰδεσιν ὡς γένος. |
| someone were to call both Callias and a piece of wood “man”, without having observed any community between them. |
| But if, in all other respects, we are to suppose that the common definitions apply to the Forms, for example that the plane figure and the other parts of the definition apply to the circle, but that “of what the Form is a Form” must be added, we should inquire if this is not absolutely meaningless. For, to what will this be added? To “center” or to “plane” or to all elements of the definition? For all the elements that exist in the substance are Forms, for example “animal” and “two-footed”; furthermore, it is clearly necessary that this is something, as in the case of the plane, i.e. some definite nature, which will be present as genus in all its species. |
OVERVIEW:

M.4 presents the Theory of Forms and its implications. Aristotle starts from his predecessors and continues with Platonists’ views expressing his objections to them and reporting on problematic points which lead to absurd assumptions. The principal points that have negative implications are, according to Aristotle, firstly that there turn out to be more Forms than physical beings, secondly that there are said to be Forms of beings’ qualities and thirdly that due to the existence of Forms both of substances and of non-substances there is serious difficulty in treating all Forms as substances, which is indispensable in order that the Theory of Forms keeps its interest.

- Aristotle points out the necessity of examining the Theory of Forms in its original form, i.e. without connecting it at all with numbers.
- The exponents of the Theory of Forms are said to have been influenced by Heraclitus -who claimed that the perceptible things are always in a flux- in assuming that there must be some things other than the perceptible ones to serve as objects of knowledge.
- Socrates was the first to formulate inductive arguments and general definitions, Democritus defined in a way “hot” and “cold” and the Pythagoreans connected their definitions to numbers. Socrates’ successors treated the universals as separately existing things and called them “Forms”.
- However, the Theory of Forms: (a) redoubles the number of beings.
- (b) Moreover, it uses proofs that do not prove anything or prove the existence of Forms of things of which the Platonists do not think there are any Forms.
- (c) Furthermore, according to the scientific arguments, there will be Forms of all the things that constitute objects of sciences.
(d) Also, according to the One-over-Many argument, there will also be Forms of negations.

(e) Further, according to the argument of "conceiving what has perished", there will be Forms of perishable things.

(f) Finally, there will turn out to be Forms of relative terms, while other arguments posit the "third man".

In general, the Theory of Forms negates what the exponents of Forms treat as more important than the Forms. For, it follows that it is number and not the Dyad that comes first, which means that the relative is prior to the thing "itself".

In addition, according to the Theory of Forms, there will be Forms not only of substances, but also of many other things.

However, there can be Forms only of substances, provided (a) that the Forms are participated in by substances and (b) that the same names must indicate substance both in the world of Forms and in the world of perceptible things.

Finally, if it is accepted that the Forms and the perceptible things which participate in them share the same definition and only the name of "that which it is the Form of" has to be added, attention should be paid: (a) to the fact that each and every element of the definition of a Form has to be also a Form and (b) to the fact that there will be a Form of "formness" which will be present in all definitions of Forms.
COMMENTARY:

1078b7-12: Aristotle is now ready to deal with the Forms without connecting them at all with numbers. Furthermore, he announces his intention to study Forms in the original way in which previous thinkers investigated them. By “οἱ πρῶτοι τὰς ἰδέας φήσαντες εἶναι”, Aristotle means Plato and his followers. Aristotle is about to refer to the previous thinkers and their contribution to the final formulation of the Theory of Forms by the Platonists.

1078b12-1078b23: As Annas states, the passage starting from 1078b12 and ending at 1078b32 is related to the passage 987a29-987b10 from Metaphysics’ A.6. However, the emphasis here is very different, since in Metaphysics’ M Aristotle focuses on the material relevant to the theory of Forms.

a) The first group of previous thinkers to whom Aristotle refers, are those who had been persuaded by the Heraclitean theory and thus believed in the existence of some immovable and generally unchangeable things apart from the physical beings. According to these thinkers, the existence of some stable objects is indispensable, since they are more appropriate as objects of scientific study. Not being in a constant state of flux, they can be studied more accurately and thus more efficiently. Given that Aristotle has the scientific knowledge and study in mind, the term “φρόνησις” is used here in the Platonic sense in which it is not distinguished from “ἐπιστήμη” and thus not restricted to moral questions.

b) The second group of previous thinkers that Aristotle has in mind is presented in comparison to Socrates. In contrast to Socrates, who dealt with moral virtues, the physicists (or natural scientists) dealt with physical objects. It

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63 Annas (1976), 152
64 In Book A (987a32) of Metaphysics, Cratylus is mentioned as the Heraclitean who was Plato’s first master in philosophy. So, Cratylus may be included in the first group of previous thinkers Aristotle refers to.
65 Ross (1924), vol. 2, 422
should be clarified here, as Annas notes, that “natural scientists” are not like modern scientists. We have to keep in mind that the Presocratic cosmologists were philosophers as much as scientists. Aristotle compares natural scientists with Socrates mainly as regards the extent to which they dealt with the problem of definition. Democritus, who is the representative of physicists referred to by Aristotle, is said to have touched the subject superficially.

c) The third group of previous thinkers referred to by Aristotle is that of the Pythagoreans. Pythagoreans are said to have linked definitions of things on numbers. These thinkers reduced things to numbers ending up at identifying, according to Alexander, “καιρός” with number 7, “δίκαιον” with numbers 4 or 9 and “γάμος” with number 5.

1078b23-32: Having made reference to Heraclitus’, Democritus’ and Pythagoreans’ thought, Aristotle goes on to give details of Socrates’ thought.

a) According to Aristotle, dialectic did not exist yet in Socrates’ time. However, as Ross notes, Aristotle is quoted as having called Zeno the Eleatic the inventor of dialectic. Zeno was senior to Socrates. As a result, dialectic in all probability did exist in Socrates’ time. But it did not exist in the same way in which it evolved due to Socrates. The factor which differentiates dialectic before Socrates with dialectic after him is the fact that Socrates was the first to formulate the “what is”, i.e. definitions of things. Definitions of things were necessary, because having agreed upon the “what is” of things it would be easier to define and study their contraries and thus assume if it is the same science that deals with a kind of things and their contraries. So, Socrates contributed a lot to dialectic, as he sought for definitions.

b) Moreover, Socrates is said to be the first who used inductive reasoning. By “induction” Aristotle means here the generalization from specific examples. It seems dubious that Socrates invented inductive arguments and general

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66 Annas (1976), 154
67 Alex., 28.24-29.21
68 Ross (1924) v. 2, 422
definitions. What Aristotle may mean here is that Socrates was the first who recognized their importance and used them as a method of systematic investigation⁶⁹.

1078b31-1079a2: Aristotle distinguishes Socrates from Plato and his followers by the fact that, while Socrates did not regard universals and definitions as existing separately from the things to which they refer, Plato thought of them as separate entities.

a) Aristotle seems to imply that the Platonists, trying to overcome some difficulties in studying physical things, introduced Forms, which in fact aggravated the problems.

b) One first instance of the undesirable implications the Theory of Forms led to, is that the Forms turn out to be too many in number. According to Aristotle, the introduction of Forms has to be conceived as introduction of even more things than those which already exist in an attempt to study and understand the things which already exist.

c) In addition, Aristotle remarks, the Forms turn out to be more in number than individual beings. Crubellier⁷⁰ reasonably thinks that what Aristotle really means here by the term καθ᾽ ἔκαστα is not the individual sensible things, but the different classes of things, i.e. the class of physical objects, the class of qualities, the class of actions etc. According to Platonists, Forms are more in number than types of things and not more in number than separate physical things. So, it would be more appropriate to say that each Form applies to one class of individuals than say that each Form applies to one individual. Finally, Forms can be proven to be more in number than types of sensible things, if the One-over-Many argument applies not only to general terms for types of physical objects, but also to general terms for qualities of physical objects.

1079a2-4: Aristotle introduces here the One-over-Many argument:

⁶⁹ Crubellier (1994), 164
⁷⁰ Crubellier (1994), 167-168
a) According to this argument, there is one Form, or else one general term, applying to many individual things.

b) Aristotle notes that there is one name over many things not only in the case of individuals existing around us and being experienced through the senses, but also in the case of eternal things.71

c) So, the One-over-Many argument seems to have an unrestricted application.

1079a4-7: The greater number of Forms in comparison to the number of classes of things is not the only problem Aristotle raises as far as the Theory of Forms is concerned.

a) One second difficulty he refers to is that there is no way of demonstrating the existence of the Forms. For, we cannot say that all the Forms can function as premises beginning from which we can be led to deductions.

b) Moreover, Aristotle says, there turn out to be Forms of things of which the Academy does not think there are Forms. Even though the Platonists believe there are Forms only of objects, they end up in saying that there are Forms of qualities, like beauty, equality, justice etc.

1079a7-11: Given that the scientific definitions and arguments include numerous things which -though not physical objects- constitute objects of scientific study:

a) there have to be Forms of all the things which are objects of scientific study.

b) Furthermore, there have to be Forms of negations. If, for example, there is a Form of the Just, there must be a Form for the Not Just as well.

c) What is more, there have to be Forms of things that have perished, since, though they are not around us any more, we can have their image in our mind.

d) So, we are led to the assumption that the Platonists thought that there is a Form of anything which is conceivable by our mind.

71 Crubellier [(1994), 169] notes that the eternal things are not necessarily sensible eternal things. They could also be mathematical objects of which there are also Forms.
1079a11-13: By talking about “relations” Aristotle means those attributes (like double, more, less etc.) which require a complement in relation to which they are understood. According to Platonists, there must be Forms of these attributes, too.

a) In his attempt to prove that the Platonists contradict themselves in believing that there are Forms only of physical objects and saying that there also are Forms of things which are not physical objects, Aristotle says that Platonists state there are Forms of qualities.

b) Furthermore, Aristotle makes reference to the Third-Man argument. This argument is based on the fact that, since the original “man” (or the genus “man”) and the particular “man” (or the species “man”) share the same name, there has to be one third “man” nature predicated of both the original and the particular “man”. By participating in this “third man” Form, the generic “man” and the specific “man” share the same name, i.e. “man”. According to this argument, any Form seems to presuppose the existence of at least another Form which functions as a link between the Form and its participant. However, as already said, this leads to the existence of more Forms than species of physical objects.

1079a14-19: Aristotle says that argumentation in favor of the Forms leads its adherents to totally absurd assumptions:

a) On such assumption is that number is first and not the dyad. Number is said to be first instead of dyad, in the sense that number is a more general term in comparison to dyad.

b) If there is Form of number it has to be conceived as more general than the Form of dyad, because the notion of “number” includes the notions of “dyad”, “triad” etc. As genus, the Form of number is prior to dyad, at least in definition.

c) Therefore, number comes first, which is absurd, since number is just a quality of dyad and dyad is the individual object that is undeniably prior in being.

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72 Syr., 111.33-37
1079a23-30: Aristotle’s argument here goes like this:

a) Suppose that an individual object “x” participates in the Form of Double. The Form of Double is eternal. It follows that “x” participates in something that is eternal.

b) However, being eternal is an incidental property of the Forms and not an essential aspect of their nature.

c) So, “x” participates in the Form of Double and also participates accidentally in an accidental property of this Form, i.e. the characteristic of being eternal.

d) Such kind of “accidental participation” is not like participation in a Form, for the accident of a Form is not itself a Form.

e) Therefore, the accidental participation can be thought of either as not participation at all or as participation in a different sense.\(^{73}\)

If it is assumed by Platonists that there are Forms of incidental properties, it seems that saying that something which participates in the Form of Double also participates in the Form of Eternality will have negative consequences. First of all, there has to be one Form for every genus of physical objects, so that every physical object participates in the Form of it. If the same physical object participates in more than one Form, then there is not only one Form for every being. Therefore, if one being participates in many Forms, which Form among all these should be characterized as the being “itself”?

We should also take into account here that, if a physical object participates in Forms of its incidental properties, these Forms of incidental properties are accidentally participated in by the objects. Given that the qualities of things are incidental, a thing can be said to participate in the Form of its quality only accidentally. But Forms, Aristotle says, cannot be participated in incidentally.

So, it is necessary that there are Forms only of substances. In case there are Forms of accidents, they will not be Forms of the same kind with Forms of substances, because substance terms and non-substance terms cannot pick out Forms of the same

\(^{73}\) Annas (1976), 157
kind\textsuperscript{74} and also because Forms of substances and Forms of accidents cannot be in the same sense participated in by physical objects.

Furthermore, Forms of substances have to be substances as well. For, how could they be participated in by something not having the same nature with them?

\textbf{1079a31-1079b3:}

a) Aristotle wonders how saying that many perishable or imperishable dyads bear the same name differs from saying that the original “Two” and a particular “Two” bear the same name. The name has to be the same according to the One-over-Many argument. And it must have the same sense when applied to both the original and the particular object.

b) A dilemma is introduced here: either Forms and objects which participate in them have the same form or they do not. If they have the same form, then they will have something else in common apart from their name. If they do not have the same form, then they will not be of the same type, but they will just be homonyms, just as the person Callias and a wooden model of Callias can both bear the name “man”. The person Callias and the wooden statue are not of the same type and, as a result there is no community between them. However they can both bear the name “man” owing to their bearing man’s outline.

There has to be community between a Form and its participant, for the Theory of Forms to keep its interest. A Platonist would probably be expected to agree with the first view, i.e. that Forms and participants have the same form, or else that the same name applies in the same sense both to a Form and to the object participating in it. If the same name applies in the same sense both to the Form and to the particular physical object participating in it, then both Forms and particulars are substances, for, as Annas\textsuperscript{75} notes, “a difference of categorical application of a word amounts to a difference of sense”. Therefore, it seems that according to Aristotle the Platonist who thought that Forms and objects share common names in the same sense, would be

\textsuperscript{74} Annas (1976), 158
\textsuperscript{75} Annas (1976), 159
obliged to agree that Forms are substances and that there are Forms only of substances.

1079b3-1079b11: Aristotle has already said that a physical object and its Form bear the same name.

a) Provided that an object and its Form bear the same name, it is also reasonable that they share the same definition. For example, the Form of circle and a particular circle can both be defined as “a plane figure whose boundary consists of points equidistant from a fixed point that is the center”.

b) Moreover, in the case of the definition of the Form of circle, we need to say something more, i.e. that the Form of circle is “the Form of a plane figure whose boundary consists of points equidistant from a fixed point that is the center”. Thus, the definition of the circle is integral part of the definition of the Form of circle.

c) Nevertheless, according to the Theory of Forms, in the case of the definition of the Form of circle, there have to be Forms also of “plane” and of “center”, just as, in the case of the definition of the Form of man, there have to be Forms of “animal” and “two-leggedness”, for they constitute essential aspects of the thing that is defined. In general, any part of a substance’s definition has its Form.

d) So, in the case of the Form of circle, it is a matter of question to which part of its definition should the phrase “Form of” be added, given that all the elements of the definition are themselves Forms. It turns out that the phrase “Form of” would seem to be a redundant addition.

e) One further difficulty is that, due to the fact that “Form” is part of definition, it is necessary that “Form” can also be defined. According to Aristotle, “Form” has to be thought of as some sort of generic nature. This means that there has to be an original “Form” present in all particular Forms as their genus, or else a Form of “Form”, which is totally absurd.
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| 1079b12-15 | Πάντων δὲ μάλιστα διαπορήσεις αὐτὲς τὰ ἐιδῆ ἢ τοῖς ἀιδίοις τῶν 
| | αἰσθητῶν ἢ τοῖς γιγνομένοις καὶ τοῖς 
| | φθειρομένοις· οὕτω γὰρ κινήσεως ἐστιν οὕτω 
| | μεταβολῆς οὐδεμᾶς αἰτία αὐτοῖς. | But the question somebody might above all examine is what on 
| | earth the Forms contribute to sensible things, either to those that 
| | are eternal or to those which are generated and decayed; for the 
| | Forms are neither causes of any motion nor causes of any change 
| | in them. |
| 1079b15-18 | Ἀλλὰ μὴν οὕτω πρὸς τὴν ἑπιστήμην οὐθέν 
| | βοηθεῖ τὴν τῶν ἄλλων (οὔδὲ γὰρ οὕσια ἕκεινα 
| | τούτων· ἐν τούτοις γὰρ ἢν), οὕτῳ εἰς τὸ 
| | εἶναι, μὴ ἐνυπάρχοντά γε τοῖς μετέχουσιν. | But again the Forms help in no way towards the knowledge of 
| | other things (for those are not the substance of these things; 
| | otherwise they would have been in them), or towards their 
| | existence, since they are not in the individuals which participate 
| | in them; |
| 1079b18-20 | οὗτω μὲν γὰρ ἰσῶς αἴτια δόξειν ἄν εἶναι ἃς 
| | τὸ λευκὸν μειμιγμένον τῷ λευκῷ. | in that case, the Forms might be thought perhaps to be causes, 
| | since white is cause of the white thing, in which it has been 
| | mixed. |
| 1079b20-23 | Ἀλλ᾽ ἀλλ᾽ οὗτος μὲν ὁ λόγος λίπαν εὐκίνητος, 
| | ὃν Ἀναξαγόρας μὲν πρῶτος Ἐυδοξὸς δὲ 
| | Nevertheless, this argument, which was stated firstly by 
| | Anaxagoras and later by Eudoxus in his discussion of difficulties |
| 1079b23-26 | ἀλλὰ μὴν οὐδὲ ἐκ τῶν εἰδῶν ἐστὶ τάλλα κατ᾿ οὕθενα τρόπον τῶν εἰωθότων λέγεσθαι. Τὸ δὲ λέγειν παραδείγματα εἶναι καὶ μετέχειν αὐτῶν τὰ ἄλλα κενολογεῖν ἐστὶ καὶ μεταφορὰς λέγειν ποιητικὰς. | But it is also impossible to say in any ordinary manner of speaking that the other things come from the Forms. And to say that the Forms are patterns and that other things participate in them is to use empty phrases and poetical metaphors. |
| 1079b26-30 | Τί γάρ ἐστι τὸ ἐργαζόμενον πρὸς τὰς ἱδέας ἀποβλέπον; Ἐνδέχεται τε καὶ εἶναι καὶ γίγνεσθαι ὅτι τὰς ἱδέας ἀποβλέπουν καὶ μὴ ἐκακώμενοι, ἄστε καὶ ὄντος Σωκράτους καὶ μὴ ὄντος γένοιτ' ἂν οίος Σωκράτης· ὁμοίως δὲ δήλω ὦτι κἂν εἰ ἦν ὁ Σωκράτης ἀδιός. | For what is that works looking to the Forms? And it is possible for anything both to be and to come into being without being imitated from something else, so that whether Socrates exists or not, a man like Socrates might come to be; and evidently the case might be the same, even if Socrates was eternal. |
| 1079b31-33 | Ἐσται τε πλείω παραδείγματα τοῦ αὐτοῦ, ἡστε καὶ εἰδη, οἶον τοῦ ἀνθρώπου τὸ ζῶν καὶ τὸ δίπου, ἀμα δὲ καὶ αὐτοανθρώπου. | And there will be several patterns and therefore several Forms of the same thing, for example Animal and Two-Footed will be patterns and thus Forms of human being, and so too will the original Human Being. |
Furthermore, the Forms will be patterns not only of sensible things, but also of things-themselves, as for example the Form of Genus is the pattern of Forms that are genera; therefore, the same thing will be pattern and copy.

Moreover, it might seem impossible that the substance and that whose substance it is should exist separately; thus how could the Forms, being substances of things, exist apart from the things?

But in the *Phaedo* it is thus said, namely that the Forms are causes both of being and of coming-into-being. However, assuming that the Forms exist, things are still not generated unless that which will set the whole process in motion exists, and also many other things come into being, such as a house and a ring, of which Platonists do not claim there are any Forms. Therefore, it is clearly possible that those things of which they say that there are Forms may exist and be generated due to the same kind of causes as those of the things we have just talked about, and not because of the Forms.

But as regards the Forms we can draw many conclusions similar
Σύμφωνα και διά λογικώτερων και ακριβεστέρων λόγων ἐστι πολλά συναγαγείν ὁμοία τοῖς θεωρημένοις.

| τρόπον και διὰ λογικώτερων καὶ ακριβεστέρων λόγων ἐστὶ πολλὰ συναγαγεῖν ὁμοία τοῖς τεθεωρημένοις. |
| to those which we have already considered, both by the foregoing methods and through more logical and more accurate arguments. |
OVERVIEW:

The material of M.5 logically fits into Book N, because it constitutes part of the third inquiry Aristotle has promised to deal with in the first chapter of Book M. The reason why Aristotle deals with the Theory of Forms here may be that he intends to deal with the Theory of Forms together with the previous arguments concerning number, following the order which is kept in A.9 of Metaphysics.

- One important issue that needs to be discussed is the way in which Forms contribute to perceptible things, given that: (a) they are not causes of any change in them, (b) they do not contribute anything to the knowledge of them and (c) they contribute nothing to the being of them.

- Moreover, it cannot be true that the Forms are participated in by the other things, for: (a) there is nothing that that works looking to the Forms, (b) it is possible for anything to exist without being the imitation of an original being, (c) there will be many different patterns and thus many different Forms of the same thing and (d) the same thing will turn out to be both pattern and copy.

- Furthermore, it seems impossible that the substance and the thing of which the substance is should exist separately.

- Finally, even if it is true that the Forms exist, (a) things cannot come into being without a moving cause and (b) many other things (e.g. houses, rings) come into being, even though the Platonists do not claim there are any Forms of them.
COMMENTARY:

1079b12-15: This section announces Aristotle’s intention to examine the contribution of Forms to sensible things:

a) Two different categories of sensible things are referred to here: the first category is that of eternal sensible things, while the second one is that of generable and corruptible sensible things.

b) It goes without saying for Aristotle that Forms cannot contribute anything to any category of sensible things. The reason is that Forms cannot contribute to any motion or change which sensible things undergo.

c) Forms could be said to contribute something to sensible things which are generated and decay, provided that they contribute something to their generable and corruptible nature.

d) However, how could Forms play such a role without somehow possessing the feature of generability and corruptibility? It really seems that Forms are of a radically different nature when compared to sensible things.

e) As a result, Forms seem unable to contribute anything to sensible things, at least insofar as the sensible things are movable and changeable.

1079b15-18: This section puts forward the view that Forms could be said to contribute to eternal sensible things, provided that they contribute to their knowledge as well as to their being.

It is a matter of question here if, for Aristotle, contribution to being is somehow linked with contribution to knowledge. Should the idea that like is known by like be accepted by Aristotle, then contribution to the being of something would be a precondition for contribution to the knowledge of it. Therefore, it seems that the cause of knowing something and the cause of being are in fact two different ways to describe one and the same cause76.

76 Crubellier (1994), 193
So, it could be assumed that, in order to acquire knowledge of eternal things by means of Forms, there has to be some link between the essence of Forms and the essence of eternals. There has to be some compatibility with each other, if Forms are to be thought of as means for understanding eternal things. Forms cannot contribute to knowing things, since they do not contribute to the essence of things and exist in separation of them\textsuperscript{77}. Moreover they cannot contribute to their existence, for they are not inherent in their participants. It is noteworthy that Aristotle stresses twice that Forms do not exist inside eternal things. This indicates that existing inside things is a precondition of Forms’ ability to contribute anything to sensible things. Thus, Forms cannot be thought of as first principles of eternal things, since they cannot be the nature or reality of things, given that they do not exist inside them.

\textit{1079b18-20:} The use of the adverb \textit{oūtω} (i.e. in this case) at the beginning of this section refers to the previously denied case that Forms exist inside sensible things. If Forms existed inside sensible things, they could be approached not only as a means of acquiring knowledge of sensible things, but also as causes of them. For example, when the white color is mixed with something, it gives this thing the property of whiteness, which means that white color is a cause of the whiteness of the thing in which it exists.

\textit{1079b20-23:} The argument to which this section refers is the one mentioned above, i.e. that Forms could be thought of as causes of sensible things, if they existed in them. Aristotle recognizes that the absurdities of this theory lead to the conclusion that the theory is refutable. Therefore, Forms cannot be thought of as existing inside sensible things.

\textit{1079b23-26:} In this section Aristotle states that sensible things are not composed of Forms in any ordinary sense and that sensible things are not participants in Forms.

\textsuperscript{77} Crubellier (1994), 195
Aristotle here refers to Forms using the term “paradigm”. A “paradigm” is a model, or pattern or guide for change. Thus, if Forms were paradigms of things, then they should be able to function as models for their change. Aristotle says in *Nicomachean Ethics* (cf. 1079a3-19) that Forms seem too remote and abstract to serve as paradigms.

Furthermore, even if Forms were models of things’ change, this would not entail that Forms would also be the factors that initiate the change of things. It has already been said that Forms are not causes of the change of things.

1079b26-30: This section presents the first of the five arguments Aristotle uses in order to support his view that Forms are not in fact necessary to serve as “paradigms” of things. According to this first argument:

a) any sensible thing coming into existence does not need to be copied from any model. In other words, it is possible to be or become anything without imitating an original pattern.

b) This means that a sensible particular thing can resemble something else, without having a relation of model-copy to it.

c) So, even if one were to grant that sensible things are similar to what Platonists introduce as Forms, it would not follow that their existence results from their imitating Forms.

1079b31-33: This section presents Aristotle’s second argument:

a) The absurdity of the Theory of Forms which is mentioned here is that there have to be several Forms of the same thing. For example, Socrates must participate in the Form of Man, in the Form of Animal and in the Form of Two-Footed.

b) The existence of many Forms of the same thing could be interpreted as existence of many patterns for the same thing, which is absurd, since it is

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78 Annas (1976), 160
reasonable that each thing imitates only one original thing serving as the paradigm of it.

c) Moreover, it seems that, in the case of Socrates, the Form of Man, which is the original being of the particular human being bearing the name “Socrates”, can be analyzed into the Form of Animal and the Form of Two-Footed.

d) If the Forms of Animal and Two-Footed were thought of as paradigms of Socrates in the same way the Form of Man is considered to be, then they would turn out to be redundant paradigms, given that Animal and Two-Footed are in fact parts of Man.

e) So, what would be the necessity of the Form of Animal and the Form of Two-Footed, provided that there is the Form of Man that includes the aspects “animal” and “two-footed”?

One further difficulty arising from the conviction that one Form can be analyzed into many other Forms is that the Form of a being, for instance the Form of Man, and the Forms of the parts of the being’s definition, for example the Forms of Two-Footed and of Animal, can exist separately from each other, given that Forms are distinct and by themselves existing entities. Therefore, it turns out that a Form is able to exist separately from its features, which is absurd.79

1079b33-35: This section presents Aristotle’s third argument:

a) The third absurdity the Theory of Forms leads to, according to Aristotle, is that the same thing will be pattern and copy.

b) If there is genus Form of species Form, then the species Form will be both the paradigm of sensible things and the copy of another Form.

c) However, it is inappropriate to attribute two opposite roles to the same thing, since it is the role of a paradigm which is attributed to Forms and not that of a copy.

79 Crubellier (1994), 201
d) Furthermore, if, apart from the Forms of sensible things, there are also Forms of the Forms of sensible things, there will turn out to be an accumulation of Forms which is impractical and thus unreasonable.

1079b35- 1080a2: This section presents Aristotle's fourth argument:

a) By this argument Aristotle explains why Forms could not be thought of as substances of things.

b) If they were the substances of sensible things, they should exist inside them.

c) However, according to the Theory of Forms, Forms exist in separation from sensible things.

d) As a result, Forms cannot be the substances of things.  

1080a2-8: This section presents Aristotle's fifth argument:

a) In the *Phaedo* (99e-105c), Forms are said to be causes both of being and of coming into being.

b) However, according to Aristotle, Forms are inadequate as a means of explaining the generation and existence of things. This point has already been made at the beginning of this chapter (1079b12-18). In other words, Forms cannot be considered as causes of things’ existence. Things cannot come into being without a moving cause, but Forms are not the factors leading to the generation of things.

c) So, it could be assumed that sensible things have come into being owing to the causes which have already been mentioned by Aristotle in A.2, 983a26-32 of *Metaphysics*, and not due to the Forms.

d) Furthermore there are many things, like houses and rings, which exist, even though Platonists say there are no Forms of them. This fact supports the argument that Forms are not the causes of things’ coming into existence.

e) So, Forms are proven to be both insufficient and unnecessary as causes.

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80 The point made here reminds of the point made previously, in M.5, 1079b15-17.
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<tr>
<td>1080a12-14</td>
<td>Ἐπεὶ δὲ διώρισται περὶ τούτων, καλῶς ἔχει πάλιν θεωρῆσαι τὰ περὶ τοὺς ἀριθμοὺς συμβαίνοντα τοῖς λέγουσιν οὐσίας αὐτοὺς εἶναι χαριστάς καὶ τῶν ὀντὸν αἴτιας πρώτας.</td>
<td>But now that we have drawn distinctions about the Forms, it is well to consider again the problems connected with numbers which confront those who say that numbers are separate substances and first causes of existing beings.</td>
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<tr>
<td>1080a15-21</td>
<td>Ἀνάγκη δ’, εἴπερ ἐστὶν ὁ ἀριθμὸς φύσις τις καὶ μὴ ἄλλη τίς ἔστιν αὐτοῦ ἡ οὐσία ἄλλα τούτ’ αὐτό, ὡσπερ φασί τινες, ήτοι εἶναι τὸ μὲν πρῶτόν τι αὐτοῦ τὸ δ’ ἐχόμενον, ἐτερον ὁν τῷ εἴδει ἑκατον, καὶ τοῦτο ἦ ἐπὶ τῶν μονάδων εὐθὺς ὑπάρχει καὶ ἐστὶν ἀσύμβλητος ὁποια腹部 μονάς ὁποιαὶ μονάδι, ἡ εὐθὺς ἐφεξῆς πάσας καὶ συμβληται ὁποιαὶ ὁποιαὶ ὁποιαὶ, οἶον λέγουσιν εἶναι τὸν μαθηματικὸν ἀριθμὸν·</td>
<td>But, if the number is some kind of nature and its being is nothing other than number itself, as some people maintain, it follows that some numbers are primary, while some are derivative, since each number is different in kind. And this is either directly true of the units, and any unit whatsoever is non-combinable with any other unit, or they are all directly successive and all units are combinable with each and every other unit, as people claim the mathematical number to be.</td>
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<tr>
<td>1080a21-30</td>
<td>ἐν γὰρ τῷ μαθηματικῷ οὐδὲν διαφέρει οὐδεμία μονάς ἐτέρα ἐτέρας· ἢ τὰς μὲν συμβλητὰς τὰς</td>
<td>For in the mathematical number no unit differs in any way from any other unit. Or some units are combinable, while</td>
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δὲ μὴ (οἷον εἰ ἐστὶ μετὰ τὸ ἐν πρώτῃ ἡ δυάς, ἔπειτα ἡ τριάς καὶ οὗτῳ δὴ ὁ ἄλλος ἀριθμός, εἰς δὲ συμβληται αἱ ἐν ἑκάστῳ ἀριθμῷ μονάδες, οἷον αἱ ἐν τῇ δυάδι τῇ πρώτῇ αὐταῖς, καὶ αἱ ἐν τῇ τριάδι τῇ πρώτῃ αὐταῖς, καὶ οὗτῳ δὴ ἐπὶ τῶν ἄλλων ἀριθμῶν: αἱ δ᾽ ἐν τῇ δυάδι αὐτῇ πρὸς τὰς ἐν τῇ τριάδι αὐτῇ ἀσύμβλητοι, ὁμοίως δὲ καὶ ἐπὶ τῶν ἄλλων τῶν ἑφεξῆς ἀριθμῶν.

Hence, while the mathematical number is counted thus, i.e. what comes after the one is counted as two, i.e. consisting of another one added to the former one, and three is thought of as consisting of another one besides these two, and the other numbers similarly; but the Form number is counted thus, i.e. after one comes a distinct two that does not include the original one, and the triad is counted not including the dyad, and the rest of the numbers similarly. Or, one sort of number must be such as we first described, and another such as the
mathematicians maintain, and the sort of number we have last named must be a third sort of number.

Furthermore, these numbers must exist either in separation from things or not in separation, but in sensible things, not however in the manner in which we first considered, but in the sense that sensible things are composed of the numbers existing in them; or else, some of them must be separate, while others not, or all of them must be separate. So, these are necessarily the only ways in which numbers can exist.

Now, almost all those who say that one is the beginning and substance and element of all things, and that number is (derived) from it and something else, have, each, described number in one of these ways, only no one has said that all units are non-combinable. And this happened reasonably enough; for the numbers cannot exist in some other way apart from those already mentioned.

Some thinkers hold that both kinds of number exist - the first
ἀριθμούς, τὸν μὲν ἔχοντα τὸ πρότερον καὶ ὑστερόν τὰς ἑ диάς, τὸν δὲ μαθηματικὸν παρὰ τὰς ἑ διάς καὶ τὰ αἰσθήτα, καὶ χωριστοὺς ἀμφοτέρους τῶν αἰσθητῶν· οἱ δὲ τὸν μαθηματικὸν μόνον ἀριθμόν εἶναι, τὸν πρῶτον τῶν ὄντων, κεχωρισμένον τῶν αἰσθητῶν. Καὶ οἱ Πυθαγόρειοι δ᾽ ἔνα, τὸν μαθηματικὸν, πλὴν οὐ κεχωρισμένον ἀλλὰ ἐκ τούτου τὰς αἰσθητὰς οὐσίας συνεστάναι φασίν· τὸν γὰρ ὅλον οὐρανὸν κατασκευάζουσιν εἰς ἀριθμῶν, πλὴν οὐ μονάδων, ἀλλὰ τὰς μονάδας ὑπολαμβάνουσιν ἐχειν μέγεθος· ὅπως δὲ τὸ πρῶτον ἐν συνεστή ἔχον μέγεθος, ἀπορείν ἐνίκασιν. Ἀλλος δὲ τις τὸν πρῶτον ἀριθμὸν τὸν τῶν εἰδῶν ἕνα εἶναι, ἐνιοί δὲ καὶ τὸν μαθηματικὸν τὸν αὐτὸν τούτον εἶναι. Ὡμοίως δὲ καὶ περὶ τὰ μήκη καὶ περὶ τὰ ἐπίπεδα καὶ περὶ τὰ στερεά. Οἱ μὲν γὰρ ἕτερα τὰ of which involves priority and posteriority being identical with Forms, while the mathematical number is distinct from the Forms and the sensible things- and also that both kinds of number exist in separation from the sensible things; however, some other thinkers state that the mathematical number alone exists, which comes first among the beings and exists in separation from the sensible things. And Pythagoreans also believe that there is one kind of number, that is the mathematical; only they say that it is not separate, but sensible substances have been made up of it; for they construct the whole universe out of numbers, however not of numbers made up of units; for they assume that the units have spatial magnitude; but they seem to be at a loss as for how the first unit was formed having spatial magnitude. Another thinker assumes that the primary kind of number, that is Form number, is the only that exists, while some others assume that the mathematical number is identical with this. The same assumption applies in the case of lines, planes
Some thinkers distinguish the objects of mathematics from those which come after the Forms; and among those treating this subject otherwise, some thinkers treat the mathematical objects and speak about them in a mathematical way, namely those who neither regard the Forms as numbers nor hold that the Forms exist, and others discuss the mathematical objects, however not in a mathematical way; for they say that neither every spatial magnitude is divisible into extended magnitudes, nor does the dyad come from any two units. But all thinkers who hold that unity is an element and principle of existing things consider numbers to be composed of units, except the Pythagoreans; but they consider numbers as having spatial magnitude, as it has been previously stated.

So, it is clear from this in how many ways it is possible to speak of numbers, and that all the ways have been mentioned; all of them are impossible, but perhaps some more so than others.
OVERVIEW:

In M.6, Aristotle discusses and points out weaknesses of theories that already exist about numbers and their units as regards their combinability or not.

- If number is an entity the essence of which is just number, then (a) either the units of number are non-combinable with each other or (b) they are combinable, as in the case of mathematical number or (c) some units are combinable, while some others not or (d) there are all three aforementioned sorts of number.

- Moreover, numbers must be (a) either all separate from things or (b) all inside things or (c) some of them separate while some other non-separate from things.

- Almost all those who treat the One as an element and a substance have adopted one or another of the aforementioned views. However, no one has ever said that all units of numbers are non-combinable with each other.

- Now, (a) there are some thinkers who claim that both Form number and mathematical number exist and also that they are separate from perceptible things. (b) There are some others who believe in the existence of only mathematical number. A group of them treats it as separate from perceptible things, while Pythagoreans treat it as component of the sensible objects. (c) Another opinion is that only Form number exists and some of the thinkers who have adopted it identify Form number with mathematical number.

- Similarly, there are various views as far as the objects of geometry are concerned.
COMMENTARY:

1080a12-14: This section announces Aristotle’s intention to move to numbers, with the aim of undermining the arguments of thinkers who treat them as separate substances and causes of things.

1080a15-21: In this section, a distinction begins to be made by Aristotle between the mathematical number and the Form number.

a) If number is an entity whose essence is number, then there are two possibilities: (1) Either there is an order of priority among different numbers and also a difference in species both between the numbers and between their units, which are therefore incomparable, or (2) all units are comparable.

b) The first possibility corresponds to the case of Form number, while the second possibility corresponds to the case of mathematical number.

c) In the case of mathematical number all units, even of different numbers, are comparable, while in the case of Form number all units are specifically different and thus incomparable.

The first possibility that is described by Aristotle presupposes an order of priority ("τὸ μὲν πρωτὸν τι αὐτὸν τὸ δ’ ἑκάστον" (1080a17)) among numbers that are also different in species ("ἐτερον ὑπὸ τῶν εἰδεὶ ἑκατον" (1080a17-18)). As far as the order of priority is concerned, it is a matter of question whether it refers to the succession of numbers ([1], 2, 3 … etc.) or it indicates an ontological hierarchy among numbers, according to which the existence of the numbers that are posterior in the numerical series depends on the existence of the numbers that are prior and, as a result, the elimination of the number that comes first in the series entails the elimination of the number that comes next81. Given that numbers are also said here to differ in kind, it seems more probable that the priority among them is of ontological nature.

The usage of the term ἀσυμβλῆτος in Aristotle shows that the word must mean “incomparable”. The verb συμβάλλω has the meaning of “bringing together”,

81 Crubellier (1994), 221-223
“uniting” or “combining”. The units of ideal numbers are ἀσύμβλητοι in the sense that they cannot be brought together or united, so that they enter into arithmetical relations with one another. Things are comparable only if they belong to the same kind. Thus, as Ross\(^\text{82}\) says, ἀσύμβλητος is practically equivalent to ἕτερ ὑπὲρ ἐδει (cf. 1080a17).

1080a21-30: This section explains extensively the difference between the mathematical and the Form number as far as their units are concerned.

a) As far as the Form number is concerned, units of the same number are combinable, but units of different numbers are non-combinable.

b) As far as the mathematical number is concerned, units of all numbers are combinable.

c) So, taking into account the two possibilities mentioned in M.6, 1080a15-21, it seems that in fact Aristotle divides numbers into three categories as regards the combinability or non-combinability of their units: (a) Numbers whose units are specifically different and non-combinable in any way, (b) numbers whose units are all combinable and (c) numbers whose units are combinable, but only within the same number. The first class cannot be identified with any sort of number, for in this class the units are not combinable even in the same number. The second class can be identified with that of mathematical numbers, while the third class can be identified with that of Form numbers.

1080a30-37: In this section, Aristotle explains the way in which mathematical numbers and Form numbers are formed, and summarizes his views about the division of numbers into three classes:\(^\text{83}\)

a) (i) Numbers specifically different, whose units are non-combinable, (ii) mathematical numbers, whose units are all combinable and (iii) “that named

\(^\text{82}\) Ross (1924), v. 2, 427
\(^\text{83}\) Annas (1976), 163
"last", i.e. Form numbers, whose units are combinable within the same number.

b) In the case of mathematical numbers, each number comes from the previous one after the addition of one unit, which is combinable to the units of the previous number. So, in the case of mathematical numbers, all units are of the same kind.

c) On the contrary, as far as Form numbers are concerned, although they constitute an ordered series, they do not come from each other in the way mathematical numbers do, i.e. by addition of one more unit to the units of the number that is previous in the series. The reason for this fact is that Form numbers are different in species. Thus, in the case of Form numbers, units are of different kinds.

d) The third kind of numbers mentioned by Aristotle is presented as “that named last”. It has already been said that there are three possibilities concerning units, i.e. (i) to be non-combinable, (ii) to be combinable and (iii) to be combinable, but only within the same number. It has also been said that the second possibility concerns mathematical numbers, while the third one concerns Form numbers. So, the question is what sort of numbers the first possibility concerns.

It is probable that the first kind of number Aristotle makes reference to is the Form number. However, the Form numbers in this case are not to be considered as made up of units. We had better think of each Form number as one and indivisible thing, or else as one unit. In this way it makes sense that any unit is non-combinable with any other unit. For, how would it be possible for any number to be composed of units that are non-combinable with each other? The third kind of number that is mentioned is again the Form number, but here it is thought of as consisting of units that are combinable within the same number and non-combinable among different numbers. So, it could be assumed that Aristotle has in fact two different categories of number in mind, i.e. Form number and mathematical number, but three different ways in which units can be thought of, i.e. as non-combinable at all, as combinable or
as partly combinable. Or else, two ways in which Form numbers can be conceptualized with respect to their units.

1080a37-1080b5: This section refers to the ways in which “these” numbers, i.e. the numbers talked about beforehand, namely the Form numbers and the mathematical numbers, can be thought of.

a) They can be said to exist either in separation or inside things. And, in case they are inherent in things, either all things or just some of them can be said to have numbers inside them as components. In other words, either all things or just some of them can be said to be composed of numbers.

b) Aristotle is probably speaking about Pythagoreans here, who - though looking like Platonists in a way, since they believe in the reality of numbers - are unlike Platonists in another way, for they do not take numbers to be separate and distinct entities.

1080b6-11: This section deals with One as beginning and element of number. Aristotle points out here that no thinker who considered One as beginning and element has ever claimed that all units are non-combinable. It is also pointed out that something else is also needed for number to be made up, apart from unit. This could be the ἄπειρον in the case of Pythagoreans and the “great and small” or Indefinite Dyad in the case of the Platonists. It is noteworthy that the second principle is not presented with a name here due to the fact that many different names have been ascribed to it by different thinkers.84

1080b11-1080b33: In the beginning of this section Aristotle speaks of “both” kinds of number. This passage reinforces our claim that there are in fact two different kinds of number, i.e. mathematical number and Form number. Aristotle presents three different views concerning numbers.

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84 Crubellier (1994), 231
a) First, some thinkers hold that both Form number and mathematical number exists, of which Form number is the same as Form, while mathematical number is distinct from Forms. According to them, both kinds of numbers exist in separation from sensible things. This group of thinkers probably consists of Plato and his followers.

b) The second view is expressed by those thinkers who believe only in the existence of mathematical number. By ὦ ἄ θ Aristotle in all probability means Speusippus. The Pythagoreans, like Speusippus, believed only in mathematical number. However, they differed from Speusippus in treating mathematical number not in separation from sensible things, but as a constituent part of them. Moreover, they did not think of number as made up of units, since units have spatial magnitude. In other words, they had not distinguished arithmetic from geometry. This is not peculiar, for Pythagoreans had not reached the notion of non-corporeal reality. It is a matter of question, though, how numbers came to be in such a case. The inability of the Pythagoreans to answer this question could result from their inability to reach the position of a fully developed atomism, according to which the ultimate and eternal fact is an indefinite plurality of atoms or monads. Numbers are described by the Pythagoreans as “indivisible magnitudes” and things are identified with numbers composed of indivisible magnitudes or monads.

c) The third view is expressed by thinkers who, like Xenocrates, think that mathematical number is identified with ideal number.

d) Thinkers who hold that only Form number exists are never named by Aristotle. It could be a Platonist, but we do not know anything further.

The same classification applies to geometry, since all theories of number have analogues for geometry. The objects of geometry are presented as “τὰ μετὰ τὰς ἱδέας” in the sense that the Forms of numbers are prior to quasi-Forms of lines, planes and solids.

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85 Ross (1924), v. 2, 429
86 Ross (1924), v. 2, 428
So, first of all we must inquire whether the units are combinable or non-combinable and, if they are non-combinable, in which of the two ways we have distinguished. For it is possible that any unit is non-combinable with any other unit, and it is also possible that the units existing in the original Two are non-combinable with the units existing in the original Three, and in this way the units existing in each primary number are non-combinable with one another.

However, if all the units are combinable and undifferentiated, then we get the mathematical number and this alone and the Forms cannot be the numbers. For, what sort of number will the original human being or the original animal or any other Form be? For, there is one Form of each being, for example one Form of the original human being and another Form of the original animal; but the similar and undifferentiated numbers
are infinitely many, so that this particular three is not the original human being any more than any other three. But if the Forms are not numbers, then it is impossible for them to exist at all. For, from what principles will the Forms come? For, the number comes from the One and the Indefinite Dyad, and the principles and the elements are said to be principles and elements of the number, and the Forms cannot be placed either before or after the numbers.

| 1081a17-21 | Εἰ δ᾽ ἀσύμβλητοι αἱ μονάδες, καὶ οὕτως ἀσύμβλητοι ὠστε ἠτίουν ἠτινιοῦν, οὕτε τὸν μαθηματικὸν ἐνδέχεται εἶναι τούτον τὸν ἀριθμὸν (ὅ μὲν γὰρ μαθηματικῶς ἐξ ἀδιαφόρων, καὶ τὰ δεικτομένα κατ᾽ αὐτοῦ ὡς ἐπὶ τοιούτου ἀρμόττει) οὕτε τὸν τῶν εἰδών. But, if the units are non-combinable, and if they are non-combinable in the sense that any one is non-combinable with any other, then this number can be neither the mathematical number (for the mathematical number consists of undifferentiated units, and the facts demonstrated of it suit it as such) nor the Form number.

| 1081a21-29 | Οὐ γὰρ ἐστι τῆς, δύας πρώτη ἐκ τοῦ ἑνὸς καὶ τῆς ἀριστότου δύας, ἐπειτα οἱ ἔξις ἄριθμοι, ὡς λέγεται δύας, τριάς, τετράς ἀμα γὰρ αἱ ἐν τῇ δύαδι τῇ πρώτῃ μονάδες γεννάνται, εἰτε ἄστερ οἱ πρώτος εἰτο βι ἐξ For the two will not be the first number generated from the One and the Indefinite Dyad and then the other numbers in succession, as we say, two, three, four; this is because the units existing in the first two are generated simultaneously, either as
| 1081a29-35 | Ἐτι ἐπειδή ἦστι πρῶτον μὲν αὐτὸ τὸ ἐν, ἐπειτὰ τῶν ἄλλων ἦστι τι πρῶτον ἐν δεύτερον δὲ μετ᾽ ἐκείνον, καὶ πάλιν τρίτον τὸ δεύτερον μὲν μετὰ τὸ δεύτερον τρίτον δὲ μετὰ τὸ πρῶτον ἐν, ὡστε πρῶτεραι ἢν εἶναι αἱ μονάδες ἢ οἱ ἀριθμοὶ ἢ ἔν πέρενται, οἷον ἐν τῇ δύαδι τρίτῃ μονάς ἦστι πρὶν τὰ τρία εἶναι, καὶ ἐν τῇ τριάδι τετάρτῃ καὶ ἡ πέμπτη πρὶν τοὺς ἀριθμοὺς τούτους. | Furthermore, since the original one is first, and then there is a one which is first among the others and second after the original one, and again a third one, that is second after the second one and third after the first one; therefore, the units should be prior to the numbers after which they are called, for instance there will be one third unit in the two before the existence of the three, and there will be one fourth unit in the three and also the fifth unit before these numbers exist. |
| 1081a35-1081b10 | Οὐδέσις μὲν οὖν τὸν τρόπον τούτον ἐφήκεν αὐτῶν τὰς μονάδας ἀσυμβλήτους, ἦστι δὲ κατὰ μὲν τὰς ἑκείνους ἀρχὰς εὐλόγων καὶ οὕτως, κατὰ μέντοι τὴν ἀλήθειαν | But nobody has said that the units of these numbers are non-combinable in this way, but even this is reasonable according to their principles, although according to the truth it is |
And it is impossible for the first unit to be prior and posterior provided that there is a first unit or a first one, and similarly in the case of twos, if there is a first two; for it is reasonable and necessary that after the first there should be a second and, if a second, a third and so on with the others in succession. But, it is impossible to say both things simultaneously, namely that a unit is first after the one and that another unit is second, and also that there is a two that is first. However, they make a first unit or a first one, but not a second and a third, and also they make a first two, but not a second and a third.

And it is clear that, if all units are non-combinable, it is not possible for the original two and the original three to be two or three and so on with the rest of numbers. For, both in case the units are undifferentiated and in case they differ from each other, it is necessary that the numbers are counted by addition, the two, for example, by adding another one to one, and the three by adding another one to the two, and the four likewise;
δυσὶ προστεθέντος, καὶ τὴν τετράδα ὡς αὐτός· τούτων δὲ ὅντων ἀδύνατον τὴν γένεσιν εἶναι τῶν ἀριθμῶν, ὡς γεννώσιν ἐκ τῆς δυάδος καὶ τοῦ ἕνος. Μόνιμον γὰρ γίγνεται ἢ δυάς τῆς τριάδος καὶ αὐτὴ τῆς τετράδος, τὸν αὐτὸν δὲ τρόπον συμβαίνει καὶ ἕπι τῶν ἐχομένων.

but, if this is so, it is impossible that the number is generated as they generate it, from the Dyad and the One; for the two becomes part of the three and the three becomes part of the four; and this happens in the same way to the succeeding numbers.

1081b20-37 Αλλ᾽ ἐκ τῆς δυάδος τῆς πρώτης καὶ τῆς ἀδύνατου δυάδος ἐγένετο ἢ τετράς, δύο δυάδες παρ᾽ αὐτὴν τὴν δυάδα· ἐι δὲ μὴ, μόνιμον ἦσται αὐτή ἢ δυάς, ἔτερα δὲ προσέσται μία δυάς. Καὶ ἢ δυάς ἦσται ἐκ τοῦ ἕνος αὐτοῦ καὶ ἄλλου ἕνος. Εἰ δὲ τούτῳ, οὐχ οἷον τ᾽ ἐγένετο τὸ ἔτερον στοιχεῖον δυάδα ἀδύνατον· μονάδα γὰρ μίαν γεννά ἄλλ᾽ ὡς δυάδα ἀδύνατον. Έτι παρ᾽ αὐτὴν τὴν τριάδα καὶ αὐτὴν τὴν δυάδα πῶς ἐσονται ἄλλαι τριάδες καὶ δυάδες; Καὶ τίνα τρόπον ἐκ προτέρων μονάδων καὶ ὕστερων σύγκειται; Πάντα γὰρ ταῦτ᾽ ἄποτά ἐστι καὶ πλασματῶδη, καὶ ἀδύνατον εἶναι πρώτην δυάδα, εἰτ ἀυτὴν τριάδα. Ανάγκη δ᾽, ἐπείπερ ἦσται τὸ ἐν καὶ ἢ ἀδύνατος δυάς στοιχεία. Εἰ δ᾽

However, from the first two and from the Indefinite Dyad there came the four, namely two twos existing besides the original Two; otherwise, the original Two will be part of it and one more two will be added. And the two will consist of the original One and of another one. But, if so, it is not possible for the other element to be the Indefinite Dyad; for it generates one unit, but not a definite two. Again, how will there be other Threes and Twos besides the original number Three and the original number Two? And in what way are they composed out of prior and posterior units? For all these things are absurd and fictitious, and it is impossible for a first Two and then for an original Three to exist. However, it is necessary for them to exist, if the One and the Indefinite Dyad are to be the elements.
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|---|
| ἀδύνατα τὰ συμβαίνοντα, καὶ τὰς ἀρχὰς εἶναι ταύτας ἀδύνατον. Εἰ μὲν οὐν διάφοροι αἱ μονάδες ὀποιαιοῦν ὀποιαιούν, ταῦτα καὶ τουαθ’ ἐτερα συμβαίνει εξ ἀνάγκης. Εἰ δ’ αἱ μὲν ἐν ἀλλω διάφοροι αἱ δ’ ἐν τῷ αὐτῷ ἀριθμῷ ἀδιάφοροι ἀλλήλαις μόναι, καὶ οὕτως οὐθὲν ἐλάττω συμβαίνει τὰ δυσχερὴ. |
| However, if the consequences are impossible, then it is also impossible for these to be the principles. So, if any units are different from any other units, these and other similar consequences necessarily follow. And, if the units existing in different numbers are of different kind, while the units existing in the same number are alone undifferentiated from one another, even so the difficulties which follow are no less. |
| 1082a1-5 Οἷον γὰρ ἐν τῇ δεκάδι αὐτῇ ἐνείσι δέκα μονάδες, σύγκειται δὲ καὶ ἐκ τούτων καὶ ἐκ δύο πεντάδων ἢ δεκάς. Επεὶ δ’ οὐχ ὁ τυχὼν ἀριθμὸς αὐτῇ ἢ δεκάς οὐδὲ σύγκειται ἐκ τῶν τυχοῦσών πεντάδων, ἦσσερ οὐδὲ μονάδων, ἀνάγκη διαφέρειν τὰς μονάδας τὰς ἐν τῇ δεκάδι ταύτῃ. |
| For example, in the original Ten there are ten units and the ten is composed both of these and of two fives. And, since the original Ten is not any chance number, and it is not composed of any chance fives, just as it is not composed of any chance units, it is necessary that the units which exist in this ten are different. |
| 1082a5-11 Ἄν γὰρ μὴ διαφέρωσιν, οὐδ’ αἱ πεντάδες διοίσουσιν εξ ὑπὲρ ἢ δεκάς· ἐπεὶ δὲ διαφέρουσι, καὶ αἱ μονάδες διοίσουσιν. Εἰ δὲ διαφέρουσι, πότερον οὐκ ἐνέσσονται πεντάδες ἀλλαὶ ἀλλὰ μὸνον αὐτὰι αἱ δύο, ἢ ἐσσονται; Εἴτε δὲ μὴ ἐνέσσονται, ἄτοπον· εἴτ’ ἐνέσσονται, ποία |
| For, if they are not different, then the fives of which the ten consists will not differ either; but, since they are different, the units will also be different. However, if they are different, will there not be any other fives in the ten, but only these two, or will there be others? And, if there are not other fives, this is
アルバム καὶ ἀνάγκη γε μὴ ἐκ τῶν τυχουσῶν δυάδων· ἡ γὰρ ἀόριστος δυάς, ὡς φασι, λαβόντα τὴν ἀφισμένην δυάδα δύο δυάδας ἐποίησεν· τοῦ γὰρ ληφθέντος ἦν δυσποίος.

But it is indeed necessary that the four does not consist of any chance twos; for the Indefinite Dyad, as they say, made two twos after taking the definite two; for it duplicated what was received by it.

Besides, how can the two be a certain nature besides the two units, and the three besides the three units? Either by participation, just as the white human being exists besides the white and the human being (for it participates in them), or when the one is some difference of the other, just as the human being exists besides the animal and the two-footed.

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87 1082a17: Codices Ab and E read “θατέρου θατέρον” and consider it to be “vulgate” text. Christ claims that “θατέρου θατέρον” is more suitable in comparison to “θατέρου θατέρον”, given that μεθέξει is not a verbal type of which θατέρον is the subject and θατέρον is the object. Nevertheless, he chooses to correct the passage by totally removing these two words. I would agree with Christ, because in the example that follows “white” and “human being” cannot be said to participate the one into the other. Moreover, as regards the locution θατέρον (subjective genitive) θατέρον (objective genitive), it would indeed be odd and look as a dittography.
Furthermore, some things are one by contact, while some others are one by mixture, and some others are one by position; none of which relations can apply to the units of which the two and the three consists; but, just as the two human beings are not any one thing apart from both of them, so it must be with the units.

And they will not be different due to this, namely that they are indivisible; for the points are indivisible, but yet two of them do not make up anything apart from them.

Moreover, we must not fail to notice this, namely that it follows that there are prior twos and posterior twos, and similarly with the other numbers. For, let the twos which exist inside the four are contemporaneous; yet, they are prior to the twos which exist inside the eight, and they generated the fours which exist inside the original Eight, just as the two has generated them. Therefore, if the first two is a Form, then these will also be Forms of some sort.

And the same argument also applies to the units; for the units
1082b2-8 'Όλως δὲ τὸ ποιεῖν τὰς μονάδας διαφόρους ὑποσχόν ἀτοπον καὶ πλασματάδες (λέγω δὲ πλασματάδες τὸ πρὸς ὑπόθεσιν βεβιασμένον): οὕτε γὰρ κατὰ τὸ ποσὸν οὕτε κατὰ τὸ ποιον ὀρῷμεν διαφέρουσαν μονάδα μονάδος, ἀνάγκη τε ἢ ἴσον ἢ ἄνισον εἶναι ἄριθμόν, πάντα μὲν ἀλλὰ μάλιστα τὸν μοναδικόν, ὡστ' εἰ μήτε πλεῖων μήτ' ἐλάττων, ἰσος: τὰ δὲ ἰσα καὶ ἀλλως ἄδιάφορα ταύτα ὑπολαμβάνομεν ἐν τοῖς ἀριθμοῖς.

In general, making the units different from each other in any way is totally absurd and fictitious; and by fictitious I mean that which is forced in order to fit one’s hypothesis. For we do not see any unit differing from any other unit either in quantity or in quality, and it is necessary that all the numbers are either equal or unequal - all the numbers but especially the number which consists of abstract units - so that, if a number is neither greater nor less than another, it is equal to it; but those which are equal and totally undifferentiated we conceive them as identical in the sphere of the numbers.

1082b9-16 Εἰ δὲ μή, οὐδ' αἱ ἐν αὐτῇ τῇ δεκάδι δυάδες ἄδιάφοροι ἔσονται ἵσαι οὕσαι τίνα γὰρ αἰτίαν ἔξει λέγειν ὅ

Otherwise, the twos existing inside the ten itself will not be indifferent either, although they are equal; for, what reason
φάσκων ἀδιαφόρους εἶναι; Ἐτι εἰ ἀπασα μονὰς καὶ μονὰς ἄλλη δύο, ἢ ἐκ τῆς δυάδος αὐτῆς μονὰς καὶ ἢ ἐκ τῆς τριάδος αὐτῆς δυάς ἔσται ἐκ διαφερουσῶν τε, καὶ πότερον προτέρα τῆς τριάδος ἢ ὑστέρας Μᾶλλον γὰρ ἐσκε προτέραν ἀναγκαίον εἶναι: ἢ μὲν γὰρ ἅμα τῇ τριάδι ἢ δ’ ἅμα τῇ δυάδι τῶν μονάδων.

**1082b16-29**

Καὶ ἡμεῖς μὲν ὑπολαμβάνομεν ὅλας ἕν καὶ ἕν, καὶ ἕαν ἢ ἴσα ἢ ἄνισα, δύο εἶναι, οἷον τὸ ἀγαθόν καὶ τὸ κακόν, καὶ ἀνθρώποι καὶ ἱπποί· οἳ δ’ οὔτως λέγοντες οὐδὲ τὰς μονάδας. Εἰτε δὲ μὴ ἐστὶ πλείων ἁρμίμως ὁ τῆς τριάδος αὐτῆς ἢ ὁ τῆς δυάδος, θαυμαστόν· εἰτε ἐστὶν πλείων, δήλων ὅτι καὶ ἱσος ἑννηκοτῇ δυάδι, ὡστε οὔτως ἀδιαφόρος αὐτῇ τῇ δυάδι. Ἀλλ’ οὐκ ἐνδέχεται, εἰ πρώτος τις ἐστιν ἁρμίμως καὶ δεύτερος. Οὐδὲ ἐσονται αἱ ἱδέαι ἁρμίμωι. Τοῦτο μὲν γὰρ αὐτὸ ὀρθῶς λέγουσιν οἱ διαφόροις τὰς μονάδας ἀξιοῦντες εἶναι, εἰτε ιδέαι will anyone be able to allege who claims that they are indifferent? Furthermore, if any unit plus another unit makes two, the unit coming from the original Two and another unit coming from the original Three will make a two, which is composed of different units and will it be prior to the three or posterior to it? For it rather seems that it must be prior; for one of the units is simultaneous with the three, while the other unit is simultaneous with the two.

And we assume generally that one and one, whether they are equal or unequal, make two, for instance the good and the bad or the human being and the horse; but those who hold these views say that not even the two units make two. And it is surprising if the number of the original Three is not greater than that of the original Two; and if it is greater, then it is clear that there is a number in it equal to the two, so that this number is indifferent from the original Two. But it is not possible, if there is a first and a second number of some sort. Nor will the Forms be numbers. Those who claim that the units
must be different are right on this particular point, if there are to be Forms, just as has already been stated; for the Form is unique. But, if the units are undifferentiated, the twos and the threes will also be undifferentiated. Therefore, they have to say that when we count in this way, namely one, two, we do not add to the already existing number;

For, if we do, neither will the number be generated from the Indefinite Dyad, nor can a number be a Form; for one Form will exist in another Form, and all the Forms will be parts of one Form. Therefore, as far as their assumption is concerned, they speak rightly, but in general they are wrong; for their theory is very destructive, because they will admit that this point presents some difficulty, whether, when we count and say one, two, three, we count by addition or by enumerating portions. But we do both ways; therefore, it is absurd to make this difference draw on so great a difference of being.
OVERVIEW:

In M.7, Aristotle discusses the difficulties arising from the theories about combinability or non-combinability of the units that comprise numbers.

- Aristotle firstly points out that it is necessary to examine whether the units are combinable or non-combinable and, if they are non-combinable in which sense they are so.
- If all the units are combinable and not different, then we get only the mathematical number and the Forms cannot be numbers. However, if the Forms are not numbers, it is impossible for them to exist.
- And, if all the units are non-combinable, then (a) the number we get can neither be the mathematical number nor the Form number. Moreover, (b) the units will be prior to the numbers after which they are called. For example, the second unit in number two will be prior to number three.
- If there is a first unit, then there will be prior and posterior units and similarly there will be prior and posterior twos. However, although these thinkers recognize the existence of a first unit and a first two, they do not recognize a second or a third.
- Furthermore, (c) if all the units are non-combinable, it is not possible for the original two, the original three etc. to exist. For, it is necessary that the numbers are generated by addition. Number two, for example, is part of number three, which is generated after the addition of a unit to number two and number three is part of number four, which is generated after the addition of a unit to number three. However, if this is so, it is impossible that the number is generated, as they claim, from the Indefinite Dyad and the One. For, the Indefinite Dyad duplicates the One and produces one unit. It cannot produce a definite two.
(d) It is also a matter of question how is it possible for other twos to exist besides number two and how can they be composed of prior and posterior units.

If the units in different numbers are different from each other and thus non-combinable, but the units in the same number are not different from each other and thus combinable, equal difficulties follow:

(a) Firstly, the original Ten is composed of two fives. And, since the original Ten is not an ordinary number, and it cannot thus be composed of two ordinary fives, it is necessary that the units in the one five are different from the units in the other five. However, if the units of the one five are different from the units in the other five, then it follows that the fives existing in the original Ten are different from each other. Nevertheless, if there are two different sorts of five in Ten, what sort of ten will be composed of them?

Secondly, (b) how can number two exist as a distinct nature apart from the two units that constitute it, and similarly the three apart from the three units of which it is composed?

Furthermore, (c) some things are one by contact, while some others are one by mixture, and some others are one by position. However, none of the aforementioned ways of being one thing can apply to the units of which the number two and the number three consists.

Moreover, (d) there are prior and posterior twos, threes etc. For, even though the twos which exist inside number four are contemporaneous, they are prior to the twos which exist inside number eight. It is also noteworthy that, since the first two is a Form, then these twos will also have to be Forms of some sort.

In general, (e) saying that there is a difference of any sort between units is totally absurd and fictitious. For, units do not differ from each other either in quantity or in quality. It is also necessary that the numbers which are equal to each other and identical with each other.
In addition, (f) if any unit and another unit make two, then the unit coming from the original Two and another unit coming from the original Three will also make two. This two will be composed of units that are different in kind and it is a matter of question whether it will be prior to number three or posterior to it.

(g) The original number Three must be greater than that of the original number Two. If it is greater, then it contains a number that is equal to number Two. The number that exists inside Three is equal to and has no difference from Two. However, this is impossible.

Nevertheless, if this is so, then (h) the Forms will not be numbers. Those who claim that the units are different from each other are right, provided that there are Forms. For, each Form is unique. But, if the units are identical, the twos and the threes will be identical, too.
1080b37-1081a5: In this section, Aristotle uses the notion of “combinability”. The problem of combinability or non-combinability of the units has already been introduced at 1080a18-30. The difference between the previous section and this one is that Aristotle here focuses his account on the Form numbers.

In the previous section, Aristotle has already alluded to three options as regards the (non-)combinability of the units. The first alternative, which is also referred to in this section, is that all the units are combinable with one another. The second alternative, which is also referred to here, is that the units are all non-combinable with one another. The third alternative is that the units are combinable with one another, but only within the same number.

According to Annas\textsuperscript{88}, none of the options mentioned above is compatible with the belief in Form number. The reason for this difficulty is that the Platonists treated Form numbers as entities being unique of their kind. As a result, the units that exist inside each Form number should also, Platonically speaking, be unique of their kind. Given that the units are different, they cannot be said to be combinable. So, the first alternative is not possible.

As far as the second and the third alternatives are concerned, some contradictions are produced from these. If all the units are different, and thus non-combinable, then how is it possible for them to be added to one another in order to construct numbers? And, if the units are undifferentiated, but only within the same number, then how is it possible to claim that the number which is greater comes from the number which is smaller? It becomes clear that the preconditions for the application of two principal rules concerning numbers, namely the rule that each number is composed of units and the rule that each number is generated after the addition of one unit to the immediately previous number, cannot be fulfilled. In the following sections, Aristotle elaborates on his arguments in favor of this point.

\textsuperscript{88} Annas (1976), 166-167
In this section, Aristotle criticizes the view that all the units are combinable:

a) If all the units were to be combinable -which presupposes their being specifically undifferentiated- then any two units would be able to make the number two.

b) In such a case there would be many twos and, as a consequence, the original two would no longer be unique.

c) This repeatability of the numbers would cause serious difficulties, since, in case they were treated as Form numbers, there would be more than one identical Form (actually, infinitely many) for one kind of beings.

d) However, the Form number is unique.

e) So, on this option, the Forms would not be numbers, for each number would not be unique.

f) But, if the Forms were not numbers, then their existence would not be able to be explained. For, the first principles, according to the Platonists, are the One and the Indefinite Dyad, which are elements of numerical nature and which generate the numbers.

g) Therefore, if the Forms are not numbers, it is not possible for them to exist at all, because no suitable principles have been introduced for them.

Aristotle passes now on to the view according to which even the units in one number are non-combinable with one another. As Ross\textsuperscript{89} claims, given that this theory had no supporters, the space that Aristotle devotes to its treatment is disproportionate.

In this section, Aristotle criticizes the option that all the units in one number are non-combinable with one another. If this is the case, then neither the existence of the mathematical number, nor the existence of the Form number is possible. The reason why the mathematical number cannot exist, if all the units are

\textsuperscript{89} Ross (1924), v. 2, 435
different and thus non-combinable, is evident, given that all the theorems concerning the mathematical number are founded on its consisting of undifferentiated units as well as on its being repeatable. The reason why the Form number cannot exist either is that the units are non-combinable, which entails that any unit cannot be placed next to another such (non-combinable) unit in order to form with it a number. Therefore, number cannot be fitted together out of such units.

1081a21-1081b35: ARISTOTLE’S FOUR ARGUMENTS:

1081a21-29: In the first argument,

a) Aristotle makes reference to the fact that the natural numbers create a series in which number two is the direct successor of one, number three is the direct successor of two and so on.

b) Then, Aristotle makes reference to the generation of the units in the first Two, saying that they are generated simultaneously. Annas claims that there are two different ways of conceiving this simultaneous generation of the two units in the first Two. We can say either that the units are generated simultaneously with each other or that they are generated simultaneously with that number to which they belong. The second alternative is more likely, since it better suits a theory of numbers treating them as consisting of non-combinable units. Non-combinability implies inaddibility. As a result, the units cannot easily be conceived as put together at the same time to create a number. It seems more reasonable to say that the units are produced at the same time with the number.

c) However, if the units are thought of as produced at the same time with the number, then it is not possible for them to be created one by one before the creation of the numbers. So, if we admit that the units are simultaneously generated, we also have to admit that there cannot be a sequence of prior and posterior numbers.

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90 Annas (1976), 168
d) And, given that the reason for the simultaneous generation is that the units are non-combinable with one another, we infer that the non-combinability of the units excludes the possibility of numbers' being prior and posterior to one another. For, due to the non-combinability of the units, they are inadddible and therefore they cannot be said to come before or after one another.

It is a matter of discussion whether number can really be conceived as the product of a process of composition of units, on account of the different ontological status that is ascribed to units by the Platonists. Moreover, if number is considered as the whole that is composed of essentially different constituent parts to which it is posterior, one further question that arises is how is it possible, for instance, that in the original Dyad there is an element that is prior to the first Two\textsuperscript{91}.

1081a29-35: In the second argument, Aristotle refers again to the existence of a sequence of numbers:

a) If the units are not generated at the same time with their numbers, they are generated one after another in a series.

b) Further, Aristotle introduces a general principle not confined to the units, namely that when something is prior and something else is posterior, then the compound of these two will be prior to its part which is posterior to the other part and posterior to its part which is prior to the other part.

c) This leads to the absurd result that the number sequence will go as follows: 1, first unit in 2, 2, second unit in 2 and so on. This result is paradoxical, since it violates the number series, thus implying the falsity of the non-combinability of the units.

1081a35-1081b10: In his third argument, it becomes clear that, even if the series of numbers and the series of units are kept distinct, there is one further problem:

a) According to Aristotle, it is reasonable to say that after the One, which is the first principle, a first unit comes and then a second unit comes and so on.

\textsuperscript{91} Crubellier (1994), 249
b) The two is composed of these two units which come after the One. So, there are two units before we have the two, three units before we have the three and so on.

c) However, given that we start with the One, then the two will correspond to the unit coming after the One, or else to the first unit after the One, and the three will correspond to the unit coming after the first unit after the One, or else to the second unit coming after the One.

d) Therefore, from the perspective of the units, the two will be placed in the second position after the One, while, from the perspective of the numbers, the two will be placed in the first position after the One.

e) This difference reflects a conflict with the concept of the Form number\(^2\). For, according to the Theory of Forms, the original Two cannot come after two units following the original One.

f) Besides, the Form numbers, as already said, cannot be thought of as consisting of units of the same kind. This fact also makes the existence of many original Ones or many original Twos impossible.

g) Here Aristotle points out that if there is a series of different and distinct units (or ones) there will also be a series of different and distinct twos and threes and so on; it seems that the concept of ontological hierarchy among successive numbers is an essential aspect of the Theory of Forms that is based on the doctrine of non-combinable units\(^3\). However, the existence of numerous twos, threes etc. contradicts the assumption of the uniqueness of the Form numbers.

h) This is the reason why the Platonists, who are the holders of the Theory of Forms, make a first one, but not a second and a third, and similarly they make a first two, but not a second and a third. For, one cannot have it both ways and say both that the two comes first after the one and that there is a first and a second unit after the One. In other words, one has to choose between the two and a unit as the successor of the One.

\(^2\)Annas (1976), 169
\(^3\)Crubellier (1994), 251
1081b10-20: In the fourth argument, Aristotle claims that,

a) whether the units are differentiated or not, the numbers are created by adding on one unit every time.

b) So, each number is differentiated from the previous one in the series due to having one extra unit.

c) In this sense, the two, for instance, will be part of the three, since the two units existing in the number two are also needed to count to the number three. According to this additive model of generation, the numbers cannot be non-combinable.

d) However, the Platonists state that the Form numbers are produced by doubling, from the One and the Indefinite Dyad. According to this duplicative model of generation, the Form numbers seem to be distinct, unique and non-combinable, for each Form number is not made up of another number.

e) Nevertheless, the smaller numbers constitute part of the greater ones.

f) Therefore, the mode of number generation which reasonably prevails, according to Aristotle, is that of adding one more unit to the already existing number. Aristotle seems to appeal to common sense here.

1081b20-35: In this section, Aristotle repeats the idea that, if the Form numbers are generated by the addition of one extra unit to the already existing number, then it is impossible for them to be produced from the One and the Indefinite Dyad, which are the first principles, according to the Platonists. If numbers were generated by addition, then the one number would be part of the other number and their units would be all combinable. On the contrary, if numbers were generated by duplication from the One and the Indefinite Dyad, then number Four, for instance, would be the result of the duplication of the original number Two by the Indefinite Dyad. This process of duplication would lead to the existence of four distinct twos, i.e. the Indefinite Dyad, the original Two and the two Twos resulting from the duplication of
the original Two by the Indefinite Dyad to constitute the original Four. All these distinct Twos would be non-combinable and they would be linked with each other only through a relation of ontological priority and posteriority.

Furthermore, if the Form numbers were produced through a process of duplication from the One and the Indefinite Dyad, then each number would be a special creation, differing in kind from all the others. Nevertheless, the existence of many distinct twos and threes as well as the existence of units that would be prior and units that would be posterior could not be reasonably explained, especially if it was ascribed to a process of duplication performed by only two specific principles, i.e. the One and the Indefinite Dyad.

1081b35-1083a20: In this long section, Aristotle deals with Plato’s actual theory of numbers, namely that the units are combinable, but only within the same number. Aristotle tries to support his point using the following eight arguments:

1081b35-1082a14: First argument:
Aristotle has already shown that it is necessary for the Form numbers to be generated by addition, whether their units are thought of as undifferentiated or as differentiated. In this section, he tries to show that the units of numbers are necessarily undifferentiated. In order to prove that the units are undifferentiated, Aristotle uses the example of the original number Ten.

a) The number Ten is composed of ten units which have to differ from each other, given that the Ten also differs in kind from any other Form number.

b) These ten units, out of which the number Ten is made, can also be considered as two fives.

c) The question is whether these two fives are the only fives that can exist or there are others, too. Moreover, it is a matter of question what sort of fives is included in the original Ten. There are two possibilities: either (a) the two fives that belong to the original Ten do not differ from each other (which is explicitly negated by Aristotle at 1082a7: “ἐπεὶ δὲ διαφέρονσι”) or (b) the two
fives are different from each other provided that the units that exist in each five are different from the units that comprise the other five.\textsuperscript{94}

d) However, there are many ways of dividing ten given and differentiated units into two fives. If you take, say, three units from the first five and two units from the second five, you will get a new five that will be different from the original two\textsuperscript{95}. If there are more than these two particular fives, they can produce more than one ten. So, in such a case, the original Ten will not be the unique ten.

e) One further problem that arises is that the Platonists, who believe in the existence of distinct and unique Form numbers, also claim, according to Aristotle, that the Indefinite Dyad, after receiving the definite two, produces two twos and thus number Four formed. However, somebody has to be inconsistent to stand for the unique Form numbers (e.g. number Ten), but also to accept the double appearance of other numbers (e.g. number Two).

1082a15-26: Second argument:

In this section, Aristotle wonders how any Form number can be a unified entity, or else how the units of any number can form a unity, using the example of number Two analogously to the example of the definition of “human being”:

a) Just as any other number, number Two is a collection of units.

b) A collection of units cannot easily be thought of as an entity that exists separately from the units that comprise it. If number Two could be considered as such a distinct unified entity, then the relation of this entity to the units that comprise it would be either accidental -just as the white human being is an entity that participates in the accidental properties of “white” and “human being”- or essential -just as “animal” and “two-footed” are necessarily involved in the definition of “human being”.

c) However, the units comprising a number cannot be related to number in the conceptual way in which the terms “white”, “animal” and “two-footed” are

\textsuperscript{94} Crubellier (1994), 263
\textsuperscript{95} Ross (1924), v. 2, 437
related to human being. For, the terms “white”, “animal” and “two-footed” do not belong all to the same logical or ontological level, since the one term is logically or ontologically prior or posterior to the other and also there is no sense in which all these terms could be numerically thought of as “one” thing.66

d) Furthermore, it is a matter of question in what way the units of any number can form a unity. Is it by contact or by mixture or by position? This cannot happen in any of the aforementioned ways of physical or geometrical combination, just as two distinct human beings cannot produce one unified entity that exists over and above them.

1082a26-1082b1: Third argument:

Aristotle has already formulated two arguments in favor of the idea that the units out of which the numbers are made cannot be different from each other. The first one is the argument of the non-existence of unique numbers, while the second one is the argument of the non-existence of unified numbers.

In the third argument, which is presented in this section, Aristotle depends on his first argument, according to which some numbers make up and thus produce other numbers, to show one further absurdity the Platonists are reasonably led to:

a) It turns out that many Twos exist, some of which will be prior, while some others will be posterior. This will also be true of the other numbers.

b) If we accept that the numbers are Forms and also that some numbers produce other numbers, then it has to be said that some Forms produce other Forms.

c) And, given that the units existing inside numbers must reasonably behave like the numbers which they make up and thus produce other numbers, they will be Forms too.

d) Therefore, it turns out that the Forms will be composed of other Forms.

66 Crubellier (1994), 265-266
e) However, if the Forms are made up of other Forms, then the things that participate in these Forms must also be thought of as composed of other things.

f) For instance, the human being will be partly composed of the animal, if it is accepted that the Form of Human is composed of the Form of Animal.

Crubellier claims it is not necessary that the units have all the properties of numbers—and thus should be considered as Forms—just because they behave like numbers (i.e. they duplicate so as to produce numbers). I think that Aristotle treats units as necessarily having the properties of numbers composed of them on the basis of his general train of thought, according to which the principle or constituent part of something must definitely be of the same nature with its product.

1082b2-11: Fourth argument:

Aristotle returns to his main attempt to show that the units of the numbers cannot be differentiated. Firstly, we should take into account that the concepts of equality and inequality hold true in case of any number, no matter what is the sort of the objects that are counted. However, the two aforementioned concepts are valid especially in the case of monadic numbers, i.e. numbers that are made up of units. Aristotle claims that we cannot see any difference among the units either in quantity or in quality and he intends to show that the difference (or inequality) between numbers cannot entail a difference (or inequality) between the units of which they are made up. For example, the five twos of which the ten is composed cannot be said to be differentiated. For, each of them consists of two units that are identical and thus equal to each other. Given that the units from which the five twos are composed do not possess any quality that could alter the status of equality of these numbers, the five twos existing in the ten are equal to each other. So, on what grounds could they be differentiated?

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Crubellier (1994), 269
1082b11-19: Fifth argument:

Aristotle formulates this argument, trying to show that the units should not be considered as differentiated from one another:

a) Aristotle gives an example here: the number two and the number three are two different numbers.

b) If we take one unit from the number two and one unit from the number three, what will be the number made up of these two units?

c) Given that the one unit comes from the two, which is prior to the three, and the other unit comes from the three, which is posterior to the two, the number made up of them ought to come between the two and the three and it is more likely to be prior to the three.

d) However, what will be the number coming between the two and the three? There is no such number, at least in the Aristotelian sense of number.

e) Therefore, given also that most people think that two heterogeneous items make up a two, the number made up of the unit from the number two and the unit from the number three ought to be a two.

f) Nevertheless, the Platonists seem unwilling to state both that two beings differing in kind, for example a human being and a horse, can make a number two and that two non-combinable units make a number two. But, while doing so, they can give no account of why the entity made up of two things that are different in kind is not a number.

The foundation of the Platonists’ view is easy to find: two entirely different things cannot be made to make a two unless there is a common element in both, say an “E”, according to which they can be added and be counted as two “Es”. The good and the bad are two as contraries in the range of excellence as much as the cold and the hot are two as extreme contraries in the range of temperature. But a thought and a rose are not two unless they are counted as two things in the most general sense of “thing”.

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98 Ross (1924), 2, 439
**1082b19-23: Sixth argument:**

According to this argument, if the Form numbers are considered as a sequence, there cannot be a number identical with the number two that exists in the three. And this is due to the fact that a Platonist thinks of the numbers as distinct and unique, and thus he cannot treat any two numbers as equal or say that any two numbers contain an equal number. Again, like Aristotle’s third argument, this argument seems to have nothing to do with the units.

**1082b23-37: Seventh argument:**

In this seventh argument, Aristotle claims that the Forms cannot be numbers and he gives as a reason that, if the units are undifferentiated from one another, we are led to the absurd conclusion that the one Form exists inside the other. Aristotle’s argument seems to run this way:

a) Let us assume that the numbers are composed of undifferentiated units, since we have seen the difficulties of the opposite view.

b) Let us further assume that Forms are numbers. We could give two examples:
   (a) the Form of Justice is number 4 and (b) the Form of Animal is number 7.

c) It would follow that since the seven undifferentiated units of number 7 can be represented as forming two groups (of 3 units and of 4 units, respectively) the Form of Justice (= number 4) is part of the Form of Animal (= number 7).

d) And in general, if Forms are numbers and the units of those numbers are undifferentiated, any Form having less units will be part of any Form consisting of more units.

e) So, the Platonists, by saying that the numbers are unique and composed of differentiated units, are consistent to their theory about the Form numbers. However, given that the number cannot be like this, their whole theory is flagrantly false.
But first of all it is good to define which the difference between one number and another number, one unit and another unit is, if there is a difference. It is necessary that the units differ either in quantity or in quality; however, neither of these alternatives seems to be possible. Nevertheless, units *qua* number differ in quantity. If the units (*qua* units) did differ in quantity, too, then even equal numbers in quantity of units would differ from one another.

Besides, are the first units greater or smaller and the latter units increase in size, or the reverse? For all these are absurd. But they cannot differ in quality either. For no attribute can apply to them; for even in the case of numbers they say that the quality applies to them after the quantity.
Furthermore, neither from the One nor from the Two could this come to them; for the former is no quality, while the latter produces quantity; for this nature is the cause of the things’ being many. And if something else is the case, this must be said right at the beginning and the difference of the unit must be determined, and especially why it is necessary that it exists.

Otherwise, which difference do they refer to?

So, it is clear that, if the Forms are numbers, neither can all the units be combinable with each other nor can they be non-combinable with one another in either of the two ways. But in the case of numbers things are not correctly said, in the way in which some other people speak about them. But these are those who do not believe that there are the Forms either without qualification or as numbers of some sort, while they believe that the mathematical objects exist and that the numbers are primary among the existing things and that the original One is their principle. Now it is absurd that a one is a beginning of the ones, as they say, while a two is not a
Now if this is how things stand with regard to number, and only the mathematical number is supposed to exist, then the One will not be a principle (for it is necessary that the One which is of such a nature is different from the other units; and if so, then there must be some two which is first among the twos; and similarly with the other numbers in succession); however, if the One is a principle, then it is necessary that the facts about the numbers are rather as Plato used to say; that is, there is some first two and some first three, and the numbers cannot be combinable with each other. But then again, if one assumes this, we have stated that many impossibilities result. However, it is surely necessary that either the one or the other is the case, so if neither is, it is not possible for number to be separable.

It is also clear from these considerations that the third view is the worst, namely that the Ideal number and the Mathematical...
τὸν μαθηματικὸν. Άναγκη γὰρ εἰς μίαν δόξαν συμβαίνειν δύο ἀμαρτίαι: οὔτε γὰρ μαθηματικὸν ἀριθμὸν ἐνδέχεται τοῦτον εἶναι τὸν τρόπον, ἀλλὰ ἰδίας ὑποθέσεις ὑποθέμενον ἀνάγκη μηκύνειν, ὥσα τε τοῖς ἐνδέχεται τὸν ἀριθμὸν λέγουσι συμβαίνει, καὶ ταὐτα ἀναγκαῖον λέγειν.

Ο δὲ τῶν Πυθαγορείων τρόπος τῇ μὲν ἐλάττως ἔχει δυσχερείας τῶν πρῶτων εἰρημένων, τῇ δὲ ἰδίᾳ ἐτέρας. Τοῦ μὲν γὰρ μὴ χωριστὸν ποιεῖν τὸν ἀριθμὸν ἀφαιρεῖται πολλὰ τῶν ἀδυνάτων· τὸ δὲ τὰ σώματα ἐξ ἀριθμῶν εἶναι συγκείμενα, καὶ τὸν ἀριθμὸν τοῦτον εἶναι μαθηματικὸν, ἀδυνατὸν ἔστιν. Οὔτε γὰρ ἄτομα μεγέθη λέγειν ἄληθες· εἰ ἃ ἔχει τὸν τρόπον, οὐχ ἂ ν ἡ μονὰς μέγεθος ἐχουσιν· μέγεθος δὲ ἐξ ἀδυνάτων συγκείσθαι πᾶς δυνατόν; Ἀλλὰ μὴν ὁ γ᾽ ἀριθμητικὸς ἀριθμὸς μοναδικὸς ἔστιν. Εἰκόνις δὲ τὸν ἀριθμὸν τὰ ὅντα λέγουσι· τὰ γοῦν θεωρήσατα προσάπτουσι τοὺς σώμασιν ἐξ ἐκεῖνων number is the same. For two errors have to meet in the same opinion. For it is not possible for the mathematical number to exist in this way, but the holder of this view has to spin things out by making peculiar assumptions. And he also has to admit all the consequences happening to those who hold that the numbers are Forms.

1083b8-19

Ο δὲ τῶν Πυθαγορείων τρόπος τῇ μὲν ἐλάττως ἔχει δυσχερείας τῶν πρῶτων εἰρημένων, τῇ δὲ ἰδίᾳ ἐτέρας. Τοῦ μὲν γὰρ μὴ χωριστὸν ποιεῖν τὸν ἀριθμὸν ἀφαιρεῖται πολλὰ τῶν ἀδυνάτων· τὸ δὲ τὰ σώματα ἐξ ἀριθμῶν εἶναι συγκείμενα, καὶ τὸν ἀριθμὸν τοῦτον εἶναι μαθηματικὸν, ἀδυνατὸν ἔστιν. Οὔτε γὰρ ἄτομα μεγέθη λέγειν ἄληθες· εἰ ἃ ἔχει τὸν τρόπον, οὐχ ἂ ν ἡ μονὰς μέγεθος ἐχουσιν· μέγεθος δὲ ἐξ ἀδυνάτων συγκείσθαι πᾶς δυνατόν; ΑΛλὰ μὴν ὁ γ᾽ ἀριθμητικὸς ἀριθμὸς μοναδικὸς ἔστιν. Εἰκόνις δὲ τὸν ἀριθμὸν τὰ ὅντα λέγουσι· τὰ γοῦν θεωρήσατα προσάπτουσι τοὺς σώμασιν ἐξ ἐκεῖνων

But the manner of the Pythagoreans has fewer difficulties than the theories mentioned above, but from another point of view it has others of its own. For not making the number separate removes many impossible consequences; but for the bodies to be composed of numbers, and for this number to be the mathematical one, is impossible. Neither is it true to say that there are indivisible magnitudes; and, even if this is totally true, still the units at least do not have any magnitude; and how is it possible for a magnitude to be composed of indivisible things? But the arithmetical number, at any rate, is composed of units, but the Pythagoreans say that the beings are numbers; at least they apply their mathematical
1083b19-23

| ὡς ἔστων τῶν ἀριθμῶν.
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So, if it is necessary for the number to exist in some of the ways mentioned above, provided that it is a self-subsistent real thing, but none of these ways of conceiving the number is possible, it is evident that the number has no such nature as is constructed by those who make it separate.

1083b23-36

| Ἐτι πότερον ἐκάστη μονὰς ἐκ τοῦ μεγάλου καὶ μικροῦ ἰσασθέντων ἐστίν, ἢ ἡ μὲν ἐκ τοῦ μικροῦ ἡ δ᾽ ἐκ τοῦ μεγάλου; Ἔτι, ἐκατέρων ἐκάστη μονὰς ἐκ τοῦ μεγάλου καὶ μικροῦ ἰσασθέντων ἐστίν, ἢ ἡ μὲν ἐκ τοῦ μικροῦ ἡ δ᾽ ἐκ τοῦ μεγάλου; Ἐι μὲν δὴ ἐκατέρων ἐς ἐς ἐκατέρων ἐκάστη μονὰς ἐκ τοῦ μεγάλου καὶ μικροῦ ἰσασθέντων ἐστίν, ἢ ἡ μὲν ἐκ τοῦ μικροῦ ἡ δ᾽ ἐκ τοῦ μεγάλου; Ἐτι, ἐκατέρων ἐκάστη μονὰς ἐκ τοῦ μεγάλου καὶ μικροῦ ἰσασθέντων ἐστίν, ἢ ἡ μὲν ἐκ τοῦ μικροῦ ἡ δ᾽ ἐκ τοῦ μεγάλου; Ἐτι, ἐκατέρων ἐκάστη μονὰς ἐκ τοῦ μεγάλου καὶ μικροῦ ἰσασθέντων ἐστίν, ἢ ἡ μὲν ἐκ τοῦ μικροῦ ἡ δ᾽ ἐκ τοῦ μεγάλου; |

Furthermore, does each unit come from the great and the small after being equalized, or the one unit comes from the small and the other unit from the great? If the latter is true, then neither is each thing composed of all the elements, nor are the units undifferentiated; for in the one unit there is the great, while in the other unit there is the small, which is contrary by nature to the great. Moreover, what about the units in the original Three? For there is one over. But perhaps it is due to this that in the odd number they make the original one the middle unit. And, if each of the two units comes from the great and the small after their being equalized, how will the Two, which is a
(ἀναιρουμένης γάρ ἀναιρεῖται ἡ δυάς): ἰδέαν οὖν ἰδέας ἀναγκαἰον αὐτήν εἶναι, προτέραν γ᾽ οὔσαν ἰδέας, καὶ γεγονέναι προτέραν. Ἐκ τίνος οὖν; Ἡ γὰρ ἀόριστος δύας δυοποιὸς ἦν.

| 1083b36-1084a7 | single nature, come from the great and the small? Or, in what way will it differ from the unit? Besides, the unit is prior to the Two; for, when the unit destroyed, the Two is destroyed, too. So, it is necessary that the unit is the Form of a Form, since it is prior to a Form, and also it must have been generated before it. From what, then? For the Indefinite Dyad in their view produces the twos. |
| 1083b36-1084a7 | Ἐτι ἀνάγκη ἦτοι ἀπειρον τὸν ἀριθμόν εἶναι ἢ πεπερασμένον. χωριστὸν γὰρ ποιοῦσι τὸν ἀριθμόν, ὥστε οὐχ οὐδὲν τε μή οὐχὶ τούτων θάτερον ὑπάρχειν. Ὄτι μὲν τοῖνυν ἄπειρον οὐκ ἐνδέχεται, δήλον (οὔτε γὰρ περιττὸς ὁ ἀπειρός έστιν οὔτ’ ἀρτίος, ἢ δὲ γένεσις τῶν ἄριθμῶν ἢ περιττοῦ ἀριθμοῦ ἢ ἀρτίου αἰε ἐστιν· ὥδε μὲν τοῦ ἕνου εἰς τὸν ἀρτίον πίπτοντος περιττός, ὥδε δὲ τῆς μὲν ὑμᾶς ἐμπιπτούσης ὁ ἁφ’ ἕνος διπλασιαζόμενος, ὥδε δὲ τῶν περιττῶν ὁ ἀλλὸς ἀρτίος. Besides, the number must be either infinite or finite; for they make the number separate, so that it is not possible that neither of these alternatives is true. It is clear that it cannot be infinite; for the infinite number is neither odd nor even, but the generation of the numbers is always a generation of either an odd number or an even number, and in one way an odd number is generated when the one is added to an even number, while in another way the number which is generated from the one is doubled when the two is added, and in another way the other even numbers are generated when the odd numbers are added. |
Furthermore, if every Form is a Form of something, and the numbers are Forms, the infinite number will also be a Form of something, either of something among the perceptible things or of something else. However, this is not possible either in view of their assumption or in view of reason, though they regard the Forms in this way.

Nevertheless, if the number is finite, how far does it go? For in answering this question, not only the fact, but also the reason should be stated. But, if the number goes up to the ten, as some say, firstly the Forms will soon run short; for instance, if the three is the original Human being, what number will be the original Horse? Each number which goes up to the ten is a Form. So, it must be one of the numbers in this series; for these numbers are substances and Forms. But they will still run short; for the kinds of animals will exceed them.

At the same time it is clear that, if the three is the original Human being in this way, then so are the other threes; for the units existing in the same numbers are similar, and therefore

| 1084a7-10 | Έτι εί πάσα ιδέα τινός οί δέ ἀριθμοί ιδέαι και ο ἀπειρος ἐσται ιδέα τινός ή τῶν αἰσθητῶν ή ἄλλων τινός· καίτοι οὔτε κατά τὴν θέσιν ἐνδέχεται οὔτε κατὰ λόγον τάττουσι γ᾽ οὔτω τὰς ιδέας). | 158 |
| 1084a10-17 | Εἰ δὲ πεπερασμένος, μέχρι πόσου; Τοῦτο γὰρ δεῖ λέγεσθαι οὐ μόνον ὃτι ἄλλα καὶ διότι. Ἀλλὰ μὴν εἰ μέχρι τῆς δεκάδος ὁ ἀριθμός· ὀσπερ τινὲς φασιν, πρῶτον μὲν ταχὺ ἐπιλείψει τὰ εἴδη -οίον εἰ ἔστιν ἢ τριᾶς αὐτοάνθρωπος, τίς ἐσται ἀριθμός αὐτόσπος; Αὐτὸ γὰρ ἐκαστὸς ἀριθμὸς μέχρι δεκάδος· ἀνάγκη δὴ τῶν ἐν τούτως ἀριθμῶν τινά εἶναι (οὐσία γὰρ καὶ ἱδέας οὕτω); ἄλλ᾽ ὁμας ἐπιλείψει (τὰ τοῦ ζώου γὰρ εἴδη ύπερεξε). | |
| 1084a18-21 | Ἀμα δὲ δῆλον ὃτι εἰ οὔτως ἢ τριᾶς αὐτοάνθρωπος καὶ αἱ ἄλλαι τριάδες (ὁμοίαι γὰρ αἱ ἐν τοῖς αὐτοῖς ἀριθμοῖς) ὡστε ἀπειροὶ ἐσονται ἀνθρωποι, εἰ μὲν ιδέα | |
| 1084a21-29 | Καὶ εἰ μέρος ὁ ἔλαττων τοῦ μείζονος, ὁ ἐκ τῶν συμβλητῶν μονάδων τῶν ἐν τῷ αὐτῷ ἀριθμῷ, εἰ δὴ ἢ τετράς αὐτῇ ἰδέα τινὸς ἐστίν, οἰον ῥποῦ ἢ λευκοῦ, ὁ ἄνθρωπος ἐσταὶ μέρος ῥπου, εἰ δύας ὁ ἄνθρωπος. Αὐτοῦ δὲ καὶ τὸ τῆς μὲν δεκάδος εἶναι ἰδέαν ἐνδεκάδος δὲ μὴ, μηδὲ τῶν ἔχομένων ἀριθμῶν. Ἐτεὶ δὲ καὶ ἔστι καὶ γίγνεται ἕνα καὶ ὁν καὶ ἔδη οὐκ ἐστίν, ὡστε διὰ τὸ ἐκάκεινον ἔδη ἐστίν; Οὐκ ἀρα αἰτία τὰ ἔδη ἐστίν. | And if a smaller number is part of a greater number, being a number composed of units that are combinable in the same number, then if the original Four is the Form of something, for example of the horse or of the white, then the Human Being will be part of the Horse, if the Human Being is Two. It is also absurd that there is a Form of the ten, but not of the eleven, nor of the succeeding numbers. Besides, some things both exist and are generated, of which there are no Forms; so, why are there not any Forms of those, too? It follows that the Forms are not causes. |
| 1084a29-1084b2 | Ετεὶ ἀποτομὴν εἰ ὁ ἀριθμὸς ὁ μέχρι τῆς δεκάδος μᾶλλον τι ὁν καὶ ἔδος αὐτῆς τῆς δεκάδος, καίτοι τοῦ μέν οὐκ ἐστι γένεσες ὡς ἔνος, τῆς δὲ ἐστίν. Πειρώνται δ᾽ ὡς τοῦ μέχρι τῆς δεκάδος τελείου ὄντος ἀριθμοῦ. Γεννάσι | Furthermore, it is absurd that number up to ten should be more of one substantial being and more of a Form than the original Ten, although there is no generation of the former as a single thing, whereas there is of the latter. However, they try to |
make a theory on the assumption that the number series up to ten is a complete number. At least they generate the derivative things, for example the void, the proportion, the odd, and the other things of this kind, within the ten. And they assign some things to the principles, for instance change, rest, the good and the bad, but they assign other things to the numbers. This is why they say that the one is the odd; for, if the oddness depended on three, how could five be odd? Besides, they hold that the magnitudes and the other things of that sort go up to a certain quantity, for example the first line - namely that which is indivisible - then comes the two and then come these too that extend up to ten.

Besides, if the number is separate, somebody might raise the question whether the one or the three or the two is prior. And, inasmuch as the number is composite, the one is prior, while inasmuch as the universal and the Form is prior, the number is prior; for each of the units is part of the number as its matter, but the number acts as its Form.
| 1084b7-13 | Καὶ ἐστι μὲν ὡς ἢ ὀρθὴ προτέρα τῆς ὀξείας, ὅτι ἀφισταὶ καὶ τῷ λόγῳ ἐστι δ’, ὡς ἢ ὀξεία, ὅτι μέρος καὶ εἰς ταύτην διαφέρει. Ὡς μὲν δὴ ὦλη ἢ ὀξεία καὶ τὸ στοιχεῖον καὶ ἡ μονὰς πρότερον, ὡς δὲ κατὰ τὸ εἶδος καὶ τὴν ὑσίαν τὴν κατὰ τὸν λόγον ἢ ὀρθὴ καὶ τὸ ἀλλο τὸ ἐκ τῆς ὦλης καὶ τοῦ εἶδους ἐγγύτερον γάρ τοῦ εἶδους καὶ οὗ ὁ λόγος τὸ ἁμφω, γενέσει δ’ ὑστερον. | And in a sense the right angle is prior to the acute angle, since the right angle has been defined first and is prior in definition; however, in a sense the acute angle is prior, because it is a part and the right angle is divided into it. Hence, the acute angle and the element and the unit are prior as matter, while, as regards the Form and the being in the sense of the formula, the right angle and the whole made up from the matter and the form are prior; for the compound thing is nearer to the form and it is the object of formula/definition, but in generation it is later. |
| 1084b13-23 | Πῶς οὖν ἄρχη τὸ ἔν; Ὅτι οὐ διαφέτον, φασίν· ἀλλ’ ἀδιαφέτον καὶ τὸ καθόλου καὶ τὸ ἐπί μέρους καὶ τὸ στοιχεῖον. Ἀλλὰ τρόπον ἀλλον, τὸ μὲν κατὰ λόγον τὸ δὲ κατὰ χρόνον. Ποτέρως οὖν τὸ ἐν ἄρχη; Ὡσπερ γὰρ εἰρήται, καὶ ἡ ὀρθὴ τῆς ὀξείας καὶ αὐτή ἐκείνη δοκεῖ προτέρα εἶναι, καὶ ἐκατέρα μία. Αμφοτέρως δὴ ποιοῦσι τὸ ἐν ἄρχην. Ἐστι δὲ ἀδύνατον. Τὸ μὲν γὰρ ὡς εἶδος καὶ ἡ ὑσία τὸ δ’ ὡς μέρος καὶ ὡς ὦλη. Ἐστι γὰρ So, in what way is the one a principle? Because it is not divisible, they say. However, the universal and the particular and the element are also indivisible, however in a different way, namely the one in definition and the other in time. In which of the two senses is, then, the one a principle? For, as it has been said, both the right angle seems to be prior to the acute angle and the acute angle seems prior to the right angle and each of them is one. For they make the one a principle in |
both ways. However, this is impossible. For in one way the one is a principle *qua* Form and as the substance, while in another way it is *qua* part and as matter. For, in a way, each of them is one, though in truth it is one only potentially (at least if the number is a sort of unity and not like a heap, but the different numbers consist of different units, as they say), whereas in actuality each of them is not one.

<table>
<thead>
<tr>
<th>1084b23-32</th>
<th>The cause of the error they made is that at one and the same time they made their inquiry from the point of view of the mathematics and from that of the universal definitions, so that from the former point of view they regarded the one, namely their principle, as a point; for the unit is a point without position. So, just as other thinkers have also done, these ones put the existing things together from minimum parts. Therefore, the unit becomes the matter of the number and at the same time it is prior to the two; and again it is posterior to</th>
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<tr>
<td>πως ἐν ἐκάτερον -τῇ μὲν ἀληθείᾳ δυνάμει (ἐὰν γε ὁ ἀριθμὸς ἐν τι καὶ μή ὡς σωφρός ἀλλ᾽ ἐπερεψ ἐξ ἑτέρων μονάδων, ἀπὸ τῆς φασίν), ἐντελεχείᾳ δ᾽ ὡς ἔστι μονὰς ἐκάτερα.⁹⁹</td>
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⁹⁹ 1084b23: Ross chooses to correct the passage by adopting the reading “ἐντελεχεία δ᾽ ὡς ἔστι μονὰς ἐκάτερα”. Christ’s choice, i.e. “ἐντελεχεία δ᾽ ὡς ἔστι μονὰς ἐκάτερα” seems preferable to me, because Ross’s reading repeats what has already been said in 1084b20-21 (“ἔστι γὰρ πως ἐν ἐκάτερον”).
| 8 1084b33-1085a1 | ἐν καὶ οὔτας ὡς μέρος ἔλεγον. Ταύτα δ᾽ ἀμα τῷ αὐτῷ ἀδύνατον ὑπάρχειν. | the two, for the two is regarded as a whole of some sort and as a one and a form. But, since they were looking for the universal, they treated the predicated one as also being a part in this sense. Nevertheless, these two characteristics cannot belong at the same time to the same thing. | But, if the original One alone must be without position (for there is no difference between it and any other one, apart from the fact that it is a principle), and the two is divisible, while the unit not, then the unit is more similar to the original One. But, if the unit is more similar to it, then it must be more similar to the unit than to the two; so, each of the units in the two must be prior to the two. But they deny this; at any rate, they generate the two first. |
OVERVIEW:

In M.8, Aristotle examines the Platonists’ and Pythagoreans’ views as far as the One and the way of numbers’ existence are concerned.

- In the first argument of this section, Aristotle says that, if the units are different from each other, it is necessary that they differ either in quantity or in quality; however, both of these alternatives are impossible:
  - If the units did differ in quantity, then numbers which are equal in quantity of units would differ from one another.
  - However, the units cannot differ in quality either. Units have no qualities, given that quality in numbers depends on quantity.
  - If units differ from each other in some other way, then this difference and the reason why it exists ought to have been determined at the beginning by those who support it.
  - The way in which some other thinkers have spoken about numbers is equally bad. These (i.e. Speusippus) do not believe that there are Forms, but they believe that the numbers are primary entities and that the One is their principle. However, it is absurd that a One is a beginning of the ones, as they say, while a Two is not a beginning of the twos, or a Three of the threes.
  - Nevertheless, the following view (i.e. that of Xenocrates) is the worst of all, namely that the Form number and the mathematical number are the same. For there are two errors in the same opinion.
  - As regards the manner of Pythagoreans, serious difficulties arise from the treatment of bodies as being composed of mathematical numbers.
  - One further question is whether each unit comes from the Great and the Small after their being equalized, or the one unit comes from the Small and the other unit comes from the Great.
If the latter is true, then neither all the elements contribute to the generation of each thing, nor are the units identical to each other.

If the former is true, the question is firstly how will number two be a single nature that comes from the Great and the Small and secondly in what way will two be distinguished from the one.

Moreover, the number must be either infinite or finite, if we accept that it exists separately.

However, number cannot be infinite; for the infinite number is neither odd nor even. Furthermore, the generation of numbers is always a generation of either an odd or an even number, and numbers are generated either by addition or by multiplication. Also, if every Form is a Form of something, and the numbers are Forms, the infinite number will also be a Form of something, which is neither possible nor reasonable.

If number is finite, how far does the series of numbers go? If the series of numbers goes up to the ten, then the Forms will soon run short. Further, if the number three is the Form of Human being, then the other threes will also be Forms of Human Being or they will at least be human beings and thus there will be an infinite number either of Forms of Human Being or of human beings.

Moreover, if the smaller number is a part of the greater number, then, if the number four is the Form of Horse and the number two is the Form of Human Being, the Human Being will be part of the Horse.

It is also absurd that there is a Form of number ten, but not a Form of number eleven, twelve etc.

Besides, some things both exist and come into being, of which there are not Forms. However, why are there not any Forms of those things? It follows that the Forms are not causes of existing things.

So, in what way is the One a first principle? Is it a first principle “because it is not divisible”, as they say? However, the universal, the particular and
the element are all indivisible and they are first principles in different
senses.

- They make the one a principle in both ways. However, this is impossible. For in one way the One is a principle *qua* Form, while in another way it is a principle *qua* matter.

- The cause of their error is that at one and the same time they considered the One from two points of view. Firstly, from the point of view of mathematics, they treated the One as a point, namely as a material element. Secondly, from the point of view of universal definitions, they regarded the One as a formal element in the number. Nevertheless, these two characteristics cannot belong at the same time to the same thing.

- If the original One alone must be without position and if the number two is divisible -while the unit is not- then the unit is more similar to the original One. But, if the unit is more similar to the One than it is to number two, then each of the units in number two must be prior to number two. However, they generate the number two first.
COMMENTARY:

1083a1-17: Eighth argument (cf. continuance of the list of arguments that has begun in M.7):

In the eighth and final point of his argument against the theory of non-combinable units, Aristotle undermines and finally demolishes the idea that there are differentiated units:

a) This argument begins with the claim that the numbers cannot differ in quality, but only in quantity, since a number can be greater or smaller than or equal to another number.

b) However, if the units were also different in quantity, i.e. if one unit was greater than the other, then two equal numbers whose units were in 1:1 correlation would differ if the one had greater units than the other. Therefore, the units cannot differ in quantity.

c) But the units also can not differ in quality. For, the numbers which can be said to differ in quality, have qualitative differences, such as being prime or composite, even or odd, that result from their being different in quantity. The units do not differ in quantity, as it has already been shown. So, they cannot be qualitatively different, too.

d) Furthermore, if there is any other qualitative differentiation among the units, then it must be rationally accounted for by the Platonists.

Special attention should be paid on the Aristotelian question that is formulated at 1083a6-8, i.e. whether the first units are greater or smaller and the latter units increase in size, or the reverse. Alexander\textsuperscript{100} and Syrianus\textsuperscript{101} give prominence to the view that the first units are smaller, while the latter ones are greater, just as it happens with numbers. Nevertheless, Crubellier\textsuperscript{102} considers that it makes more sense if the units of a small number are greater than those of a great number; for, if the units are treated as aliquot parts of numbers, for instance as the one third of the

\textsuperscript{100} [Alex.], 741.22-25
\textsuperscript{101} Syr., 139.31-32
\textsuperscript{102} Crubellier (1994), 280
number three and the one fourth of the number four, then it is true that the units become smaller and smaller as numbers grow, given that, for example, the one third is greater than the one fourth. Crubellier’s claim is probably weakened by the fact that the unit, which is a single entity, could hardly be thought of as a fractional part, like $1/3$, $1/4$ etc. Therefore, it seems that in both cases the inequality of units is equally impossible.

1083a20-1083b1: The ἕτεροι τινες, about whom Aristotle speaks in this section, are Speusippus and the other thinkers who do not believe in the existence of Form numbers, but only in the existence of mathematical numbers. In particular:

a) Speusippus, in accepting only the mathematical number, assumes that there are infinitely many units, which combine with each other into infinitely many twos, threes etc.

b) Aristotle insists on the fact that, if all the units come from an original One, then there should also be many twos and threes deriving from an original Two and an original Three respectively, provided that something “first” should be followed by something second, something third etc.

Although Speusippus rejects the Platonic theory of Forms, he holds that the numbers are derived from the One and the Indefinite Dyad, i.e. the principles supposed to produce the Forms. So, according to Aristotle, by admitting the existence of an original One, Speusippus, probably without realizing it, is committed to an original Two, Three etc., which is inconsistent with his rejection of the Forms. However, what applies to the One, which is not number, should not be considered as necessarily applying to the number two or the number three. If, on Speusippus’ hypothesis, the One and the Indefinite Dyad are principles of things, why should their products necessarily be principles, too? Even in the original theory of Forms this does not hold true: the products of Forms, i.e. the sensible particulars, are not Forms. Therefore, Speusippus is not reasonably obliged to assume that apart from the many units there are also many twos and threes.
1083b1-8: By referring to those who identify the Form number with the mathematical number, Aristotle has in all probability Xenocrates in his mind. Xenocrates does not in fact need a separate critique, since he inherits the difficulties of both the Platonic and the Speusippean theory of numbers.

1083b8-19: The Pythagoreans are also dealt with briefly by Aristotle, since their refusal to separate the numbers from the things makes things less difficult in their case. However,

a) Aristotle attributes to them the fault of treating the numbers as constituent parts of the bodies.

b) Numbers of such a nature cannot be identified with the mathematical numbers because mathematical numbers do not have magnitude. However, the Pythagoreans say that the beings are numbers; at least they apply their mathematical propositions to the bodies as if the latter were made up of those numbers.

c) Furthermore, Aristotle claims that there are no indivisible magnitudes. But, even if one believes that there are indivisible magnitudes, neither units nor points would be classified among them. For, neither units nor points are magnitudes. If units were classified among magnitudes, then mathematical numbers, which consist of units, would be conceived as magnitudes, too, which is impossible. Moreover, points are not exactly magnitudes given that, according to Aristotle, the point is in fact a unit holding a position (cf. De Anima, A 409). Hence, given that both units and points do not have size, they are not magnitudes at all and as a consequence they cannot be classified among the things that are indivisible. Nevertheless, numbers, which comprise units, and magnitudes, which are composed of points, are irrefutably divisible. The question that arises is how it is possible for any divisible magnitude to be composed of indivisible things. Similarly, how can any divisible arithmetical number be composed of indivisible units?
1983b19-23: This is the conclusion of Aristotle’s whole argument, starting from the beginning of the sixth chapter of M.

1083b23-1084b2: This section is the first part of an even greater section which contrasts with the previous one in that it is more of an unconnected series of arguments with no connecting topic. However, they are suitable for the Book M, because they continue the attacks on Platonist theories of mathematical objects.

1083b23-36: Aristotle begins with showing the difficulties in combining two Platonist views, namely the concept of numbers as groups of units and the concept of the generation of numbers from the One and the Indefinite Dyad. In particular, Aristotle asks in what way the units in the numbers are to be produced from these principles:

a) Firstly, Aristotle asks whether each unit comes both from the Great and from the Small after their being equalized or either from the Great or from the Small.

b) In the second case, namely if the units come either from the Great or from the Small, two problems arise: first, each unit will not come from all the elements and there will be both quantitative and qualitative difference among the units. Second, in the case of odd numbers, such as the three, there is one extra unit left over, which is the one making the number odd. The question is what produces this unit. According to Aristotle, the One, i.e. the first principle, has to step in here.

c) Then, Aristotle touches on the problems resulting from supposing that each unit comes from both the Great and the Small. If each unit is generated from both the Great and the Small, then the first product of the doubling element will not be the original number Two, but the original Unit. Moreover, the first Unit in the original number Two is prior to the Two. So, given that the Two is a Form and that the first Unit is also a Form, the first Unit must be the Form of a Form. Also, the Indefinite Dyad only doubles, and therefore it produces only twos. So, from what will the First unit that is prior to the first Two be
produced? Therefore, even if the first principles of the numbers - namely the One and the Indefinite Dyad - are said to be also principles of the units, there will be the awkward result that the units have to be produced before the numbers.

d) Hence, Aristotle in fact shows that the concept of the numbers as sets of units and the concept of the derivation of the numbers from the One and the Indefinite Dyad have no obvious connection with each other and the most obvious ways of connecting them lead to absurdities.

1083b36-1084a2: Aristotle states that the Platonists who take the numbers to be separate from the things of which they are numbers are committed either to the existence of an infinite plurality of numbers or to the existence of a finite number-series which has to stop at some arbitrary limit. Aristotle himself, as his treatment of the Infinite in the Third Book of the Physics (207a33-207b15) shows, believes that the number-series goes on to infinity, in the sense that you can always go on adding one more unit to any number that you reach. Furthermore, Aristotle seems to insist that on his interpretation the numbers that go on forever are not separate from the process of adding.

1084a2-7: Aristotle holds that, if a Platonist accepts the existence of infinitely many numbers, then he faces two problems:

a) First, the infinite number is neither odd nor even, as any number has to be. It is a matter of question why the infinite number is neither odd nor even. This question could be answered on the basis of the fact that the infinite number will be qualitatively different from the ordinary numbers we know. So, it is not certain that we would be able to show it to be odd or even, due to our inability to make sense of it.

b) Moreover, both the numbers that are odd and the numbers that are even consist of a limited number of units, in relation to which they are classified as odd or as even. An infinite number would reasonably consist of an unlimited
number of units. Such a number would have no limits. Therefore, depending on what would we assume that an infinite number is either odd or even?

Attention should be paid at the difficult passage about the Platonic derivation of even and odd numbers. It seems that, according to Aristotle, there are three ways in which the generation of a number can happen:

a) Firstly, an odd number can be generated after the addition of a unit to an even number, i.e. “x” + 1 (where “x” is any even number, namely 2, 4, 6, 8, 10, 12 etc.).

b) Secondly, the number which is generated from the one (ὁ ἀφ’ ἕνος), i.e. the number two, is doubled when the two is added. It is a matter of question in what sense “τῆς δύνας ἐμπιστοὺς” should be conceived. Should it be understood as an involvement of the two in an operation of adding numbers or in a multiplying procedure? It seems more reasonable that the mathematical operation which Aristotle has in mind here is that of multiplication, given that the number is said to be “doubled”. Therefore, the Platonists are said to derive even numbers from the multiplication of the two (and its powers) by the two, i.e. “n^2” X 2 (where “n^2” is any power of number two).

c) Finally, another way in which the Platonists are said to derive even numbers is by addition of odd numbers, i.e. “y” + “y” (where “y” is any odd number, namely 3, 5, 7, 9, 11, 13 etc.)

Consequently, according to Crubellier\textsuperscript{103}, the three Platonic ways of deriving even and odd numbers could be briefly described as follows:

a) production of the powers of 2 by the repeated application of the operation “X2”,

b) production of odd numbers by an operation of the form: “n^2 + 1”

c) production of even numbers by an operation of the form: “2 (n^2 + 1)”

\textsuperscript{103} Crubellier (1994), 306
1084a7-10: Aristotle holds that the second problem a Platonist would face, if he accepted the existence of infinitely many numbers, is that, if the numbers are Forms, the infinite number will be the Form of something. However, such a Form would be a Form of an infinite plurality. Nevertheless, this is unfortunate for a Platonist, for a Form is essentially a principle of limit, given that it is a knowable and therefore strictly determined entity.

1084a12-17: Aristotle deals with the obviously unacceptable alternative of the number-series being finite, making here reference to the claim that the Platonist ideal number-series stops at ten. Aristotle points out that, if the Forms are to be identified with the numbers, then there have to be more than ten numbers, because there are more than ten Forms.

1084a18-21: Aristotle argues that provided that we accept that the original number Three is the Form of Human Being then all of the infinitely many threes that exist will be human beings, too. And, given that according to the Platonists the numbers are Forms, every human being will be an original Human Being, namely a Form of Human Being. Or, at least, there will still be infinitely many particular human beings.

It is a matter of question how the phrase “ἐν τοῖς αὐτοῖς ἀριθμοῖς” should be conceived. In Physics (Δ 12 221a11-13), Aristotle explains the phrase “ἐν ἀριθμοῖς εἶναι” in two ways. The first sense of “ἐν ἀριθμοῖς εἶναι” is “being part or property of number or being, in general, anything that belongs in a sense to number”. In this sense, “the 3s existing in the same numbers” should be thought of as the 3 that is included in the original 4, the 3 that is included in the original 5, the 3s that are included in the original 6 and so on up to the original 10, but also as the 3 that is included in the 4 that is included in the original 5, the 3 that is included in the 4 which is in the original 6, the 3 that is included in the 5 which is in the original 6 etc. The second sense of “ἐν ἀριθμοῖς εἶναι” is “being something of which there is a

104 Ross (1924), v. 2, 449
number”. In this sense, “the 3s existing in the same numbers” should be conceived in fact as “the (mathematical) 3s which participate in the same (Form) numbers”\textsuperscript{105}.

The two aforementioned senses in which the phrase “ἐν ἀριθμῷ εἶναι” can be understood are equally applicable here. In any of the two cases, if we accept that the Form number Three is the Form of Human Being, then we are obliged to also accept that all the other infinitely many threes that exist are either infinitely many original or infinitely many particular human beings.

\textbf{1084a21-25}: Aristotle makes reference to another absurdity resulting from the identification of the Forms with the numbers: if, for instance, the two is part of the four and the two is the original Human Being, while the four is the original Horse, then logically the original Human Being is part of the original Horse.

\textbf{1084a25-27}: Aristotle holds that the non-existence of any Form numbers beyond the ten is absurd. He does not imply here that the Platonists did not recognize any number going beyond the number ten. The Platonists thought that there are Form Numbers but set a limit to them and the limit was the Form number Ten. They did not claim that there are no mathematical numbers besides the Form numbers. Mathematical numbers go well beyond the number ten, possibly to infinity. But this does not entail that the Form numbers should also be more than ten and possibly infinite.

The limitation of Form numbers to ten must have something to do with the decadic system of numbering; after ten we number for the most part by repeating numbers that are already present in the 1-10 series (ἐν-δεκα, διώ-δεκα [= δώδεκα], τριά καὶ εἴκοσι, πέντε καὶ ἑνενήκοντα etc.). Furthermore, the numbers from 2 to 9 are numbers that consist only of units and not of tens\textsuperscript{106}. This fact makes numbers from 2 to 9 more original numbers that those which follow.

\textsuperscript{105} Crubellier (1994), 310
\textsuperscript{106} Crubellier (1994), 313
Aristotle raises a question that better suits the fifth chapter, namely that there are some things (i.e. the artificial ones), of which there are no Forms. Then, he once more reaches the conclusion that the Forms are not the causes of the things.

1084a29-1084b2: Aristotle criticizes the way in which the Platonists talk of the number ten and its properties. According to Aristotle, the Platonists treat the numbers going up to ten as a complete entity. However, they also treat the number ten as a unitary object. In his attempt to prove the absurdity of such a way of considering the number ten, Aristotle claims that the Platonists hold that some things like the void or the odd are generated inside the ten, while they also assign various such things to the principles on a mystical basis.

Given that some instances of “the derivative things” (τὰ ἐπόμενα) are said by Aristotle to be the void, the proportion and the odd, while the examples that are given for the things that “they (i.e. Platonists) assign to the principles” (ταῖς ἀρχαῖς ἐποδιδόσασιν) are the change, the rest, the good and the bad, it is a matter of question if the status of these and ἐπόμενα things could be further determined. The examples of the πρῶτα are couples of opposite things that are general predicates (i.e. movement-rest, good-bad). On account of the fact that the principles form a couple of opposite elements (i.e. One-[Indefinite] Dyad) it seems reasonable that they generate things which are also opposite to each other.

As far as the ἐπόμενα are concerned, the examples that are given by Aristotle form an incongruous list (void, proportion, odd). The only feature these heterogeneous things have in common is, according to Crubellier107, their relation to the aspect of place. Indeed, the void (κενόν) is described by Alexander108 as represented by the interval that exists between two successive even Form numbers and also between two successive odd Form numbers. Moreover, the proportion (ἀναλογία) is said by Syrianus109 to exist in many cases between different numbers from 2 to 10. As regards the odd (περὶ τῶν), I think it could be given a regional status

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107 Crubellier (1994), 314-315
108 [Alex.], 750.12-24
109 Syr., 150.4-7
if it was described as the sort of number that is preceded by an even number and also succeeded by an even number. Nevertheless, in my opinion the involvement of the proportion and the odd in the list complicates Crubellier’s interpretation of ἐπόμενον. For, in what way the examples that are given -apart from the void- could be clearly related to the aspect of place? Perhaps, the relation to a structure that consists of more than one numbers is more suitable as common feature of the aforementioned heterogeneous examples.

Then Aristotle claims that the Platonists identify the odd with the One. According to Ross\textsuperscript{110}, the force of διό (“therefore”) here seems to be the following one: the Platonists derived all the derivative entities either from the first principles or from the numbers up to ten. Now, the oddness could not be derived from any particular number, since an odd number could not explain the oddness of any other number, given that the numbers are independent of each other. Therefore, the oddness had to be derived from one of the principles and of the two the One was more appropriate for this purpose.

In the last sentence, the “first indivisible line” (ἡ πρώτη γραμμή, ἡ ἄτομος) seems to be Plato’s way of describing the point. It is worth discussing the relation of the “first, indivisible line” to the two subsequent terms, i.e. “the Dyad” and “these (things) that extend up to ten”. Given that the “first, indivisible line” is a term that describes the point, which is the geometrical counterpart of the One, the “Dyad” could probably describe the “second line”, i.e. the ordinary line that extends between two points, and the things “that extend up to ten” could be the Three, which has as geometrical counterpart the plane figure that is defined by three points (i.e. the triangle), and the Four, which has as geometrical counterpart the solid body that is defined by four points (i.e. the tetrahedron). It is noteworthy that the sum of the above mentioned numbers (1, 2, 3 and 4) is equal to ten.

1084b2-32: In this long section, Aristotle formulates his criticism on the Academy’s way of deriving the number.

\textsuperscript{110} Ross (1924), v. 2, 451
1084b3-7: The first question that Aristotle raises is whether the one is prior to the number or vice versa. Then he remarks that the ones (or units) of which a number is composed stand to the number as the matter stands to the form. So, the answer to Aristotle’s first question depends on the point of view from which it is asked. From one point of view, the ones (or units) come first, since the number is the product of their addition to each other. But, from another point of view, the number comes first, since the number is the means of identifying a group of units as a group of “so many” units. Therefore, the Platonists are proven wrong in failing to distinguish the two senses in which the question can be answered and in making their “one” prior both as matter, i.e. as the units of which a number is composed, and as form, i.e. as the formal unity of a set of units.

1084b7-20: Aristotle formulates the idea that of the two senses mentioned above, that of form is more likely to be prior to the other. In order to support this point, he uses the example of the right and the acute angles, of which the latter can be thought of as matter of the former, while the former has an independent definition which is presupposed in the definition of an acute angle.

Ross\(^{111}\) refers to a difficulty arising from the fact that there are three things mentioned, namely the universal, the particular and the element (1084b14-15), but then Aristotle talks about only two things that are opposed to each other (1084b15-16). According to Ross, the difficulty may be overcome either by taking τὰ δὲ as referring to both the particular and the element or by supposing that καὶ τὸ ὑστοχεῖον is explicative of τὸ ἐπὶ μέρος.

As far as the meaning of κατὰ λόγον and κατὰ χρόνον is concerned, Alexander\(^{112}\) explains that the universal is indivisible in λόγος, for the definition of something is not divisible into other forms and formulae. However, this seems impossible. Let us take the definition of “human being” as “living thing endowed with language”. Are not “living” and “thing” and “language” the parts into which “human being” can be

\(^{111}\) Ross (1924), v. 2, 452
\(^{112}\) [Alex.], 752.29-753.9
divided? Moreover, according to Alexander, the particular is said to be indivisible in χρόνος, since the Form of something is not prior in time to the thing of which it is a Form. Nevertheless, this is impossible, too. The Form of X is prior in time to any particular x since (i) the Form is eternal or atemporal or sempiternal, whereas the x exists for a limited span of time, and (ii) the destruction of x does not entail the destruction of X. The particular being (e.g. this rose plant, this cat, this stone) can only be indivisible in time in the sense that it is one thing for as long as it endures being one. Therefore, I would agree with Ross’ proposal to take “τρόπον ἄλλον” as qualifying “ἀρχή”\textsuperscript{113}.

1084b20-23: Aristotle supports the previous point also in another way. The form is actuality, while the matter is the range of a thing’s potentialities. So, the number considered as a unity is prior to the units which exist in it. For, there has to be a number with the formal properties of, say, the number three, for us to be able to count up to the number three and in thus actualize the three units existing in this number.

1084b25-28: When Aristotle says that the unit is a point lacking position, he may mean either that the Platonists treated the points as the matter of the lines or that they -while thinking of the units as the matter of the numbers- somehow considered the units as mysteriously disembodied points\textsuperscript{114}. Aristotle makes an inference as to the status of the unit from what the Platonists described as the status of the point. They defined the point as a unit in position, whence it would follow that the unit is a point without position.

1084b28-32: Aristotle claims that the Platonists combined the two viewpoints mentioned above, treating the one both as matter, since the units are parts of numbers, and as form, since every number is characterized by a formal unity. However, such a combination is illicit, according to Aristotle.

\textsuperscript{113} Ross (1924), v. 2, 452
\textsuperscript{114} Annas (1976), 183
In this section Aristotle, among some quite obscure arguments he formulates, also claims that the unit should be considered as prior to the two, since the units are more similar to the original One (which is the first principle) in that they are indivisible. Ross\textsuperscript{115} makes here the suggestion that probably the term ἄθετον is used by Aristotle because each particular unit has a setting (or θέσις) in some particular number. All that distinguishes the One (which is the formal principle of number) from the particular units, is that it has no such particular setting (i.e. it is ἄθετον).

\textsuperscript{115} Ross (1924), v. 2, 454
M.9 (1085a1-1086b13)

“Introduction to the discussion about principles”

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<tr>
<td>1085a1-6</td>
<td>Ἐτι εἰ ἐστὶν ἡ δυάς ἐν τῷ αὐτῷ καὶ ἡ τριάς αὐτή, ἀμφό τις δυάς. Εἴ τινος οὖν αὐτὴ ἡ δυάς; Ἀπορήσεις δ᾽ ἐν τῷ καὶ ἐπεὶ ἀρχὴ μὲν οὐκ ἐστὶν ἐν τοῖς ἁριθμοῖς, τὸ δ᾽ ἑρεξῆς, διὸν μὴ ἐστι μεταξὺ μονάδων (οἰούν τῶν ἐν τῇ δυάδι ἢ τῇ τριάδι), πότερον ἑρεξῆς τῷ ἐν αὐτῷ ἢ οὕ, καὶ πότερον ἡ δυάς προτέρα τῶν ἑρεξῆς ἢ τῶν μονάδων ὀπτεράσων.</td>
<td>Further, if the original Two is one thing and so is the original Three, they both together are two. What thing, then, does this two come from? Someone might also raise this question: since there is no contact in numbers, but only succession, as far as those units are concerned between which there is nothing (e.g. the units in the two or in the three) - do they succeed the original One or not, and is the two or either of the units in the two prior to the numbers which succeed it?</td>
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<tr>
<td>1085a7-12</td>
<td>Ὅμως δὲ καὶ περὶ τῶν ὑστερον γενῶν τοῦ ἁριθμοῦ συμβαίνει τὰ δυσχερή, γραμμῆς τε καὶ ἔπιπεδου καὶ σώματος. Οἱ μὲν γὰρ ἐκ τῶν εἴδων τοῦ μεγάλου καὶ τοῦ μικροῦ ποιοῦσιν, οἰοὺς ἐκ μακροῦ μὲν καὶ βραχέος τὰ μήκη, πλατέος δὲ καὶ στενοῦ τὰ ὑπόπεδα, ἐκ βαθεός δὲ καὶ ταπεινοῦ τοὺς ὁγκοὺς ταῦτα δὲ ἐστὶν εἴδη τοῦ μεγάλου καὶ μικροῦ.</td>
<td>Also in the case of the classes of things that are posterior to number, namely the line and the plane and the body, similar difficulties occur. For some people produce them out of the species of the Great-and-the Small: for example they derive lengths from the Long-and-Short, and planes from the Broad-and-Narrow, and masses from the Deep-and Shallow; and these are species of the Great-and-Small.</td>
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1085a13-23  
Τὴν δὲ κατὰ τὸ ἐν ἄρχῃν ἄλλοι ἄλλας τιθέσαι τῶν τοιούτων. Καὶ ἐν τούτοις δὲ μυρία φαίνεται τὰ τέ ἀδύνατα καὶ τὰ πλασματώδη καὶ τὰ ύπεναντία πάσι τοῖς εὐλόγοις. Ἀπολελυμένα τε γὰρ ἄλληλων 
συμβαίνει, εἰ μὴ συνακολουθοῦσι καὶ αἱ ἄρχαι ὡστὲ εἶναι τὸ πλατὺ καὶ στενὸν καὶ μακρὸν καὶ βραχὺ (εἰ 
δὲ τούτο, ἐστὶ τὸ ἐπίπεδον γραμμὴ καὶ τὸ στερεὸν ἐπίπεδον· ἐτὶ δὲ γωνίαυ καὶ σχήμαtau καὶ τὰ τουάτα 
pᾶς ἀποδοθήσεται), ταύτῳ τε συμβαίνει τοῖς περὶ τὸν ἀριθμόν· ταύτα γὰρ πάθη μεγέθους ἐστίν, ἀλλὰ 
οὐκ ἐκ τούτων τὸ μέγεθος, ἀλλὰ ὡστεν τὰ ἐν τῷ ἐπίπεδῳ ἐκ λείου καὶ τραχείου τὰ 
στερεά.  

And as regards the principle which answers to the One, 
different people among such thinkers posit it in different 
ways. In these things, too, the impossibilities and the 
contrivances and the beliefs that are contrary to all 
reasonableness appear to be innumerable. For things turn 
out to be cut off from one another, unless the principles also 
imply one another so that the Broad-and-Narrow be also long 
and short; but if so, the plane will be a line and the solid will 
be a plane. (Besides, how can the angles and the figures and 
such things be explained?) And the same thing happens as 
with the properties of number; for these things are attributes 
of magnitude, not what magnitude is generated from, much 
as neither length is generated from the straight and the 
curved nor the solids from the smooth and the rough.

1085a23-31  
Πάντων δὲ κοινῶν τοιῶν ὅπερ ἑπὶ τῶν εἰδῶν τῶν ἂν γένους συμβαίνει διαπορεῖν, ὅταν τις θῇ τὰ 
καθόλου, πότερον τὸ ζῷον αὐτὸ ἐν τῷ ζῷῳ ἢ ἐτέρου 

Common to all these cases is precisely the perplexity arising 
with the species of a genus, if one posits universals: is the 
original Animal or something other than the original Animal
αὐτοῦ ἑσυχῶν. Τούτῳ γὰρ μὴ χαριστοῦ μὲν ὅτες
οὐδεμίαν ποιήσει ἀπορίαν χαριστοῦ δὲ, ἀσπερ ὦι
ταῦτα λέγωντες φασί, τοῦ ἑνός καὶ τῶν ἀριθμῶν οὐ
ῥάδιον λύσα, εἰ μὴ ῥάδιον δεῖ λέγειν τὸ ἀδύνατον.
Ὅταν γὰρ νοῇ τις ἐν τῇ δύνατι τὸ ἑν καὶ ὅλως ἐν
ἀριθμῷ, πότερον αὐτῷ νοεῖ τι ἐτερον;

inside the particular animal? For, if it is not separate, this will
cause no perplexity; but, if the One and the Numbers are
separate, as those who express these views
claim, then it is not easy to solve the problem, if one must call
what is impossible “not easy”. For, when somebody thinks of
the one in the two, and generally in a number, do they think
of a Form or of something else?

| 1085a32-1085b4 | Οἱ μὲν οὖν τὰ μεγέθη γεννῶσιν ἐκ τοιαύτης ὀλης,
ἐτεροι δὲ ἐκ τῆς στιγμῆς (ἢ δὲ στιγμὴ αὐτοῖς δοκεῖ
eῖναι οὐχ ἐν ἄλλῳ ὁδὸν τὸ ἑν) καὶ ἄλλης ὀλης οὐκ τὸ
πλῆθος, ἄλλῳ ὀὐ πλῆθος.| Some people, then, generate magnitudes from matter of this
sort, while others from the point (and the point seems to
them to be not One, but something like the One) and from
another matter which is similar to Plurality but not Plurality.
The same perplexities occur about them none the less. For, if
the matter is one, then the line and the plane and the solid
will be the same; for what comes from the same principles is
one and the same. But if the matters are more than one – i.e.
one for the line, another of the plane, and another of the solid
– then they either imply each other or not, and therefore the
same results will follow in this way too: either the plane will

| 182 | Ὅταν γὰρ νοῇ τις ἐν τῇ δυάδι τὸ ἑν καὶ ἄλλως ἐν
ἀριθμῷ, πότερον αὐτῷ νοεῖ τι ἐτερον; | inside the particular animal? For, if it is not separate, this will
cause no perplexity; but, if the One and the Numbers are
separate, as those who express these views
claim, then it is not easy to solve the problem, if one must call
what is impossible “not easy”. For, when somebody thinks of
the one in the two, and generally in a number, do they think
of a Form or of something else? |
Furthermore, no attempt is made to explain how number can come from the One and Plurality; but in whatever way they speak, the same difficulties arise as for those people who generate number from the One and the Indefinite Dyad. For while one person generates number from that which is universally predicated and not from a particular plurality, the other person generates it from a particular plurality, namely the first – two being for him, as it were, the first plurality. Therefore, there is no difference, so to speak, between the two views, but the same perplexities will follow, namely “is the generation mixture or composition or blending?” and the like.

| 1085b5-13 | Έτι πως μὲν ἐνδέχεται εἶναι ἐκ τοῦ ἕνος καὶ πλήθους τὸν ἀριθμὸν οὐθὲν ἐπιχειρεῖται ὡς δ᾽ οὖν λέγουσι ταῦτα συμβαίνει δυσχερή ἀπερ καὶ τοις ἐκ τοῦ ἕνος καὶ ἐκ τίς δυάδος τῆς ἁριστου. Ο μὲν γὰρ ἐκ τοῦ κατηγορομένου καθόλου γεννᾶ τὸν ἀριθμὸν καὶ οὐ τινὸς πλήθους, ὡς δὲ τινὸς πλήθους τοῦ πρῶτου δὲ (τὴν γὰρ δυάδα πρῶτὸν τι εἶναι πλήθος), ὡστε διαφέρει οὐθὲν ὡς εἰπέν, ἀλλὰ αἱ ἀπορία ς αἱ αὐταὶ ἀκολουθήσουσι, μίξεις ἢ θέσεις ἢ κράσις ἢ γένεσις καὶ ὅσα ἄλλα τοιαῦτα. |

1085b13-22 | Μάλιστα δ᾽ ἂν τὶς ἐπιζητήσειςν, εἰ μία ἐκάστη μονᾶς, ἐκ τινὸς ἐστὶν οὐ γὰρ δὴ αὐτὸ γε τὸ ἐν ἐκάστῃ. Ἀνάγκη δὴ ἐκ τοῦ ἕνος αὐτοῦ εἶναι καὶ πλήθους ἢ μορίοι τοῦ πλήθους. Τὸ μὲν οὖν πλήθος τι εἶναι φάναι τὴν μονᾶδα ἀδύνατον, ἀδιαφορετῶν γ᾽ But somebody might, above all, seek to know, if each unit is one, what it comes from; for each unit is not the original One, to be sure. It is, then, necessary that it comes from the original One and Plurality or part of Plurality. However, to say that the unit is a plurality of some sort is impossible, since the unit
Further, we must inquire, in the case of the exponents of this theory too, whether the number is infinite or finite. For there was, as it seems, a finite Plurality, from which, together with the One, the finite number of units comes; but there is also another original Plurality, namely infinite Plurality. So, which kind of Plurality is an element in combination with the One? And somebody might similarly seek to know about the Point and the element from which they produce the magnitudes; for surely this is not the one and only point. So, what does each of the other points come from? Surely, not from some...
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<th>Page</th>
<th>Greek Text</th>
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| 1085b35-1086a20 | Πάντα δή ταύτα καὶ ἄλλα τοιαύτα φανερὸν ποιεῖ ὁτι ἀδύνατον εἶναι τὸν ἀριθμὸν καὶ τὰ μεγέθη χαριστά, ἂτι δὲ τὸ διαφανεῖν τοὺς τρόπους περὶ τῶν ἀριθμῶν σημείον ὅτι τὰ πράγματα αὐτὰ ὕπο ὄντα ἀληθῆ παρέχει τὴν ταχεῖαν αὐτοῖς. Οἱ μὲν γὰρ τὰ μαθηματικὰ μόνον ποιοῦντες παρὰ τὰ αἰσθήματα, ὀρῶντες τὴν περὶ τὰ εἰδη δυσχέρειαν καὶ πλάσιον, ἀπέστησαν ἀπὸ τοῦ εἰδητικοῦ ἁριθμοῦ καὶ τὸν μαθηματικὸν ἐποίησαν· οἱ δὲ τὰ εἰδη βουλόμενοι ἀμα καὶ ἀριθμοὺς ποιεῖν, σὺν ὀρῶντες δὲ, εἰ τὰς αἰχὰς τις ταύτας θησεῖται, πῶς ἐσται ὁ μαθηματικὸς ἀριθμὸς παρὰ τὸν εἰδητικὸν, τὸν αὐτὸν εἰδητικὸν καὶ μαθηματικὸν ἐποίησαν ἁριθμὸν τῷ λόγῳ, ἐπεὶ ἐργὼ γε ἀνήρηται ὁ μαθηματικὸς (ἰδίας γὰρ καὶ οὐ
| Distance and the original Point. Nor, again, can there be indivisible parts of a distance, like the parts of Plurality from which the units are composed; for number is composed of indivisible parts, while magnitudes are not. Indeed, all these considerations and others of this sort make it clear that it is impossible for number and magnitudes to be separate. Moreover, disagreement about the manners of accounting for numbers is a sign that the theories themselves, not being true, bring confusion to their exponents. For, those who make only mathematicals be besides sensibles, seeing the difficulties and fictions concerning Forms, moved away from Form Number and set up mathematical number; and those who wished to make the Forms at the same time also numbers but did not see, if one posits only these principles, how mathematical number is to be besides Form Number, made the Form Number and the mathematical number identical (in name, since in actual fact the mathematical number is done away with; for they make assumptions that |
are peculiar to them and not those of the mathematicians).

But he who first postulated that the Forms are and that the Forms are numbers and that the mathematicals are, reasonably enough, separated mathematicals from Forms. The result is that all those thinkers speak correctly in some way but not correctly over all. And they themselves confirm this since they do not say the same things but things opposite to one another. The reason of their disagreement is that their assumptions and principles are false. For it is difficult to speak correctly starting from what is incorrect, as Epicharmus says: “no sooner is it said than ‘tis directly seen being wrong”. Concerning numbers the exploration of the difficulties and the distinctions that we have made are sufficient. Someone already convinced might be further convinced with more arguments but someone not convinced will come no nearer to conviction.
| aἰσθητῆς οὐσίας διορίζοντες, τὰ μὲν ἐν τοῖς περὶ φύσεως εἴρηται, τὰ δ’ οὐκ ἔστι τῆς μεθόδου τῆς νῦν· όσα δὲ οἱ φάσκοντες εἶναι παρὰ τὰς αἰσθητὰς ἐτέρας οὐσίας, ἐχόμενον ἐστὶ θεωρῆσαι τῶν εἰσημένων. | has been said in the treatises about Nature, for one, and does not belong to our present inquiry, for another; but what those people claim who hold other substances besides sensible substances to be is a sequel to our previous discussion. |

| Ἐπεὶ οὖν λέγουσι τινες τοιαύτας εἶναι τὰς ἱδέας καὶ τοὺς ἀριθμοὺς, καὶ τὰ τούτων στοιχεία τῶν ὄντων εἶναι στοιχεία καὶ ἀρχάς, σκεπτέον περὶ τούτων τί λέγουσι καὶ πῶς λέγουσιν. Οἱ μὲν οὖν ἀριθμοὺς ποιοῦντες μόνον καὶ τούτους μαθηματικοὺς ὀστεούν ἐπισκεπτέον· τῶν δὲ τὰς ἱδέας λεγόντων ἅμα τὸν τε τρόπον θεάσαι· ἀν τις καὶ τὴν ἀπορίαν τὴν περὶ αὐτῶν. Ἀμα γὰρ καθόλου τε ὡς οὐσίας ποιούσι τὰς ἱδέας καὶ πάλιν ὡς χαριστάς καὶ τῶν καθ’ ἐκαστον. Ταῦτα δ’ ὅτι οὐκ ἐνδέχεται διηγοῦνται πρῶτον. | Now, since some people say that the Forms and the numbers are entities of such kind, and that their elements are the elements and principles of beings, it should be examined what they say about these matters and what they mean. We must discuss later those who posit only numbers, and mathematical numbers at that. But as regards those who posit Forms one might at the same time come to see their mode of speaking and their perplexity about these subjects. For at one and the same time they make Forms universal and, again, treat them as separate and as individuals. And it has already been thoroughly examined that this is not possible. |
The reason that led those who think of Forms as universal to conflate the two characteristics is that they did not make their substances identical with sensible things. They believed that each individual in the domain of sensible things is in flux and none of them is at rest, and that the universal is something besides sensible things and something different from them.

As we have said in our previous discussion, Socrates initiated this tendency by reason of his definitions, though he did not separate, to be sure, the objects of definitions from individual things; and in this thought he was right, in not making the separation. This is clear from the facts. For it is not possible to attain knowledge without universals, but separation is the cause of the difficulties that occur with respect to the Forms. But since they took it to be necessary that, if other substances besides the sensible and flowing substances are to be, they should be separate, and did not have other substances to set

116 1086a36: ύσίας Ross, ideac codd.
καθόλου καὶ τὰς καθ’ ἐκαστὸν. Αὐτὴ μὲν οὖν αὐτῇ καθ’ αὐτὴν εἰη τις ἀν δυσχέρεια τῶν εἰρημένων.

up, they introduced these universally predicated ones, the result being that universals and individuals are almost the same natures. This in itself, then, would be one difficulty with the said theory.
OVERVIEW:

In M.9, Aristotle examines the difficulties arising from the treatment of the first principles of geometrical objects. Moreover, he discusses the difficulties of the Platonic way of generating numbers and magnitudes.

- If the original Two is one thing and so is the original Three, they make together two things. From what thing is this two derived?
- Is the number two or either of the units in it prior to the numbers which succeed it?
- Similar difficulties arise also in the case of classes of things that are posterior to number, namely the line, the plane and the solid: (a) some people derive them from species of the Great-and-the Small (the lines from Long-and-Short, the planes from Broad-and-Narrow and the solids from Deep-and Shallow). However, there are different opinions as regards the formal principle answering to the One. Moreover, lines, planes and solids prove to be cut off from one another, given that their principles are different; otherwise, the plane will be a line and the solid will be a plane. The same difficulty arises in the case of numbers.
- (b) Others generate magnitudes from the point (which seems to them to be similar to the One) and from another element which is similar to Plurality. Nevertheless, the same difficulties arise. For, if the matter is of one sort, then the line, the plane and the solid will be the same; for what comes from the same principles is one and the same thing. But, if there are more than one matter—i.e. one for the line, another of the plane, and another of the solid—then the plane will not contain a line.
- The same difficulties arise for those people who generate number from the One and the Indefinite Dyad. For, while one thinker derives number from the plurality which is universally predicated, the other thinker derives it from a particular plurality, i.e. the first plurality, which is the Dyad. In
both cases we may ask whether the generation from the elements is the result of a process of mixture or composition or blending etc. It is necessary that each unit is generated from the One and the Plurality or a part of the Plurality. However, to say that the unit is a plurality of some sort is impossible, since the unit is indivisible. Furthermore, to say that it comes from a part of Plurality involves many other difficulties; for it is necessary that each of the parts of Plurality is indivisible and the One and Plurality will not be elements, given that each unit will not come from Plurality and the One. Moreover, according to this view, number is generated from a plurality of indivisible things, i.e. from another number.

- Further, as regards the exponents of this theory, it is a matter of question whether the number is infinite or finite. For there was a finite Plurality, from which, together with the One, the finite number of units comes. However, there is also another original Plurality, which is infinite Plurality. So, which kind of Plurality is the first principle?

- Similarly, a point does not come from the original Point and some interval that is composed of indivisible parts. Number is composed of indivisible parts, but magnitudes are not.

- All the aforementioned considerations show that it is impossible for number and magnitudes to exist separately.

- (1) Those who believed in the existence of only mathematical objects (i.e. Speusippus) did so since they see the difficulties concerning Forms.

- (2) Those who considered the Forms to be numbers (i.e. Xenocrates) identified the Form Number with the mathematical number, but in fact they did away with the mathematical number.

- (3) He who first postulated that the Forms exist and that they are numbers and that the mathematical objects exist (i.e. Plato) reasonably separated the mathematical objects from the Forms.

- All those thinkers speak partly correctly. However, their assumptions are false.
A far as the first principles are concerned, the views of those thinkers who claim that there are other substances besides the sensible ones must be discussed.

Those who posit only mathematical numbers may be deferred. But as regards those who posit Forms, they treat Forms as universals and at the same time they treat them as separate things and as individual objects.

The reason of this impossible combination is that they believed that each sensible individual thing is in flux and that the universal things are something different existing apart from them.

As it has already been said, Socrates initiated this view by reason of his definitions, but he did not separate the universal objects from the particular ones. He was right in not making the separation.

Nevertheless, his successors thought it was necessary that, if there are other substances besides the sensible ones, they should exist separately. Thus, they assigned separate existence to universal things, the result being that universals and individuals are practically the same natures.
COMMENTARY:

1085a1-6: This section seems to include sketches for previous arguments (cf. 1082b11-16 and 1081b6-8). Aristotle’s thought here seems to proceed as follows: if the original Two is one thing and the original Three is one thing, then, when these things are combined, make a two. The first question is what this two comes from, i.e. what are the principles of this two. A second question is whether the units forming each number succeed the original One or not and whether the units forming each number are prior to the number which comes next in the series or not, probably in the sense that the units forming number Two would reasonably be considered as prior to the units forming number Three. This argument indicates the difficulty arising from two facts: firstly from the fact that, if we add any number as one distinct entity to any other number as one distinct entity, then they seem to make a certain number two that is also one distinct entity. Secondly, the difficulty arises from the fact that there seem to be units that succeed numbers, given that two numbers that consist of units are not connected to each other through contact, but only through succession. Both of the cases that are mentioned above result in total absurdity.

1085a7-12: This section answers to a section belonging to A9 from Metaphysics (992a10-19). The difficulty that is faced, as far as the principles are concerned, in the case of numbers, also occurs in the case of the objects of geometry, namely the line, the plane and the solid. The objection to the Platonic principles seems to be the same in both sections. Given that the line is derived from the Long and the Short, the plane from the Broad and the Narrow, and the solid from the Deep and the Shallow, if the matters for the different dimensions are considered to be different from each other as well, then the line, the plane and the solid cannot be thought of as species of one and the same genus. In other words, the different geometrical objects cannot be thought of as instances of the same genus of things, namely the genus of geometrical objects, since their principles are different and therefore the objects in one dimension will be generically different from the objects in the other dimension.
Also, if the matters for the different geometrical objects are considered to be variants of one and the same matter, then the line, the plane and the solid, must be thought of as one and the same genus. In other words, they will all be classified as instances of the same sort of thing, which is not true given that line, as a sort of thing, is different from the plane, which is also different sort from the solid.

Therefore, both in case the principles of the objects of geometry are many and different and in case they are one and the same, the absurdity is unavoidable, because the relation between dimensions can neither be that of genus to species nor that of totally different genera. If, on the one hand, the Great and the Small is genus and the Short and Long, the Broad and Narrow and the Deep and Shallow are its species, then a member of, say, the Broad and the Narrow, i.e. a particular plane, will have an ontological derivation that is distinct from the derivation of a particular member of the Long and the Narrow, i.e. a particular line. It follows that there is no continuity in the dimensions. If, on the other hand, the Broad and the Narrow is made a subspecies of the Long and the Short, then a plane will also turn out to be a line; for subspecies share in the trait of the species of which they are the subspecies.

1085a13-23: In this section, Aristotle accuses the Academy of taking the various forms of the Great and Small to be what numbers and magnitudes are made from, while they are in fact attributes of numbers and magnitudes.

However, even if the principles of numbers and magnitudes were not many but only one, namely the Great and Small, this would also present a great difficulty. For from the same principle come identical things. Therefore, in this case, the line would be identical with the plane and the plane would be identical with the solid, which seems totally absurd, provided also that the figure of each geometrical object displays its difference from the rest.

Aristotle touches here the subject of the attributes of existing things. The question he makes is how the existence of these attributes can be explained, given that they are neither some kind of existing things that are separate from the other existing things nor principles of the existing things.
1085a23-31: The fact that the Platonists think of the original One as well as of any original Number as entities which exist separately from the particular things which they number is the cause of numerous perplexities. For, as regards the units comprising each number, the question that arises is whether each of these units is the original One or something else. Undoubtedly, at least according to the Platonists, the units comprising each number cannot be the original One. But, if the units should be thought of as something other than the original One, then this “something” could not be classified either as a principle or as some distinct particular thing, given that the units can exist only as members of groups, i.e. as parts of numbers.

According to Annas, in this section the following difficulty is encountered: the conversation has begun from the more general point that, if any original thing (or else any genus), for example the original Animal, is thought of as constituent part of its species, for example the particular animals, and not as separate from them, then there is no perplexity. Nevertheless, when it comes to numbers, we realize that they cannot be considered as species of the genus One, just as particular animals are considered as species of the genus Animal.

1085a31-1085b4: Aristotle formulates here an argument about the geometrical objects which resembles the argument which was presented earlier at 1085a7-20. He once more appeals to the general principle that the things which come from the same principles and are not thus differentiated are identical. Therefore, he reduces those who thought of the material principle of all geometrical objects as “something analogous to \( \pi \lambda \iota \beta \delta \alpha \)” to the same dilemma to which he reduced those who talked about “the kinds of great and small”.

Furthermore, if the material principles of the line, the plane and the solid are different from each other, then, according to Aristotle, there is no way in which the plane can have lines. So, either the plane does not contain lines or the plane has to be a line. We may also consider the similar case of Plants and Animals as species of the

\[117\] Annas (1976), 185
genus Living Things. Since they are different species of the same genus, an animal does not contain a plant but both an animal and a plant are living things. In general, an indivisible species contains all the attributes of the species and genera from which it derives.

The person who tried to generate the geometrical objects from the point and something analogous to plurality is generally considered to be Speusippus, against whom the next few arguments are all directed.

1085b5-13: In this section, Aristotle seems to claim that Speusippus’ alternative, namely that the number is not generated from the Indefinite Dyad but from the Plurality, inherits all the problems encountered by Platonists (who are referred to here as the supporters of the theory that the principles of the number are the One and the Indefinite Dyad).

The phrase “the same difficulties” seems to suggest the passage from line 1082a20. However, the point is different here. There, the question concerned the way in which the units are combined into numbers, while here the question concerns the way in which the formal and the material principle are combined to form the numbers. The similarity between the two passages is that in both cases the same question arises: how can numbers be composed of forms without being themselves forms?

It is finally worth examining the meaning of the terms μίξις, θέσις and κράσις here. “Mixture” has the meaning of complete fusion, “(com)position” implies intentional arrangement and “blending” is the kind of mixture which belongs to fluids (though sometimes μίξις and κράσις are used interchangeably). The accumulation of all these terms probably aims at laying emphasis on the serious difficulty arising from the Speusippean concept and also at exhausting all the possible ways in which the principles can coexist.

1085b13-23: Having rejected the alternative that the units of which each number consists are the original One, Aristotle deals here with the question how the units come from One and Plurality, once more addressing his critique to the theories of
Speusippus. It could be a matter of discussion whether Speusippus claimed to derive units from One and Plurality or only numbers. In any case, it seems unlikely that either Plato or Speusippus had said anything definite on this topic.

According to Aristotle, it is difficult to see how Plurality could contribute anything to the formation of units, given that a unit is unable of being pluralized. Nevertheless, it is also difficult to see a unit as a definite and indivisible part of a plurality, since the idea that a divisible plurality is composed of indivisible parts is absurd.

Aristotle rejects the idea that units can be analyzed as One and a part of Plurality for the following reasons: firstly, the elements will no more be the One and the Plurality, but the One and a part of the Plurality. Secondly, this plurality of indivisible parts, which is supposed to be an element of number, will be itself a number, given that Speusippus did not believe in ideal but only in ordinary mathematical number. However, this second objection would have force only if Speusippus had defined “number” as plurality of indivisible parts. It would then be circular to define the units in terms of the plurality of indivisible parts, which is the number.

1085b23-1086a11: In this section, Aristotle draws a distinction between an “infinite plurality” and a “finite plurality”. It should be noted here that what is in general meant by “plurality” is the indeterminate possibility of taking more. At 1083b36, Aristotle has already asked whether number is finite or infinite. Here, the question is whether the number which we have shown the original Plurality to be is finite or infinite. It follows in a rational way that it is finite. However, this seems absurd, since the Plurality is by nature infinite. Therefore, according to Aristotle, we seem obliged to accept that there are two kinds of plurality, i.e. a finite plurality from which the One and the units were derived and an infinite plurality, which is the original Plurality. But, which sort of plurality is to be considered as the material principle in number?
Furthermore, in the case of geometrical magnitudes there seems to be an even
greater difficulty than the perplexity encountered in the case of numbers. Aristotle
suggests that points could be composed of the original Point and a sort of indefinite
distance.

1085b35-1086a20: This section is a conclusion of the previous ones. Different
members of the Academy are said by Aristotle to be right to some extent. However,
Aristotle once more repeats that the theories of the Academy are wrong in
fundamentals, and that the right way to challenge them is to criticize their initial
assumptions.

At 1086a9, by oí μέν, Aristotle makes reference to Plato and his successors in the
Academy. There is a correspondence between oí μέν at 1086a9 and oí μέν at 1086a2,
oí δέ at 1086a5, ὁ δέ πρῶτος θέμενος τὰ εἴδη εἶναι at 1086a11. It is noteworthy that
the Pythagoreans seem to be left aside and that the focus is on Plato and his
successors in the Academy. Now, what are the points which Aristotle accepts as
correct in the Academy? To Aristotle’s mind, Plato is right to keep Forms and
numbers separate; Speusippus is right to reject Forms altogether; and finally
Xenocrates is right to identify Forms and numbers on the ground that both come
from the same principles and also right to claim that there are not different kinds of
principles.

1086a21-26: Having finished with numbers, which is the second inquiry, Aristotle
now goes on to deal with principles and causes, which is the third inquiry. The rest
of M is something of a parenthesis or preface before the third inquiry is taken up in
N.

In this section, Aristotle stresses that the discussion now must focus on the
statements of those who believe in non-sensible substances, namely Forms and
numbers, whose principles are considered to be the principles of all things.
According to Ross\textsuperscript{118}, the reference to the treatise that is relevant to physics implies more probably *Physics* i 4-6, *De Caelo* iii 3, 4 and *De Generatione et Corruptione* i 1, where the views of the materialists are discussed.

**1086a26-35:** Aristotle makes here reference to two different questions which arise:

1) whether the Forms and the numbers could serve as elementary principles of things and  

2) whether the account given by the Platonists of the principles of Forms and numbers is satisfactory.

Aristotle emphasizes mainly on the criticism of the erroneous belief in the separate existence of Forms that is held by the Platonists. Indeed, they treat the universal intelligible objects as beings that differ in kind from the particular sensible ones. Therefore, according to them, the universal objects cannot be properties of the sensible objects. So, the universal objects, i.e. Forms, must necessarily exist separately, in the same way in which the particular beings exist.

Διηπόρηται suggests a reference to Book B. In particular, B 1003a7 is quite relevant.

**1086a35-1086b13:** According to Annas\textsuperscript{119}, Aristotle claims here that the two intellectual antecedents he already has posited for the theory (the flux argument and Socrates’ search for definitions) together lead to a self-contradictory search for items which are both universals and particulars.

\textsuperscript{118} Ross (1924), v. 2, 462  
\textsuperscript{119} Annas (1976), 188
<table>
<thead>
<tr>
<th>SECTION</th>
<th>ANCIENT GREEK TEXT</th>
<th>TRANSLATION</th>
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<tr>
<td>1086b14-20</td>
<td>Ο δὲ καὶ τοῖς λέγουσι τὰς ἱδέας ἔχει τινὰ ἀπορίαν καὶ τοῖς μὴ λέγουσιν, καὶ κατ᾽ ἀρχὰς ἐν τοῖς διαπορήμασιν ἐλέχθη πρότερον, λέγωμεν νῦν. Εἰ μὲν γὰρ τις μὴ θήσει τὰς οὐσίας εἶναι κεχωρισμένας, καὶ τὸν τρόπον τούτον ὡς λέγεται τὰ καθ’ ἐκαστα τῶν ὄντων, ἀναφέρει τὴν οὐσίαν ὡς βουλόμεθα λέγειν· ἂν δὲ τις θῇ τὰς οὐσίας χαριστάς, πῶς θήσει τὰ στοιχεῖα καὶ τὰς ἀρχὰς αὐτῶν;</td>
<td>Let us now mention a point which presents some difficulty, both to those who believe in the Forms and to those who do not, and which was stated at the beginning in the <em>Discussion of Problems</em>. Let us now mention a point which presents some difficulty, both to those who believe in the Forms and to those who do not, and which was stated at the beginning in the <em>Discussion of Problems</em>. For, if one does not suppose the substances to be separate, and in the way in which the particular existing things are said to be separate, one will do away with the substance as we wish to describe it; but if one supposes the substances to be separate, how is one to conceive the elements and principles of them?</td>
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<tr>
<td>1086b20-31</td>
<td>Εἰ μὲν γὰρ καθ’ ἐκασταν καὶ μὴ καθόλου, τοσαυτ’ ἦσται τὰ ὄντα διασπερ τὰ στοιχεῖα, καὶ οὐκ ἐπιστητὰ τὰ στοιχεῖα (ἐστασαν γὰρ αἱ μὲν ἐν τῇ φωνῇ συλλαβαὶ οὐσία τὰ δὲ στοιχεῖα αὐτῶν στοιχεῖα τῶν οὐσιῶν· ἀνάγκη δὴ τὸ ΒΑ ἐν εἶναι καὶ ἑκάστην τῶν συλλαβῶν μίαν, εἰπερ μή</td>
<td>If they are particular and not universal, the existing things will be as many as the elements, and the elements will not be knowable. If they are particular and not universal, the existing things will be as many as the elements, and the elements will not be knowable. For, let the syllables in the speech be substances, and their elements be elements of the substances; it is then necessary that there is only one BA and one of each of the syllables, if they are not universal and the same in form, but each is one in number.</td>
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καθόλου καὶ τῷ εἴδει αἱ αὐταὶ ἀλλὰ μὲν ἐκάστη τῷ ἀριθμῷ καὶ τόδε τι καὶ μὴ ὁμόνυμον· ἐτι δ΄ αὐτὸ δ΄ ἐστιν ἐν ἐκαστὸν τιθέασιν· εἰ δ΄ αἱ συλλαβαί, οὔτω καὶ εἰ ὡς ἐστιν· οὐκ ἐσται ἀριθμὸ τινὶ ἄλφα ἕνος, οὔδε τῶν ἀλλῶν στοιχείων οὐθὲν κατὰ τὸν αὐτὸν λόγον ὅπερ οὐδὲ τῶν ἄλλων συλλαβῶν ἡ αὐτὴ ἄλλη καὶ ἄλλη· and a “this” and not homonymous (and further they suppose that each original thing is one); and, if this is true of the syllables, it is also true of the parts of which they are composed; so, there will not be more than one “a”, nor more than one of any of the other elements by the same argument, according to which there cannot be many instances of the same syllable in the case of other syllables.

1086b31-37

ἀλλὰ μὴν εἰ τοῦτο, οὐκ ἔσται παρὰ τὰ στοιχεῖα ἔτερα ὄντα, ἀλλὰ μόνον τὰ στοιχεῖα· ἐτι δὲ οὐδ’ ἐπιστητὰ τὰ στοιχεῖα· οὐ γὰρ καθόλου, ἡ δ’ ἐπιστήμη τῶν καθόλου· δῆλον δ’ εκ τῶν αποδείξεων καὶ τῶν ὁρισμῶν, οὐ γὰρ γίγνεται συλλογισμός ὅτι τόδε τὸ τρίγωνον δύο ὀρθαίς, εἰ μὴ πᾶν τρίγωνον δύο ὀρθαί, οὐδ’ ὅτι ὅδε ὁ ἄνθρωπος ζῷον, εἰ μὴ πᾶς ἄνθρωπος ζῷον· But if this is true, then there will not be other existing things besides the elements, but only the elements. Again, the elements will not be knowable; for they are not universal, but knowledge is of the universals. This is clear both from the demonstrations and from the definitions; for it is no syllogism that this triangle has its angles equal to two right angles, unless every triangle has its angles equal to two right angles, nor that this human being is an animal, unless every human being is an animal.
ἀλλὰ μὴν εἴγε καθόλου αἱ ἀρχαί, ἢ καὶ ἐκ τούτων ὄσια καθόλουⁱ²⁰, ἕσται μὴ ὄσια πρῶτερον ὄσιαςⁱ²¹. τὸ μὲν γὰρ καθόλου ὄν ὄσια, τὸ δὲ στοιχεῖον καὶ ἢ ἀρχή καθόλου, πρῶτερον δὲ τὸ στοιχεῖον καὶ ἢ ἀρχή ὃν ἀρχή καὶ στοιχεῖον ἐστίν. Ταύτα τε δὴ πάντα συμβαίνει εὐλόγως, ὅταν ἐκ στοιχείων τε ποιῶσι τὰς ἰδέας καὶ παρὰ τὰς τὸ αὐτὸ εἴδος ἑχούσας ὄσιας ἐν τι ἀξιῶσι εἶναι κεχωρισμένον.

But, if the principles are universal or substances that are universal are derived from them too, then non-substance will be prior to substance; for the universal is not a substance, while the element or the principle is universal, and the element or the principle is prior to the things of which it is the element or the principle. And all these difficulties follow reasonably, when they derive the Forms from elements and claim that there is a single separate entity apart from the substances that have the same form.

εἰ δὲ μηθὲν κωλύει ὥσπερ ἐπὶ τῶν τῆς φωνῆς στοιχείων πολλὰ εἶναι τὰ ἄλφα καὶ τὰ βῆτα καὶ μηθὲν εἶναι παρὰ τὰ πολλὰ αὐτὸ ἄλφα καὶ αὐτὸ βῆτα, ἔσονται ἀνεκά γε τούτου ἀπερσοί αἱ ἡμοῖαι συλλαβαί. Τὸ δὲ τὴν ἐπιστήμην εἶναι καθόλου πάσαι, ῥώστε ἄναγκαιον εἶναι καὶ τὰς τῶν ὄντων

And if, as with the elements of speech, there is nothing to prevent many a’s and b’s from existing without there being an original “a” and an original “b” besides the many, then due to this there may be an infinite number of similar syllables. But the statement that all knowledge is universal, and thus it is necessary that the principles of the existing things are also universal and not

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¹²⁰ 1086b37-1087a1: αἱ ἐκ τούτων ὄσιαι καθόλου Ab, Ross· ἐκ τούτων ὄσιαι καθόλου E, Jaeger
¹²¹ 1087a1-2: ἕσται μὴ ὄσια πρῶτερον ὄσιας Ross· ἕσται μὴ ὄσια πρῶτερον ὄσιας Christ. Christ’s reading seems preferable, since the meaning of this sentence makes it the main clause of the conditional that is introduced by “εἴγε καθόλου αἱ ἀρχαι ἢ καὶ ἐκ τούτων ὄσιαι καθόλου”.

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ἀρχὰς καθόλου εἶναι καὶ μὴ οὐσίας κεχωρισμένας, ἔχει μὲν μάλιστ᾽ ἀπορίαν τῶν λεχθέντων, οὐ μὴν ἄλλα ἐστὶ μὲν ὡς ἀληθές τὸ λεγόμενον, ἐστι δ᾽ ὡς οὐκ ἀληθές.

separate substances, presents indeed the greatest difficulty of all the points mentioned, but there is one sense in which what is stated is true, although there is another in which it is not true.

Ἡ γὰρ ἐπιστήμη, ὥσπερ καὶ τὸ ἐπίστασθαι, διττόν, ἧν τὸ μὲν δυνάμει τὸ δὲ ἐνεργείᾳ. Ἡ μὲν οὖν δύναμις ὡς ὕλη καθόλου οὖσα καὶ ἀόριστος τοῦ καθόλου καὶ ἀόριστον ἐστίν, ἡ δ᾽ ἐνέργεια ὡρισμένη καὶ ὡρισμένου, τόδε τι οὖσα τουτερὸν τινος, ἀλλὰ κατὰ συμβεβηκὸς ή ὁψις τοῦ καθόλου χρώμα ὁρᾷ ὅτι τόδε τὸ χρώμα ὃ ὁρᾷ χρῶμα ἐστιν, καὶ ο ἰθεωρεί ὁ γραμματικὸς, τόδε τὸ ἄλφα ἄλφα· ἐπεὶ εἰ ἀνάγκη τὰς ἀρχὰς καθόλου εἶναι, ἀνάγκη καὶ τὰ ἐκ τούτων καθόλου, ὅσπερ ἐπὶ τῶν ἀποδείξεων· εἰ δὲ τούτο, οὐκ ἔσται χωριστὸν οὐθὲν οὐδ᾽ οὐσία.

For the knowledge, like the knowing, is spoken of in two ways, one potential and one actual. The potentiality, being as matter universal and indefinite, is of what is universal and indefinite, while the actuality, being definite, is of something definite, and, being a “this”, is of a “this”. But per accidens the sight sees universal color, for the individual color which it sees is a color; and the grammarian’s object of study, namely this individual “a”, is an “a”; for, if it is necessary that the principles are universal, it is also necessary that the things derived from them are universal, as in the demonstrations; and if this is true, there will be nothing which is separate, i.e. no substance. But it is clear that the knowledge is in one sense universal, but in another sense it is not.

122 1087a17: ὡς ὕλη τοῦ καθόλου Ross· ὡς ὕλη καθόλου Bonitz, Christ
καθόλου, ἔστι δὲ ἂς οὐ.
OVERVIEW:

In M.10, Aristotle addresses his criticism to the Theory of Forms, especially because it assigns separate existence to universal things.

- There is a difficulty that is faced both by those who do not believe in the Forms and by those who believe in them: if we do not suppose the substances to exist separately, we will do away with the substance. However, if we suppose the substances to exist separately, how are we to conceive their elements?
- If they are to be particular and not universal things, then: (a) the existing things will be as many in number as their elements, and (b) the elements will be unknowable. For instance, let the syllables be substances and also let their elements (i.e. letters) be the elements of substances.
- Then: (a) there will be only one instance of each syllable and also only one instance of each letter. Hence, there will be nothing but the letters. In other words, there will be no other existing things besides the elements, but only the elements. (b) Moreover, the elements will be unknowable, given that there can be knowledge only of universal things.
- However, if the principles are universal or substances that are universal are derived from them, then non-substance will be prior to substance; for the universal is not a substance, while the element or the principle is universal, and the element is prior to the things of which it is the element.
- All these difficulties follow reasonably, when the Forms are derived from elements and the Forms are treated as existing apart from the substances that have the same form.
- However, if, there are many a’s and b’s without there being an original “a” and an original “b” besides the many instances of them, then there may be infinitely many similar syllables.
Nevertheless, the statement that all knowledge is universal, and thus it is necessary that the principles of the existing things are also universal and not separately existing substances is true only in a sense.

The potentiality is universal and indefinite, while the actuality is definite and particular. For example, the grammarian’s object of study, namely this individual “a”, is an “a”.

If it is necessary that the principles are universal, it is also necessary that the things derived from them are universal. If this is true, then there will be nothing which exist separately, i.e. no substance. It is clear that the knowledge is in one sense universal, but in another sense it is not.
COMMENTARY:

1086b14-20: In this section, Aristotle says that, if one does not suppose substances to exist separately, and in the way in which particular things are said to exist, one will destroy the substance as we know it. The question of the chapter is a general one, involving those who do not believe in the Forms as well as those who do. Aristotle dismisses the supposition that substances do not exist separately, and passes to the supposition that they do, which occupies the rest of the chapter. The question is how the proponent of the separate existence of substances is to think about the way in which the elements and principles of them exist.

1086b20-31: The main point of these lines may be put thus:

(a) If each letter of the alphabet were a unique individual, then you could never by any process of composition get any more just A or B or C etc., each occurring once. You could not have, for instance, a syllable BA and a syllable BC.

(b) Similarly, if the elements of substances were unique individuals, substances would be limited in their number which would not surpass the number of the elements but could be considerably smaller.

1086b37-1087a7: Aristotle says here that, if the principles of the particular beings are universal things, then either universal things will be more basic than particulars, or particulars will have principles that are less basic than they are. Aristotle regards both alternatives as unacceptable, in case the particulars are the Forms.

The problem is summed up at 1087a4-7, where characteristically a distinction is drawn, which enables us to see that the difficulty is only apparent and that the dilemma can thus be overcome. According to Aristotle, the trouble lies in accepting the Platonist premise that over and above the individuals sharing a common form there is a separate extra entity, the Form.
The Platonists assume that the only satisfactory way to explain the fact that things share a common form and a common name is to posit another thing over and above them, the Form.

But, if one accepts that a separate Form is necessary to explain this fact, then they are in a dilemma. For the Forms as basic particulars cannot have principles or elements, which lead to manufacturing more Forms. However, there can be elements of basic particulars without their being Forms.

1087a7-15: Things can be one in kind and share a common form without there being an extra entity over and above them to explain the fact of their sharing a common form. It is thus wrong to assume that things cannot share a common form without there being another thing to explain this. In the case of letters, for instance, the main point may be put thus: if each letter of the alphabet were a unique individual, then you could never by any process of composition get any more than just A, B, C etc., each occurring once. Similarly, if the elements of substances were unique individuals, substances would be miserably limited in their number. The numerical singleness of the Form is a concept that is actually believed in by the Platonists. Aristotle shows that, if the elements are universal, either non-substance will be prior to substance or the substances will be universals; however, if this is so, then the very notion of substances -which is that they are separate- is contradicted.

1087a15-25: In a sense, there can be knowledge of individuals, if a distinction is drawn between actuality and potentiality. This means that the knowledge of the universal is merely potential, and is only actualized when individuals are brought into the picture. For example, the grammarian actually knows an A only when he is faced by some such shape. As a consequence, we do not have to say that the elements or first principles are unknowable if they are individuals.

However, there are two problems:
a) Are we entitled to call this simple recognition an instance of knowledge at all, despite the lack of reasoning and connections between concepts? Aristotle seems to be unclear about what the conditions of knowledge are to be.

b) The second question is how knowledge of first principles can be reconciled with the claim that they are individuals. Usually knowledge is opposed to sensation as being of the universal, while sensation is of the particular. Nevertheless, occasionally Aristotle admits that knowledge is of the universal in the particular, in the sense that the knowledge of the particular should relate to the knowledge of the universal. Such a knowledge is not *per accidens* (κατὰ συμβεβηκός). On the contrary, the embodiment of the knowledge of the universal in the knowledge of the individual is *per accidens*\(^{123}\).

\(^{123}\) Crubellier (1994), 388
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<td>1087a29-1087b1</td>
<td>Περὶ μὲν οὖν τῆς οὐσίας ταύτης εἰρήσθω τοσαῦτα, πάντες δὲ ποιούσι τὰς ἀρχὰς ἑναντίας, ὡσπερ ἐν τοῖς φυσικοῖς, καὶ περὶ τὰς ἀκινήτους οὐσίας ὁμοίως. Εἰ δὲ τῆς τῶν ἀπάντων ἀρχῆς μὴ ἐνδέχεται πρῶτον τί εἶναι, ἀδύνατον ἄν εἰπή τὴν ἀρχήν ἑτέρον τι οὖσαν εἶναι ἀρχήν, οἷον εἰ τις λέγω τὸ λευκὸν ἀρχὴν εἶναι οὐχ ἢ ἑτέρων ἀλλ᾽ ἢ λευκὸν, εἶναι μέντοι καθ᾽ ὑποκειμένου καὶ ἑτέρων τι ὁν λευκὸν εἶναι ἀκινῆσαι γὰρ πρῶτον ἔσται. Ἀλλὰ μὴν γίγνεται πάντα εἰς ἑναντίον ὡς ὑποκειμένου τινὸς· ἀνάγκη ἄρα μάλιστα τοῖς ἑναντίοις τοῦθ᾽ ὑπάρχειν.</td>
<td>So, as far as this sort of substance is concerned, let what we have said suffice; now, everyone makes the principles contraries, just as in the natural things, so too as regards the unchangeable substances. However, if nothing can be prior to the first principle of everything, then it should be impossible for the first principle to be a first principle as being something else; this would be as if somebody were to say that white is a principle, not qua being something else, but qua being white, but that it is also predicatable of an underlying subject and that it is white as being something else; for that thing will be prior to it. However, everything comes into being from contraries as from an underlying subject; therefore, this underlying subject must be most of all present in the case of contraries.</td>
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<td>1087b1-4</td>
<td>Αἰ τὰ ἑναντία καθ᾽ ὑποκειμένου καὶ οὐθέν χωριστὸν, ἀλλ᾽ ὡσπερ καὶ φαίνεται οὐθέν οὐσία ἑναντίον,</td>
<td>Therefore, all the contraries are always predicatable of an underlying subject, and none of them is separate. But, as it also appears, nothing</td>
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καὶ ὁ λόγος μαρτυρεῖ. Οὕθεν ἄφα τῶν ἐναντίων κυρίως ἀρχή πάντων ἄλλ᾽ ἐτέρα.

But they make the one of the contraries matter, some opposing the unequal to the One as the equal, taking the former to be the nature of plurality, and others opposing the plurality to the One. The numbers are generated according to the former thinkers from the twoness of the unequal, i.e. from the great and small, and according to the latter from the plurality, but according to both they are generated by the being of the One; for, the thinker who says that the elements are the unequal and the One, and that the unequal is a Two composed of the great and small, treats the unequal, i.e. the great and small as one thing, and does not draw the distinction that they are one thing in definition but not in number.

But they again do not name rightly the first principles, which they call elements. Some of them speak of the great and small together with the One, taking these three to be elements of numbers, the two

| 1087b21-27 | Πλήν τοῦ αὐτοῦ γε λόγου ἐστὶ τὸ ὑπερέχον καὶ ὑπερχόμενον εἶναι ἀρχάς ἀλλὰ μὴ τὸ μέγα καὶ τὸ μικρὸν, καὶ τὸν ἀριθμὸν πρώτουν τῆς δυάδος ἐκ τῶν στοιχείων καθόλου γὰρ ἀμφότερα μᾶλλον ἐστὶν. Νῦν δὲ τὸ μὲν λέγουσι τὸ δ᾽ οὐ λέγουσιν. Οἱ δὲ τὸ ἐτερον καὶ τὸ ἄλλο πρῶς τὸ ἐν ἀντιτιθέασιν, οἱ δὲ πλῆθος καὶ τὸ ἐν. |
| 1087b27-33 | Εἰ δ᾽ ἐστίν, ὡσπερ βουλοῦνται, τὰ ὄντα εἴ, ἕναντιν, τῷ δὲ Ἐνὶ ἢ οὕθεν ἕναντιν ἢ εἴπερ ἄρα μέλλει, τὸ πλῆθος, τὸ δ᾽ ἄνισον τῷ ἰσο καὶ τὸ ἐτερον τῷ ταύτῳ καὶ τὸ ἀλλὸ αὐτῶ, μάλιστα μὲν οἱ τὸ ἐν τῷ πλῆθει ἀντιτιθέντες ἔχονται τίνος |

| 212 | та μὲν δύο ὑλήν τὸ δ᾽ ἐν τὴν μορφήν, οἱ δὲ τὸ πολὺ καὶ ὡλίγον, ὅτι τὸ μέγα καὶ τὸ μικρὸν μεγέθους οἰκειότερα τὴν φύσιν, οἱ δὲ τὸ καθόλου μᾶλλον ἐπὶ τούτων, τὸ ὑπερέχον καὶ τὸ ὑπερχόμενον. Διαφέρει δὲ τούτων οὕθεν ὡς εἰπεῖν πρὸς ἕνα τῶν συμβαινόντων, ἀλλὰ πρὸς τὰς λογικὰς μόνον δυσχερείας, ὡς φυλάττονται διὰ τὸ καὶ αὐτοὶ λογικὰς φέρειν τὰς ἀποδείξεις. |

former as matter and the One as form. Others speak of the many and few, because the great and small are more appropriate in their nature to magnitude; and others speak rather of the universal term covering these, i.e. the exceeding and the exceeded. And none of these variations makes any difference, so to speak, as far as some of the consequences are concerned, but they make a difference only with respect to the abstract difficulties, which they try to avoid because they themselves put forward abstract demonstrations.

Except that, if the exceeding and the exceeded, but not the great and the small, are the first principles, then by the same argument number should come from the elements before the Two does; for both are more universal. But as it is, they assert the one of these things, but not the other. Others, again, oppose the different and the other to the One, while others oppose the plurality to the One.

However, if the existing things come from contraries, as they like to claim, and to the One either there is nothing contrary, or, if there is to be something, it is the plurality, and the unequal is contrary to the equal and the different is contrary to the same and the other is
δόξης, οὐ μὴν οὐδ᾿ οὕτως ἰκανῶς ἐσται γὰρ τὸ ἐν ὀλίγων·
πλῆθος μὲν γὰρ ὀλιγότητι τὸ δὲ πολὺ τῷ ὀλίγῳ ἀντίκειται.

It is evident that the One means a measure. And in every case there
is something else which is underlying subject for it, as is a semi-tone
in a musical scale, a finger or foot or something of this sort in a
magnitude, a beat or a syllable in rhythms; and similarly in weight
some definite weight; and in the same way in all cases, a quality in
qualities, a quantity in quantities, and the measure is indivisible,
either in kind or with reference to the senses, for the One is not
something which is a substance in its own right.

And this is reasonable; for the One means that it is a measure of
some plurality and the number means that it is a measured plurality,
i.e. a plurality of measures. Thus, it stands to reason that the One is
not a number; for the measure is not measures, but both the One and
the measure are starting points. The measure must always be one
and the same thing applying to all alike; for instance, if the measure
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<td>ἄνθρωπος, ἄνθρωπος 125. Εἰ δὲ ἄνθρωπος καὶ ἵππος καὶ θεός, ζῷον ἵππος, καὶ ὁ ἄριθμος αὐτῶν ἔσται ἦμα. Εἰ δὲ ἄνθρωπος καὶ λευκόν καὶ βαδίζον, ἥκιστα μὲν ἄριθμος τούτων διὰ τὸ ταῦτα πάντα ὑπάρχειν καὶ ἕνι κατὰ ἄριθμόν, ὃμως δὲ γενέων ἔσται ὁ ἄριθμος ὁ τούτων, ἢ τινος ἄλλης τοιαύτης προσηγορίας.</td>
<td>is horse, it applies to horses, and if the measure is human being, it applies to human beings. And if the things to be measured are a human being and a horse and a god, the measure will perhaps be living being, and the number of them will be a number of living beings. And if the things to be measured are a human being and a white thing and a walking thing, there will hardly be a number of these, since they all belong to the same subject which is one in number, but the number of these will be a number of kinds of being, or of some other such term.</td>
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<td>1088a15-21 Οἱ δὲ τὸ ἄνισον ὡς ἐν τι, τὴν δυάδα δὲ ἀκριστον ποιούντες μεγάλου καὶ μικροῦ, ποῦρως λιὰν τῶν δοκοῦντων καὶ δυνατῶν λέγουσιν· πάθη τε γὰρ ταῦτα καὶ συμβεβηκότα μάλλον ἢ ὑποκείμενα τοῖς ἀριθμοῖς καὶ τοῖς μεγέθεσιν ἔστι, τὸ πολὺ καὶ ὅλιγον ἄριθμοῦ, καὶ μέγα καὶ μικρὸν μεγέθους, ὥσπερ ἀρτιόν καὶ περιττόν, καὶ λείον καὶ τραχύ, καὶ εὐθὺ καὶ καμπύλον.</td>
<td>Those who treat the unequal as one thing, and the Two as an indefinite compound of great and small, hold views which are very far from being plausible and possible. For these are affections and properties rather than substrates of numbers and magnitudes: the many and few are affections and accidents of number, and the great and small of magnitudes, just like even and odd are affections and attributes of number, and smooth and rough and straight and curved are affections and attributes of magnitudes.</td>
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125 1088a9-10: εἰ ἵπποι, τὸ μέτρον ἵππος, καὶ εἰ ἄνθρωποι, ἄνθρωπος Bonitz, Ross· εἰ ἵππος τὸ μέτρον, ἵππους, καὶ εἰ ἄνθρωπος, ἄνθρωπος codd.
1088a21-29 | Ετι δὲ πρὸς ταύτη τῇ ἁμαρτίᾳ καὶ πρὸς τι ἀνάγκη εἶναι τὸ μέγα καὶ τὸ μικρὸν καὶ ὅσα τοιαῦτα· τὸ δὲ πρὸς τι πάντων ἥκιστα φύσις τις ἢ οὐσία [τῶν κατηγοριῶν] ἔστι, καὶ ύστερα τοῦ ποιοῦ καὶ ποιου· καὶ πάθος τι τοῦ ποιου τὸ πρὸς τι, ἕστερο ἐλέξθη, ἀλλ᾽ οὐχ ὑλή, εἰ τι ἔτερον καὶ τὸ ὅλως κοινῷ πρὸς τι καὶ τοῖς μέρεσιν αὐτοῦ καὶ εἴδεισιν. Οὐθὲν γάρ ἐστιν οὐτε μέγα οὐτε μικρὸν, οὐτε πολὺ οὐτε ὀλίγον, οὐτε ὅλως πρὸς τι, ὃ οὐχ ἔτερον τι ὁ πολυ ἢ ὀλίγον ἢ μέγα ἢ μικρὸν ἢ πρὸς τι ἔστιν.

1088a29-1088b1 | Σημεῖον δ᾽ ὅτι ἥκιστα οὐσία τις καὶ ὅν τι τὸ πρὸς τι τὸ μόνου μὴ εἶναι γένεσιν αὐτοῦ μηδὲ φθοραν μηδὲ κίνησιν ὡσπερ κατὰ τὸ ποιοῦ αὐξήσις καὶ φθίσις, κατὰ τὸ ποιοῦ ἀλλοίωσις, κατὰ τὸ ποιοῦ φορά, κατὰ τὴν οὐσίαν ἢ ἀπλὴ γένεσις καὶ φθορά, ἀλλ᾽ οὐ κατὰ τὸ πρὸς τι· ἀνεν γὰρ τοῦ κινηθῆναι ὅτε μὲν μεῖζον ὅτε δὲ ἐλαπτὸν ἢ ἴσον ἔσται διατέρου κινηθέντος κατὰ τὸ ποιοῦ.

1088b1-4 | Ανάγκη τε ἐκάστου ὑλὴν εἶναι τὸ δυνάμει τοιοῦτον, ὡστε

Furthermore, on top of this mistake, the great and small and the like must be relative to something; and the relative is of all the things in the least degree a nature or substance of some kind, and it is posterior to both quality and quantity; and the relative is some affection of quantity, as has been said, but not matter, if there is something else that is relative for both the relative in general and its parts and kinds. For there is nothing either great or small, many or few, or relative in general, which is many or few or great or small or relative without being so as something else.

A sign that the relative is least of all a particular substance and a particular being is the fact that this alone does not have any generation or destruction or change, just as there is increase and diminution in quantity, alteration in quality, locomotion in place, simple generation and destruction in substance, but there is none of this in respect of the relative; for, without changing, a thing will be now greater and then smaller or equal if another thing changes in quantity.

It is also necessary that the matter of each thing, and thus also of a
καὶ οὐσίας· τὸ δὲ πρὸς τι οὔτε δυνάμει οὐσία οὔτε ἑνεργεία.
Ἀτοπον οὖν, μάλλον δὲ ἀδύνατον, τὸ οὐσίας μὴ οὕσιαν
ποιεῖν στοιχεῖον καὶ πρότερον· ύστερον γὰρ πᾶσαι αἱ
κατηγορίαι.

1088b5-13 Ἔτι δὲ τὰ στοιχεῖα οὐ κατηγορεῖται καθ’ ᾧ στοιχεῖα, τὸ δὲ
πολὺ καὶ ὀλίγον καὶ χαῖρε καὶ ἁμα κατηγορεῖται ἀριθμοῦ,
καὶ τὸ μακρὸν καὶ τὸ βραχὺ γραμμῆς, καὶ ἐπίπεδον ἐστὶ
καὶ πλατὺ καὶ στενὸν. Εἰ δὲ δὴ καὶ ἔστι τι πλῆθος οὗ τὸ
μὲν ἄει ὀλίγον126, οἷον ἡ δυάς (εἰ γὰρ πολὺ, τὸ ἐν ἀν ὀλίγον
εἰ), κἂν πολὺ ἀπλῶς εἰ, οἷον ἡ δεκὰς πολὺ, [καὶ] εἰ
ταύτης μὴ ἔστι πλεῖον, ἢ τὰ μύρια. Πᾶς οὖν ἕσται οὕτως ἐξ
ὀλίγου καὶ πολλοῦ ὁ ἀριθμός; Ἡ γὰρ ἁμφο ἐδει
κατηγορεῖσθαι ἡ μηδέτερον· νὸν δὲ τὸ ἑτερον μόνον
κατηγορεῖται.

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substance, is that which is potentially of such nature; however, the
relative is neither potentially nor actually a substance. It is therefore
absurd, or rather impossible, to make non-substance an element of,
and prior to, substance; for all the categories are posterior to
substance.

Furthermore, the elements are not predicated of the things of which
they are elements, but the many and few are predicated, both
separately and together, of a number, and the long and the short are
predicated of a line, and a plane can be both broad and narrow. If
there is then a plurality, of which few is always true, the Two for
example (if it were many, then the One would be few), then there
would also be one which is absolutely many, for example, Ten
would be many, if there is no number greater than ten, or Ten
Thousand. How then, in view of this, will number consist of few and
many? Either both ought to be predicated of it, or neither; but,
according to this account, only the one or the other is predicated.

126 1088a9-10: οὗ τὸ μὲν ἄει <τὸ> ὀλίγον Ross· οὗ τὸ μὲν ἄει ὀλίγον codd.
OVERVIEW:

In N.1, Aristotle argues against the theory that the two first principles are contraries. At first, he deals with both of them and then he divides the discussion into two parts, in the first of which he deals with the one contrary, i.e. the formal principle, while in the second one he makes reference to the other contrary, i.e. the material principle. The discussion of the material principle continues in N2.

It is not the first time that Aristotle deals with the first principles in the Metaphysics. In M9, especially at line 1086a21 ("περὶ δὲ τῶν πρώτων ἀρχῶν καὶ τῶν πρώτων αἰτίων"), Aristotle proceeds from the discussion of Form-numbers to the discussion of the first principles that produce them. So, it seems that the transition to the discussion of the first principles has already been made by Aristotle.

Therefore, the question is where N should begin judging from where Aristotle starts to deal with the topic of the first principles. It seems that there are two cases: either the original beginning of N was at 1086a21 and the copyist who made the division of books at 1087a29 felt it was necessary to make there an introduction to the subject of the first principles, or the sequence 1086a21-1087a25 did not originally constitute part of the main structure of M and N, but it was inserted by an editor, since it was thematically relevant. In any case, the distinction between the topics that are discussed in M and N is not well maintained.

In the beginning of N1, Aristotle claims that the previous philosophers treat the first principles as contraries. The identity of these previous philosophers is a matter of question here. According to Aristotle, every contrary must be predicated of a substrate like a property. However, the first principle cannot be a property of something else. Therefore, it is impossible for the first principle to be a contrary, given that it must be self-subsistent.

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127 Ross (1924), v. 2, 470
The Platonists are said by Aristotle to call the one of the contraries -which is the material principle- “Unequal” (or else “Dyad of the Great and Small”) or “Plurality”. The other of the contraries is called “One”, which is the formal principle. What needs to be clarified here is whether the Platonists really consider the one of the first principles as Form and the other as Matter or not. One further point that deserves to be discussed separately is whether Aristotle is fair when he claims that the Platonists’ material principle is in fact two principles instead of only one.

Moreover, the Platonists are said by Aristotle to describe the material principle either as the Great and Small or as the Many and Few or as the Exceeding and Exceeded. However, these three alternatives differ only verbally from each other. Others are said to oppose the Other or the Plurality to the One. Among the five options, according to Aristotle, the Plurality seems to be the most appropriate as contrary to the One. However, even this contrast is wrong, since the opposite of the Plurality is the Fewness, and it would follow that the One is few, which is impossible. Aristotle’s criticism here seems to be focused rather on the names used for the first principles than on the consistency of the Platonists’ arguments.

As far as the formal principle, i.e. the One, is concerned, Aristotle formulates the following points:

a) “One” evidently means a measure. For instance, the One in harmony is a semitone, in length it is a finger etc. and in general in qualities it is a quality and in quantities it is a quantity. So, there is no such substance as the “One Itself”.

b) Furthermore, the One is not a number, given that “one” means a measure of some plurality and “number” means a measured plurality or a plurality of measures.

c) Finally, a measure must necessarily be something common to the things that are measured by it. For example the One in a plurality of horses must be one horse.
It needs to be discussed here whether there is any connection between the “One” as a principle and the “One” as a measure or not and, if there is some connection between them, what this connection is. Moreover, it is a matter of question whether Aristotle’s criticism of the Platonists is fair, when he points out the difficulty in finding a common measure for a plurality of items that belong to different categories of things.

Afterwards, Aristotle points out that it is absurd to say that the Indefinite Dyad is composed of the Great and Small. For, the Great and Small are properties rather than the substrate of numbers and magnitudes. In addition, the Great and Small are relative terms and therefore belong to the least substantial of all categories. The relatives are not substances either potentially or actually. So, the relatives are non-substances and non-substances cannot be the principles of substances.

N1 ends with the assumption that, if there is to be a plurality which is merely “few”, for example the number Two, there has also to be a plurality which is merely “many”, for example the number Ten or the number Ten Thousand. But, according to Aristotle, if the Many and Few are constituent parts of the number, either both of them or neither should be predicable of it. Therefore, principles such as the Great and Small or the Many and Few cannot be thought of as constituent parts of the universe, since there are magnitudes that are considered as merely great or merely small and numbers that are considered as merely many or merely few.
COMMENTARY:

1087a29-1087b4: In this section, Aristotle points out that all the previous philosophers consider the first principles to be contraries and expresses his disagreement with this theory:

(a) All previous philosophers make the first principles contraries, both in the case of natural and thus changeable things of our experience and in the case of unchangeable things.

(b) The principles they introduce must be conceived as primary elements, to which nothing can be prior.

(c) However, in the case of contraries, there seems to be another element from which the contraries are derived, i.e. their underlying subject. This substrate is prior to the contraries. The same happens in the case of the white color. The material that bears the white color is prior to the white, because it acts as the pre-existing underlying material to which the white applies.

(d) Furthermore, nothing can be considered to be contrary to a substance.

(e) Therefore, the contraries of the previous philosophers cannot be the first principles of everything, but something else (i.e. their substrate) must.

It is noteworthy that at 1087a29, Aristotle talks about “this sort of substance”. By “this sort of substance”, Aristotle probably means the unchangeable and eternal substance which has been the subject of M (M.1, 1076a11: “ἔστι τις παρὰ τὰς αἰώνιτάς ψυχίας ἀκίνητος καὶ ἀΐδιος”). Aristotle has already discussed this sort of substance, since he has already referred to numbers and Forms as candidates for being the unchangeable and eternal substance. As far as numbers are concerned, he has reached the conclusion that the objects of mathematical sciences cannot exist separately from the individual things of our experience and thus it is difficult for them to be thought of as substances. As regards Forms, Aristotle again claims that they cannot easily be considered as entities that are distinct and exist separately from perceptible beings. Therefore, the discussion of the unchangeable and eternal
substance with reference to the numbers and the Forms has ended up in Aristotle’s reluctance to identify “this sort of substance” either with the former or with the latter. Now, he proceeds to the discussion about the first principles.

At 1087a29, by “πάντες” (“everyone”), Aristotle means all the previous philosophers who have attempted to explain the nature of things by means of models based on an opposition between the first principles. All these philosophers theorize in a similar way about the contrary elements of which the universe is composed. Aristotle’s main motive for disapproving such a belief is that, even though the contraries are not substances, these previous philosophers consider them to be capable of generating substances. This becomes clear at 1087b3. There, by the phrase “καὶ ὁ λόγος μαρτυρεῖ” (“and the argument confirms this”), Aristotle makes a reference to Physics (I.6, 189a32-33), where he wonders how it is possible for the substances to be generated from non-substances and also how non-substance can be prior to substance. Provided that the first principles are considered to be of the same nature as the beings that are generated from them, the Aristotelian point seems valid.

One possible response to Aristotle’s criticism against the opposition of the first principles might be, as Syrianus notes, that the previous philosophers do not in fact treat the opposite principles as insubstantial in the sense of being inferior to the substantial things, but as insubstantial in the sense of being superior to them. So, if one is to call the entities above the level of the natural existing things “contraries”, this means that they are not insubstantial, but in fact supra-substantial.\(^\text{128}\)

It is a matter of question whether Aristotle means by “πάντες” the previous philosophers as a whole or a particular group of them. It seems that Aristotle has a certain group of previous philosophers in mind and not all of them. Firstly, the opposites play a crucial role in the Presocratic philosophy. In particular, probably Thales, who introduced the principle of water, and certainly Anaximenes, who introduced the principle of air, introduced the contrary processes of rarefaction and condensation to account for the fact that all things are modifications of air. Moreover, Empedocles introduced Love and Strife as two opposite forces. It may be the case

\(^{128}\) Syr., 165.29-33

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that Aristotle starts with the generalization that all previous thinkers used contraries—which need not imply that there were no exceptions—in order to focus his discussion on the Platonists. Therefore, it seems that Aristotle has a particular group of the previous philosophers in mind that consists mainly of the Platonists, judging from what he says next.

The reason why a contrary cannot be a substance is, according to Aristotle, that the contraries are always attributes. For example, not being white is an attribute and the contrary to that attribute is being white, which is also an attribute. We can assume that any contrary is an attribute and thus it can be regarded as contrary only to an attribute. The substance is the material subject of which all the attributes—including the contraries—are predicated. Hence, no contrary is substance and there is no contrary to substance.

Moreover, it is against all reason to treat any substantial thing as contrary to any other substantial thing, for they cannot oppose each other in terms of being subjects or substances, but only in terms of having certain attributes. The claim that a real object has no contrary is also formulated in Categories (3b24-27), where Aristotle states that there is no contrary either to substances such as human being or to determinate quantities such as ten or two cubits long (unlike the cases of indefinite quantities indicated by terms such as “many” and “few” or “great” and “small”).

Undoubtedly, the contraries presuppose a substrate in which they are (alternately) present. The reason is that there cannot be any change from the one contrary to the other without an underlying subject which undergoes the change and remains through it. For instance, it is not the coldness that can be heated, but the subject of which it is predicated. So, there has to be an underlying subject for them, which changes from the one contrary to the other. All the previous philosophers seem to make the existing things come into being out of contraries as if the contraries themselves were the underlying subject. However, given that the contraries presuppose an underlying subject, there has to be an underlying subject which cannot be identified with the contraries, for the contraries are predicated of it. In the

129 [Alex.], 774.28-30
light of the previous philosophers’ thought, this underlying subject can be thought of as existing in the contraries themselves. This means that the contraries and their underlying subject can be thought of as being one thing. However, even in this case, the previous philosophers’ contraries cannot be the first principles of everything, for they presuppose the underlying subject which does not exist separately from them and which naturally comes first, given that the contraries are predicated of it.

In general, we can notice that, while discussing contraries, Aristotle makes a quick transition from the realm of existing things’ structure to the realm of language. It seems as if from the linguistic perspective the notions of the subject and the predicate are reflections of the primary substance and its attributes from the ontological perspective.

1087b4-12: According to the Platonists, the one contrary -which is the material principle- is the Unequal (or else the Dyad of the Great and Small) or the Plurality. The other contrary is the One, which is the formal principle. Aristotle’s criticism focuses on the fact that the Platonists treat the material principle as one element, while it is two elements.

By “οἱ δὲ” (“they”), Aristotle indicates a subclass of the original group of the previous philosophers, to whom he has referred to in the previous section. This subclass consists of the Platonists. According to Aristotle, the Platonists treat the one of the two contraries as matter.

The following “οἱ μὲν” and “οἱ δὲ” (“some” and “others”) is a further subdivision of the aforementioned subclass of the previous philosophers. Aristotle here seems to think of (at least) three persons, if not more. In particular, the subclass of the Platonists is divided into two groups of thinkers.

The first group is represented by the philosopher who claims that the elements, i.e. the first principles, are the One, which is equal, and the Unequal, which is equivalent to the duality of the Great and Small. This philosopher is undoubtedly Plato.
The second group is represented by the thinkers who claim that the first principles are the One and the Plurality. The question is whether or not we can identify them. According to the ancient commentators on Aristotle’s *Metaphysics*\(^{130}\), οἱ δὲ may indicate the Pythagoreans. However, we cannot know for sure that such a view was shared by the Pythagoreans. Moreover, it does not seem right to ascribe it to them only because they are said to have considered the number as a first principle and to have treated the first principles as contraries. As it is claimed by the modern scholars\(^{131}\), οἱ δὲ probably indicates Speusippos and his pupils and followers.

Aristotle’s objection against Plato is that, although he intends to speak of two principles in total and treats the Unequal as the one of them, he does not draw the distinction that the Unequal can be conceived as one entity in terms of definition, but it should be conceived as a plurality of entities in terms of number. The reason is that the Unequal is numerically two factors. The inequality is in fact identified with a duality of unequal entities, of which the one is the Great and the other is the Small.

It is noteworthy that, at 1087b12, Alexander chooses to interpret the passage as “ἀρθμῶ, λόγῳ δ’οὐ” (“in number but not in definition”) instead of “λόγῳ, ἀρθμῶ δ’οὐ” (“in definition but not in number”), which is the reading found in the manuscripts. Alexander’s choice, which apparently means that the Unequal is one thing in terms of number, but two things in terms of definition seems less plausible than what we read in the manuscripts. For, although the Unequal can be defined by a single definition, it contains within itself a plurality, which is the Great and Small.\(^{132}\)

Therefore, the Great and the Small are included in the same definition, i.e. the definition of the Unequal, but, given that they are two things, they can by no means be considered to be the same thing. It is noteworthy, as Crubellier\(^{133}\) points out, that for Aristotle being one “in definition” (λόγῳ) seems to be equivalent to unity “in terms of form” (ἐν τῷ εἰδέ), while being one “in number” (ἀρθμῶ) seems equivalent to being one “in matter” (τῇ ὑλῇ). Therefore, the couple of Aristotelian

\(^{130}\) [Alex.], 776.10-11 & Syr., 166.16-24  
\(^{131}\) Annas (1976), 195 & Ross (1924), v. 2, 470  
\(^{132}\) Ross (1924), v. 2, 470-471  
\(^{133}\) Crubellier (1994), 403
terms “ἐἰδεῖ-ὄλη” probably defines the couple “λόγῳ-ἀρθμῷ” (cf. Δ.6 1016b32-33, I.3 1054a33-35).

It seems that by “Great and Small” Plato refers to a single principle or else a single indeterminate duality. Alexander and Syrianus comment in favor of this point. According to Alexander, Plato calls his second principle “Unequal”, which is a more universal term than its variant, namely the “Great and Small”, and thus considers his second principle to be a single element acting as an underlying material for the first principle, which is the One\(^{134}\). According to Syrianus, the “Unequal” transcends the “Great and Small”, which are one in substrate. This shows that the Unequal is in fact thought of by Plato as one source from which multiplicity, procession, productivity and differentiation spring.\(^{135}\)

However, Aristotle seems to treat the Great and Small as two distinct factors instead of one single element. This seems to be assumed also at I.8 of *Metaphysics* (1083b23-25: “ἐτι πότερον ἐκάστη μονὰς ἐκ τοῦ μεγάλου καὶ μικρὸν ἑαυτθέντων ἑστίν, ἡ ἢ μὲν ἐκ τοῦ μικρὸν ἢ δ’ ἐκ τοῦ μεγάλου;”). This passage is usually taken as an Aristotelian misinterpretation of the fact that Plato’s material principle was thought of by the Platonists as a single indeterminate duality progressing towards the Great and Small. Nevertheless, in my opinion it is most likely that Aristotle is aware of the fact that Plato considered his second principle as one element. Besides, in the *Philebus* (24a-27d), Plato has already formulated the theory that there are four classes of things, of which the first is the Infinite and the second is the Finite. To the former class belong the things that can be infinitely more or infinitely less, such as hotter and colder. In this class we could also include greater and smaller. It seems that, according to Plato, the Infinite, which involves the opposite aspects of “more” and “less”, is one in number, but the lack of fixed quantity in it makes it movable and thus not unified. It is only by mixture and cooperation of the Finite that the Infinite acquires commensurability and harmony, due to the introduction of the number. So, Aristotle’s criticism could possibly be aimed not at pointing out that Plato had two material elements in mind instead of only one, but at remarking that Plato’s attempt

\(^{134}\) [Alex.], 776.8

\(^{135}\) Syr., 166.20-21
to establish his material principle as one single entity is unsuccessful. In *Physics* (I.9, 192a11-12), Aristotle again complains that Plato overlooks the existence of a second aspect inside the Dyad.

Aristotle seems to be right in that the term “Great and Small” in fact indicates two different and also opposite things. The material principle -or else the Dyad- could be thought of as an area of plurality in which the Great and the Small act in opposite ways and tend towards contrary directions. It seems impossible to discover any unity and cohesion in the sphere of the Great and Small. This is the reason why the effect of the formal principle -or else the One- is necessary for the order and the unity of structure to occur. Therefore, we could not think of the material principle as one unified element, given that it involves two opposite and incompatible aspects.

Furthermore, Aristotle criticizes the Academy for treating the one principle as form and the other as matter. According to Aristotle, the Unequal (or Great and Small or Plurality) is thought of as matter by the Academy. Therefore, it seems that the One is accordingly thought of as form. The reason is that the One is considered to be equal. According to Syrianus, the One can be thought of as equal in the sense that it is responsible for self-identity, permanence, uniformity and keeping the same state.\(^{136}\) So, the One could have been considered by the Academy as an agent giving a specific identity and form to the material substrate. Through the Aristotelian argumentation, it turns out that the Academy’s two opposite first principles are incapable of doing the job that Aristotle’s form and matter effectively do. However, it is a matter of question whether the treatment of the one principle as form and of the other as matter is an original Academic attempt or Aristotle’s own translation. It is likely that Aristotle ascribes to the Academy the use of two concepts, (i.e. form and matter), which he was the first to introduce. The Academics seem to have treated the One as a principle that defines the indefinite nature of the Unequal. Nevertheless, they do not seem to have had any intention of describing the One as “form” and the Unequal as “matter”. “Form” and “matter” are Aristotelian terms. Therefore,

\(^{136}\) Syr., 166.21-24
Aristotle’s criticism here is founded on what he -and probably not the Academic thinkers- believed about the distinction between the form and the matter.

1087b12-21: The Platonists describe the material principle either (a) as the Great and Small or (b) as the Many and Few or (c) as the Exceeding and Exceeded. According to Aristotle, these three alternatives differ only verbally from each other, since they all entail the same difficulties:

a) Apart from their general error of making contraries the first principles, the Platonists describe their elements in a bad way.

b) Firstly, some of them describe the material element as “the Great and Small”, which are closely associated to magnitude. But, the Great and Small are two material elements, which together with the formal element, i.e. the One, make three first principles instead of only two contraries.

c) Secondly, some of Plato’s disciples describe the material element as “the Many and Few”, altering the expression in order to make it more suitable to serve as the principle of numbers.

d) Thirdly, some other thinkers -whom we cannot safely identify- describe the material element as “the Exceeding and Exceeded”, because they are universal terms that successfully cover the other two pairs.

e) However, all these variations have to be rejected, for, in view of Aristotle’s objection in the previous section, they all oppose two instead of one first principle to the One.

Aristotle’s arguments here seem to be of a linguistic nature. The Platonists might really have used different terms in their attempt to define as clearly as possible the two opposite principles. Aristotle claims that none of these terminological variations makes any difference as regards the consequences of the Platonic theory. The reason is that the only problems they remove are the verbal ones.\(^\text{137}\) In other words, the problems that are removed by the use of the aforementioned alternative expressions concern the meaning of the terms. In particular, the meaning of the “Great and

\(^{137}\) Annas (1976), 196
Small” is restricted to magnitudes, while the meaning of the “Many and Few” applies to numbers. As for the meaning of the “Exceeding and Exceeded”, it covers both the meaning of the “Great and Small” and that of the “Many and Few”. Moreover, it involves an evaluation, since “being exceeding” means “being superior to what is exceeded”. According to the ancient commentators, the pair of Exceeding and Exceeded seems preferable to Aristotle in comparison to the pair of Great and Small and the pair of Many and Few, because it consists of more general and abstract terms instead of more specific and particular ones. Nevertheless, the problems that are removed, i.e. the abstract difficulties (“λογικὰς δυσχερείας”) are trivial, since the only thing that changes is the degree of the universality of the terms. For instance, it seems unreasonable that the Great and Small, which is used to describe the magnitudes, is the principle of everything. Consequently, the more abstract the terms the better the association of them with the abstract arguments and demonstrations used by the aforementioned philosophers. But, as far as the philosophical treatment of the principles is concerned, it remains practically the same. For, all these pairs of terms indicate a duality which makes two material principles instead of only one. Therefore, none of the alternative formulations of the Platonic theory as regards the material principle adequately supports the claim of the Academy that the first principles are contraries. For, what sort of opposition could there be between one aspect on the one hand and two aspects on the other? This fact divulges the inadequacy of the Platonic theory, according to Aristotle. However, Aristotle’s criticism does not seem to be fair, since the Platonists did not treat their material principle as two distinct elements, but as one single element of an indefinite twofold nature.

1087b21-27: If the Exceeding and Exceeded is the material principle rather than the Great and Small, then Number in general should be generated from the material principle before the number Two does. For, both the Exceeding and Exceeded and Number in general are more universal terms than the Great and Small and the

138 Syr., 167.11-12
number Two respectively. Aristotle’s criticism focuses on the Platonists’ failure to observe this fact:

a) Let us suppose that the material principle is the Exceeding and Exceeded and not the Great and Small.

b) Then, the Number as an entity should be derived from the Exceeding and Exceeded before the Two.

c) For, as the Exceeding and Exceeded is a more universal notion than the Great and Small, in the same way the general notion of Number is more universal than the particular notion of Two.

d) Therefore, Number should be treated by the Platonists as prior to the Two, since the Exceeding and Exceeded are considered to be prior to the Great and Small.

e) However, the Platonists do not treat Number as prior to the Two and this inconsistency is one more sign of the inadequacy of their theory.

f) Furthermore, there are some thinkers (“οἱ δέ”), who are probably influenced by the Platonists, but whom we cannot identify with certainty. According to these thinkers, the Different and the Other is the material principle. There are also some others (“οἱ δέ”), Speusippus being included, according to whom the Plurality is the material principle.

It is again a matter of question whether Aristotle’s criticism is fair here. It seems that Aristotle tries to point out the consequences of the Exceeding and Exceeded being preferred to the Great and Small as a more universal notion. However, it is not entailed by the aforementioned point that the Exceeding and Exceeded must necessarily produce Number in the universal sense instead of the particular numbers. The first principles need to have characteristics that can be typical of the various beings which are generated by them. Nevertheless, this does not mean that the first principles need to generate beings which are as abstract as they are. In other words, it is not true that the more universal the principles the more universal the beings which come from them. If this were true, then there should also be some particular things that would generate particular beings. However, such a theory
would not be consistent with the belief in the existence of abstract first principles out of which the totality of beings is created.

1087b27-33: Among the five options of name for the second principle, Plurality seems to be the most appropriate contrary to the One, according to Aristotle. However, even this contrast is wrong, since the opposite of Plurality is Fewness, and it would follow that the One is few, which is impossible:

a) The Platonists claim that the existing things are derived from contraries.

b) However, nothing is contrary to the One.

c) If it is assumed that there is to be a contrary to the One, this must be the Plurality.

d) For, the Unequal is the opposite of the Equal, the Different is the opposite of the Same and the Other is the opposite of the Thing-itself.

e) Therefore, the Plurality turns out to be the best candidate for the principle that is contrary to the One.

f) But, even in this case, the thinkers claiming that the first principles are the One and the Plurality seem to be in support of a weak claim. For, it is not the One that is opposite to the Plurality, but the Fewness. In numerical terms, the Fewness is represented by the number Two, which is the smallest number. However, it sounds totally absurd that the One is fewness. The One, as a principle, is thought of as one singular thing and not as more than one.

As far as the two last pairs of terminological contraries are concerned, i.e. the Same and Different and the Other and Thing-itself, Alexander notes that it is preferable for ἕτερον (i.e. “Different”) to be opposed to ταὐτό (i.e. “the Same”) and for ἄλλο (i.e. “the Other”) to be opposed to αὐτό (i.e. the Thing-itself)\(^\text{139}\). According to Alexander’s point, this is the right way to put the terms as pairs of contraries. However, earlier, in *Metaphysics* I.3 (1054b15-19: “καὶ τὸ ἄλλο μὲν ἀντικειμένως καὶ τὸ ταὐτό, διὸ ἄπαν πρὶς ἄπαν ἢ ταὐτό ἢ ἄλλο... τὸ μὲν οὖν ἕτερον ἢ ταὐτό διὰ τοῦτο πᾶν πρὶς πᾶν λέγεται”), the ἄλλο (i.e. “the Other”) is opposed to ταὐτό (i.e.

\(^{139}\) [Alex.], 777.30-778.3
“the Same”) and there is no trace of any distinction made by Aristotle between ἐτερὸν and ἄλλο, since these two terms seem to be synonymous and are alternatively used in the Aristotelian context. This is also suggested by the fact that they both are opposed to ταῦτα. So, probably one of the thinkers to whom Aristotle addresses his criticism used the terms “the Same” and “the Different”, while the other used the terms “the Thing-itself” and “the Other”, without making reference to different concepts. In conclusion, it all depends on what meaning is attributed to “ἕτερον” and “ἄλλο”. For, if the term “ἕτερον” is translated as “different”, then “ταῦτα” has to be the opposite, because it means “the same”. Similarly, if the term “ἄλλο” means “other or another”, then “ἀντί” has to be the opposite, because it means “the Thing-itself”. If the meaning attributed to both “ἕτερον” and “ἄλλο” is identical -which is most likely- then the two pairs of terms might be thought of as linguistic variants for the same pair of contraries, or else as synonyms. Aristotle once more expresses here his objection against the theory that makes the first principles contraries and focuses his criticism on the names used for the first principles by the previous philosophers rather than on the power and consistency of their arguments.

1087b33-1088a4:

As far as the other principle, i.e. the One, is concerned, Aristotle formulates some points in favor of the view that the One cannot be a first principle, since it is a measure:

a) The strictest sense of the One, both in terms of meaning and in terms of use, is “measure”. Aristotle has already expressed this view elsewhere in the Metaphysics (I.1 1052b20-22: “μέτρον γὰρ ἐστιν ὁ τὸ ποσὸν γιγνώσκεται: γιγνώσκεται δʼ ἐνὶ ἢ ἄρθρῳ τὸ ποσὸν ἢ ποσὸν, ὁ δʼ ἄρθρῳς ἅπαν ἑνὶ.” and also 1053b4-6: “ὅτι μὲν οὖν τὸ ἐνὶ εἶναι μάλιστα ἐστὶ κατὰ τὸ ὅνομα ἀφορίζοντι μέτρον τι, καὶ κυριώτατα τοῦ ποσοῦ, εἰτα τοῦ ποιοῦ, φανερὸν.”). The One is treated here by Aristotle as the minimum element, the minimal quantity of which has made it a common measure.

140 Ross (1924), v. 2, 472
b) There is something different in each class of things, which acts as a measure. The One is the principle which provides any measure with the property of being a measure.

c) The minimum quantity/thing, which operates as a measure, is, in the case of music, the semi-tone, in the case of magnitudes, the finger or the foot or something of this sort, in the case of rhythms, the beat or the syllable and, in the case of weight, some definite weight. A similar claim about the measures is put forward elsewhere in *Metaphysics* (Δ.6 1016b20-24 “ἀρχὴ οὖν τοῦ γνωστοῦ περὶ ἑκατὸν τὸ ἐν. οὐ ταῦτα δὲ ἐν πάσι τοῖς γένεσι τὸ ἐν. ἐνθα μὲν γὰρ δίειςις, ἐνθα δὲ τὸ φωνήμα ἢ ἀφωνον· βάρους δ' ἐτερον καὶ κινήσεως ἄλλον. πανταχοῦ δὲ τὸ ἐν ἢ τῷ ποσῷ ἢ τῷ εἶδει ἀδιαίρετον.”).

d) Moreover, in the case of the qualities the measure is some quality, while in the case of the quantities it is some quantity.

e) The measure is indivisible in kind on the one hand and in sense on the other.

f) Finally, the One is not a substance.

It seems that, according to Aristotle, the One does not mark out any genus of things that are actual existing things. In fact, it is some unit of measurement which can be applied to categorically different items rendering them units of measurement for the category they belong to. For example, it can be applied to a quality making it a unit of measurement for qualities and it can also be applied to a quantity making it a unit of measurement for quantities. Therefore, it seems that the One is neither substance nor quality nor quantity nor item belonging to any other category. It seems to be a factor which somehow transmits its property of being uniform and indivisible to any item that acts as a measure for the category to which it belongs. However, if it is admitted that the One acts as a unit of measurement for the existing things without it being an existing thing of some specific sort, Platonists’ consideration of the One as principle seems to be total nonsense. For, being a measure means being a representative example of some category of things.

141 Annas (1976), 197
Furthermore, Aristotle claims that the measure can be thought of both in qualitative and in quantitative terms. Let us try to clarify the Aristotelian claim by means of two examples. As regards the category of qualities, in the case of whiteness, the unit of measurement could be described as the quality with reference to which we estimate the degree of whiteness of anything. This quality, which acts as a criterion, has to be the quality of being totally white, given that being totally white means bearing all the features that describe the whiteness to the fullest possible extent. As regards the category of quantities, in the case of the quantity of ten horses the unit of measurement could again be described as the quantity with reference to which we calculate the amount of horses. This quantity, which acts as a criterion, has to be the slightest possible quantity which can keep all the features and the outline of the horse, i.e. one horse. Therefore, in the case of qualities the unit of measurement is the quality to the fullest extent, while in the case of quantities the unit of measurement is one complete item of those which comprise the quantity. In both cases, the unit of measurement is the quality or quantity in which there is neither excess nor deficiency of the features that characterize the quality or quantity which is measured (cf. Metaphysics, I.1 1052b33-36: “πανταχοῦ γὰρ τὸ μέτρον ἐν τι ζητοῦσι καὶ ἀδιαίρετου· τούτῳ δὲ τὸ ἀπλοῦν ἢ τῷ ποιῷ ἢ τῷ ποιών· ὅπως μὲν ὁνὶν δοκεῖ μὴ εἰναι ἄφελεν ἢ προσθεῖαι, τούτῳ ἀκριβεῖς τὸ μέτρον.”).

Aristotle claims that the measure is indivisible in kind on the one hand and in sense perception on the other. According to Ross142, by “indivisible in kind” Aristotle may refer either to infimae species (i.e. Socrates, cat etc.) or both to genera and to species or to that which is incapable of being divided into parts that differ in kind from the whole (i.e. to elements). In my opinion, by “indivisible in kind”, Aristotle probably means that the measure of anything cannot be divided into parts that differ in nature from the measure (cf. Metaphysics, Δ.3 1014a27: “στοιχεῖον λέγεται ἐξ ὧν σύγκειται πρῶτων ἐνυπάρχοντος, ἀδιαίρετου τῷ εἰδεί εἰς ἔτερον εἶδος”). One example could be the semi-tone, as a unit of measurement of the musical scale. The semi-tone cannot be divided into some lesser intervals that are musically relevant.

142 Ross (1924), v. 2, 472
The semi-tone in this sense is the minimal interval that makes acoustically a difference to the human ear. Therefore, each unit of measurement is of one particular unmixed kind which cannot be divided into other species (cf. Metaphysics, Γ.3 999a3: “ἐν δὲ τὸ ἀδιαίρετον, ἀδιαίρετον δὲ ἰσον ἢ κατὰ τὸ ποσὸν ἢ κατὰ τὸ εἶδος...τὰ δὲ γένη διαιρέτα εἰς εἰδη, μάλλον ἂν ἐν τῷ ἐσχάτῳ εἰς κατηγορίαν· οὐ γάρ ἐστι γένος ὁ ἄνθρωπος τῶν σώματι ἄνθρωπων”). Moreover, the semi-tone is the smallest musical interval which can be played in music. So, on the basis of our sense perception, the semi-tone is indivisible into smaller musical intervals that are relevant to our sense of music. In conclusion, the semi-tone, as a unit of measurement of the musical scale, is indivisible both in kind and in sense and the same is also true of any other measure.

According to Ross, the phrase “indivisible in kind” seems to refer to the qualitative units, whereas the phrase “indivisible in sense perception” seems to refer to the quantitative ones. However, we can notice that the property of being indivisible in sense perception might be better understood in connection with the property of being indivisible in kind. This seems quite reasonable, if we take into account that the way in which our senses perceive the reality is strongly influenced by our concepts concerning the kind.

If all magnitudes are infinitely divisible, as Aristotle believes, then any quantitative measure (a kilogram, a square metre, a decibel), should be infinitely divisible. If we regard them as indivisible, it is because we have made them into units of measurement, i.e. standards with reference to which we measure the relevant magnitudes, and this conventional unification of an essentially divisible magnitude must have something to do with the way we perceive things (e.g. one foot long, a foot being an indivisible part of the human body). The same does not apply to a qualitative measure. A qualitative measure is a measure that is indivisible in a different sense which needs to be clarified. To say that the food contains one kilo of edible matter (infinitely divisible into smaller quantities) is different from saying that the food contains meat, water, tomato juice, potatoes, pepper and salt. In the latter
case, the food has been analyzed into its ingredients, each of which is of a different kind: in terms of kinds the food cannot be further divided.

It appears as if the indivisibility in sense perception presupposes the indivisibility in kind, which entails that we cannot have the one without also having the other. Hence, it is likely that at 1088a2-3, τὸ μὲν and τὸ δὲ do not refer to qualities and quantities respectively. In fact, they are used to distinguish the two senses in which both the measures of qualities and the measures of quantities are indivisible. It should be noted in support of this claim that elsewhere Aristotle uses the terms εἶδος and αἴσθησις without making a distinction between them (Metaphysics, Δ.6 1016a19: “ἀδιάφορα δὲ ἂν ἀδιαιρετον τὸ εἶδος κατὰ τὴν αἰσθησιν”).

1088a4-14: Aristotle points out that, given that “one” means a measure of some plurality and “number” means a measured plurality or a plurality of measures, the One is not a number:

a) The One is a measure of plurality.

b) The number is a countable plurality or else a plurality of measures (cf. Δ.13 1020a13: “τούτων δὲ πληθὸς μὲν τὸ πεπερασμένον ἀριθμός” & Ζ.13 1039a12: “εἰπερ ἐστὶν ὁ ἀριθμὸς σύνθεσις μονάδων”).

c) Therefore, the One is not a number. For, the measure cannot be many measures.

d) Both the measure and the One are starting points.

e) As a measure, the One has to be of the same kind with the plurality that can be counted by it.

f) For example, the measure of a plurality of horses will be one horse, because what all horses have in common is their property of being horses, and the measure of a plurality of human beings will be one human being, because what all human beings have in common is their property of being human beings.
g) Given that the measure of a plurality must necessarily possess the property shared by all the members of this plurality, the measure of the plurality consisting of a human being, a horse and a god might be one living thing.

h) But, let us suppose that there is a plurality which consists of human being, white and walking. This plurality comprises three things which are different in kind and also capable of applying to one and the same substance. For, these three aspects cannot exist separately from the substance to which they apply.

i) So, it is both unnecessary and difficult to seek either the number of this plurality or a measure for it.

j) If there has to be a number for this plurality, it will be the number of the genera, or else the categories, to which the items of this plurality belong.

As Alexander\textsuperscript{143} points out, human being, white and walking are three attributes which can apply to one and the same substance, for instance to Socrates, who is a human being, bears the white color and walks. However, they remain three different things in kind, given that human being is a substance, white is a quality and walking is an action. Human being, walking and white are three different ways of being, i.e. instances of three different categories or else predications. However, objects, qualities, relations etc. cannot form one and the same group of existing things, because each of them is relevant to a different sense of existing. So, there cannot be a common measure for all of them. Given that only the things which belong to the same particular kind can form a group of countable things, the aforementioned things cannot be counted together, for they are different in kind\textsuperscript{144}. If the plurality consisting of human being, white and walking is to be counted, there is nothing that its members have in common except that they are categories of being. So, if there is to be any number of such an incongruous plurality, it can be no other than the number of the categories existing in it, namely three.

The One, which acts as a measure, is also treated as a starting point, or else a principle. We need to distinguish the two meanings that are ascribed to the One. The

\textsuperscript{143} [Alex.], 779.18-21

\textsuperscript{144} Annas (1976), 197
One is the pattern that is imitated by any entity which acts as a measure of a plurality of items. In this sense, the One is a measure, for, as it has already been said, it transmits its property of being uniform and indivisible to any item that acts as a measure for the category to which it belongs. This is why the One seems to be what we really have in mind when we are talking about, for instance, the one horse as a measure of a plurality of horses\textsuperscript{145}.

But also, the One is a first principle, since it is prior to all the measures imitating it. The measure of a certain plurality of entities is not this particular One, but one entity which has the properties of the One and possesses all the characteristics shared by the members of this specific plurality of entities.

However, this also entails that the one horse, which acts as a measure of a plurality of horses, can by no means be treated as a real horse. It might be treated as an ideal example which acts as an incorporeal unit of measurement and not as one more substance that belongs to the species of horses. For, if it was to be treated as such, then the same thing would act both as a measure and as something that can be measured by the measure, which is absurd.

\textbf{1088a15-21:} After having criticized the One as a principle, Aristotle goes on to address his criticism to the Platonists who make the Indefinite Dyad something composed of the Great and Small. Aristotle accuses these thinkers of assuming things that are far from plausible or possible. For, the Great and Small are properties rather than the substrate of numbers and magnitudes.

\begin{itemize}
  \item[a)] The Platonists use alternatively the phrases “Unequal”, “Indefinite Dyad” and “Great and Small” for the material principle.
  \item[b)] The use of the aforementioned terms makes the material principle incapable of acting as the material substrate of the mathematical objects. The reason is that these three expressions name affections and properties of numbers and magnitudes.
\end{itemize}

\textsuperscript{145} Syr., 168.28-30
c) The Many and Few are properties of numbers and the Great and Small are properties of magnitudes, just as any term, such as “even” and “odd”, Smooth and Rough, Straight and Curved is predicated of something.

d) Therefore, a principle bearing the name “Unequal” or “Indefinite Dyad” or “Great and Small” can only belong to the category of relatives and is unable to act as an element of existing things.

The term “Unequal” indicates the unity of the second principle, while the terms “Indefinite Dyad” and “Great and Small” divulge its twofold nature (cf. Metaphysics, N.1 1087b9-12). But, all these three phrases are names for affections and properties. Therefore, they can be used to signify aspects which apply to a material substrate, but not aspects which are themselves material and act as a substrate. Aristotle focuses his accusation on the Academic point of taking the Many and Few and the Great and Small to be constituents of numbers and magnitudes, whereas they are in fact characteristics possessed by them (cf. M.9, 1085a20-23: “τὰ ὑπάρξει καὶ τὰ ἁπάτης ἡμέρας ἐστιν, ἀλλὰ οὐκ ἐκ τούτων τὸ μέγεθος, ὡσπερ οὐδ' ἐξ εὐθείας καὶ καμπύλου τὸ μήκος, οὐδ' ἐκ λείου καὶ τραχέος τὰ στερεά’”). The relatives are always predicated of something. But it has already been stated that it is impossible for a predicate to be a first principle (cf. M.1 1087a33-35), for there has to be a substrate of which it is predicated. So, there must necessarily be an underlying material, which is prior to the relative. Thus, a relative should not be treated as a first principle. In conclusion, the idea that the affections and properties are considered to be principles and thus superior to the substances possessing them, seems totally absurd.

According to Annas146, the Indefinite Dyad cannot be thought of as a relative and this makes its inclusion in this category extremely problematic. However, the Indefinite Dyad is said to be the twoness of the Great and Small, which are relative properties. Moreover, the indefiniteness is also a property which describes the nature of this principle. Therefore, it seems that the Indefinite Dyad is not in fact matter, but a compound of properties, which must necessarily apply to some underlying material that could compose a substance.

146 Annas (1976), 197
Furthermore, Aristotle criticizes the Platonists for turning the Great and Small, which are indeed relative terms, into first principles. It is hardly credible that the Platonists would describe their principle as a πρ ός τι, given that the relative terms belong to the least substantial of all categories. For, relative is an affection of quantity or quality and always implies a substrate:

a) The Great and Small and all the similar bipolar expressions are made up of relative terms. They display relational properties of existing things and cannot be said to have a being in the primary sense in which a substance has being.

b) Moreover, Aristotle claims that the relational properties are characterized by an ontological inferiority with regard to the quality and the quantity. The Great and Small are attributes that apply to the quantity and the quality, but they are not the matter of them.

c) The relatives are relatives for both the quantity in general and its parts and kinds.

d) In general, there is no relative which is many or few or great or small without being so as something else.

The relatives are said by Aristotle to be posterior to quality and quantity. As far as quantity is concerned, the double, the half and the one-third are relational properties that are predicated of the quantity, but they are not identified with its material being, as Alexander\textsuperscript{147} states. Similarly, as far as the quality is concerned, the right and the left or the property of being a master and the property of being a slave are relational properties that are predicated of the quality, without being identified with its material being. Aristotle focuses here on the idea that the relatives are attributed of the quantities and the qualities. It shouldn’t escape our notice that the relatives are predicated of the substances. Therefore, the relatives seem to be properties, but not in the least of a material nature, which makes them inferior in terms of being, given that the underlying material (i.e. the substance) is given ontological priority by Aristotle.

\textsuperscript{147} [Alex.], 780.21-22
Moreover, the relatives are made even less substantial by the fact that the one relative cannot exist without the other. For instance, the property of the great cannot exist without the small, just as the right cannot exist without the left and the property of being a slave cannot exist without the property of being a master. On the contrary, the quality of white is capable of existing, even if the opposite quality, i.e. the black, does not exist. Similarly, the quality of the hot can exist regardless of the existence of the cold. Therefore, it could be assumed that the relatives, such as the Great and Small, are the least substantial of the categories, for they cannot exist without always being related to something else. The Great and Small cannot just be great and small without the parallel existence of the contrary relative. The Great is great because of the existence of something else, in relation to which it possesses the property of being great. The same is true of the Small, the Many, the Few and in general anything that belongs to the category of relatives. This fact makes the Great and Small so insubstantial that they cannot be considered to be a first principle. It is noteworthy that the pair of contraries is evidently treated by Aristotle as a conjunction of two distinct opposite entities and not as one single entity.

It is also important to understand the phrase “relative for both the quantity in general and its parts and kinds”. This phrase might be interpreted in the following way: it is not the same sort of relative that is predicated of both the quantity in the universal sense and the particular quantities. Just as there is the quantity in the general sense and also specific quantities (e.g. a dozen of apples), in the same way there could be a universal relative that is predicated of the universal quantity, but also the particular relatives that are predicated of particular quantities. For instance, the Many and Few could be thought of as the universal relative, which is predicated of the Quantity in the universal sense. In general, the parts and kinds of universal quantity are the particular kinds of quantity such as long and short, broad and narrow, deep and shallow, light and heavy, few and many.

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148 Ross (1924), v. 2, 473
149 [Alex.], 780.32-34
1088a29-34: Aristotle accuses the Platonists of ignoring that relative is the only category of things of which there is neither generation nor destruction nor change, as there is in respect of the other categories, such as substance, quality and quantity:

a) The Great and the Small are in the least degree substantial, for they are not generated and destroyed and do not undergo any other change in themselves.

b) As in any case of any relational property, something great or something small can be described as great or as small only in relation to something else. We could use the example that is proposed by Alexander\textsuperscript{150}: let us suppose that there are two pieces of wood, both of which are two cubits long and somebody decides to extend or shorten only the one of them by one cubit. Then, the other piece will be in the first case small in comparison to the piece that was extended and in the second case great in comparison to the piece that was shortened. Therefore, the one of the two pieces of wood proves to be capable of becoming great or small in comparison to the other without undergoing an extension or shortening in itself.

c) On the contrary, as Aristotle claims, categories such as quantity, quality, place and substance must necessarily undergo change so as to be in a different state. For example, Socrates must change in quantity to become fatter, in quality to become musical, in place to go to Megara and in substance to die and decompose.

There is no sort of change which can be called change in respect of relation, as there is change in respect of substance, quantity, quality and place. The change in respect of a relational property is always a change in respect of one of the other categories\textsuperscript{151}. For example, the relative which is an affection of quantity is determined by the changes suffered by the quantity. The decrease of twelve apples in quantity leads to the change of its relational property from “many apples” to “few apples”. This entails that the category of relatives does not undergo any change in itself, which further indicates that the category of relatives is derivative, since the relatives

\textsuperscript{150} [Alex.], 781.15-18
\textsuperscript{151} Ross (1924), v. 2, 473
always have to be considered with reference to the other categories (cf. *Physics*, E.2 225b11-13: “Κατ' οὖσιαν δ' οὐκ ἐστι κίνησις διὰ τὸ μηδὲν εἶναι οὐσία τῶν δινών ἐντὸν. Οὐδὲ δὴ τοῦ προς τι ἐνδέχεται γὰρ θατέρου μεταβάλλοντος <ἀληθεύεσθαι καὶ μὴ> ἀληθεύεσθαι θάτερ μηδὲν μεταβάλλον, ὡστε κατὰ συμβεβηκός ἢ κίνησις αὐτῶν.”). What the quotation says is that if one of the relatives changes the other one which was previously, say, smaller becomes greater without undergoing any change whatsoever itself. The expression “it is true of X that it is smaller than Y” changes its truth-value although X did not change in the least. In conclusion, in the case of relatives, the change differs in two ways from the change in the other categories: firstly, the change of a relative happens in respect of another correlative item and secondly, the change of a relative presupposes a change in respect of some other category, such as the quality, the quantity, the place and the substance. Hence, the relative has an inferior claim to primary existence.

1088b1-4: Aristotle's criticism here is based on the fact that the matter of a thing is the element which is potentially that thing. Nevertheless, the relative is neither potentially nor actually a substance. Therefore, the relative is non-substance and a non-substance cannot be a principle of substance:

a) It is necessary that the matter of each thing, and thus also of a substance, is that which is potentially of such a nature as the thing is.

b) However, a relative is not a substance either potentially or actually. A non-substance cannot be prior to a substance and an element of it.

c) Therefore, a relative cannot be prior to a substance and an element of it.

A relative - apart from not being a substance - is not the matter of a substance, for it is not of material nature152. So, a relative is not actually a substance, but, given that it is not matter either, it is not potentially a substance either. A relative cannot be an element and a starting point of a substance, since it is neither actually nor potentially a substance and also because it is posterior to substance and predicated of it as a

152 [Alex.], 781.21-22
category. Hence, it can by no means be prior to substance and thus one of the first principles.

1088b5-13: Aristotle’s criticism focuses on the fact that, if relatives like the Many and Few are constituent parts of number, either both of them or neither should be predicably of it:

a) The elements are not predicated of the thing of which they are elements.
b) But, the Great and the Small are predicated of the mathematical objects.
c) Moreover, the Great and the Small cannot simultaneously apply to the same thing.
d) However, since the Great and the Small are said by the Platonists to be the elements of which mathematical objects are composed, both have to be predicated of them. This means that the Great and the Small must necessarily apply to all mathematical objects without exception.
e) But there is a serious problem: In the case of numbers, for instance, the Academy applies both the Many and the Few to them. However, the Two is a number to which the Many cannot apply. And the Ten or the Ten Thousand or whatever is the largest number must, on the same principle, be a number to which only the Many but not the Few should apply. The numbers that are between the Two, which is the smallest number, and the greatest number can be considered to be many with reference to the Two and few with reference to the largest number.153
f) But how can the Great and the Small be elements of numbers as such if in at least one case of numbers they do not both apply? Therefore, the notions of the Great and the Small cannot at the same time be true of the same first principle, since they cannot at the same time be true of the same mathematical object.154

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153 Ross (1924), v. 2, 473
154 [Alex.], 782.8-12
We must generally inquire whether it is possible for eternal things to be composed of elements. For, if so, they will have matter; for everything which consists of elements is composite. Now, assuming that it is necessary for a thing which consists of anything to come into being out of this thing of which it consists, whether it exists forever or was generated, and since everything comes to be that which it comes to be out of something which is that in capacity (for it could neither come into being from nor consist of that which lacked that capacity), and since what is capable can either be actualized or not, then even if number or anything else having matter is everlasting, it could not exist, just as both a thing which is a single day old and a thing which is any number of years old could not exist; and if this is so, the thing which has existed for so long a time that it has no limit may also not exist. Therefore, they would not be everlasting, since that which is

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| 1088b14-28 | Ἀπλῶς ἀπλῶς δὲ δεῖ σκοπεῖν, ἀρα δυνατὸν τὰ ἄδια ἐκ στοιχείων συγκείσθαι; Ἄλλην γὰρ ἔξει· σύνθετον γὰρ πᾶν τὸ ἐκ στοιχείων. Εἰ τοίνυν ἀνάγκη, ἔξ, οὐ ἔστιν, εἰ καὶ ἄεί ἐστι, κἂν, εἰ ἐγένετο, ἐκ τούτου γίγνεσθαι, γίγνεται δὲ πᾶν ἐκ τοῦ δυνάμει ὑποτοῦ τοῦ ὅ γίγνεται (οὐ γὰρ ἂν ἐγένετο ἐκ τοῦ ἀδυνάτου οὐδὲ ἦν), τὸ δὲ δυνατὸν ἐνδέχεται καὶ ἐνεργεῖν καὶ μή, εἰ καὶ ὁ ἄλλος ἄριθμος ἢ ὁ τίος ὄλλο ὑλήν ἔχον, ἐνδέχοιτ’ ἂν μὴ εἶναι, ἄσπερ, καὶ τὸ μίαν ἡμέραν ἔχον καὶ τὸ ὑποσκούν ἐπὶ· εἰ δ’ ὀύω, καὶ τὸ τοσοῦτον χρόνον ὁμοίως συνέβη πραγματευθῆναι. Εἰ δὲ ἄστι τὸ λεγόμενον νῦν ἄληθες καθόλου, ὦτι σύνεσιν ἐστιν | We must generally inquire whether it is possible for eternal things to be composed of elements. For, if so, they will have matter; for everything which consists of elements is composite. Now, assuming that it is necessary for a thing which consists of anything to come into being out of this thing of which it consists, whether it exists forever or was generated, and since everything comes to be that which it comes to be out of something which is that in capacity (for it could neither come into being from nor consist of that which lacked that capacity), and since what is capable can either be actualized or not, then even if number or anything else having matter is everlasting, it could not exist, just as both a thing which is a single day old and a thing which is any number of years old could not exist; and if this is so, the thing which has existed for so long a time that it has no limit may also not exist. Therefore, they would not be everlasting, since that which is
ἀνάγκην ἀναγκαῖον ἀναγκαῖον ἀναγκαῖον ἀλλ᾽ ἀνάγκη

There are however some people who make the element acting with the One an Indefinite Dyad, and object to the unequal quite reasonably because of the impossibilities arising; but they have only got rid of those difficulties which necessarily arise for the proponents of that theory, due to making the unequal and the relative an element; but those which arise apart from this view, these must apply to them too, whether it is the Form number or the mathematical number that they make out of these elements.

Πολλά μὲν οὖν τὰ αἷτα τῆς ἐπὶ ταύτας τὰς αἰτίας ἐκτροπῆς, μάλιστα δὲ τὸ ἀπορήσαι ἀρχαῖως. Ἐδοξέ γὰρ αὐτοῖς πάντ᾽ ἔσεσθαι ἐν τὰ ὄντα, αὐτὸ τὸ ὄν, εἰ μὴ τις λύσει καὶ ὀμόσει βαδεῖται τῷ Παρμενίδου λόγῳ «οὐ γὰρ μὴποτε τοῦτο δαμή, εἶναι μὴ ἓόντα», ἀλλ᾽ ἀνάγκη

There are many reasons for the digression towards these explanations, the chief being that they put the problem in an archaic way. They thought that all the existing things will turn out to be one, namely the Being itself, if one does not refute but goes along with Parmenides' saying: “You shall never come to know this, that the things which are
εἰναι τὸ μὴ ὧν δεῖξαι ὃτι ἐστιν. Οὕτω γὰρ, ἐκ τοῦ ὄντος καὶ ἄλλου τινός, τὰ ὄντα ἔσωθα, εἰ πολλὰ ἐστιν. not, are.”; however they thought that it is necessary to prove that what is not, is; for thus, of that which is and of something else, they thought that the existing things will be composed, if they are many.

| 1089a6-14 | Καίτοι πρὸτον μὲν, εἰ τὸ ὄν πολλαχῶς (τὸ μὲν γὰρ [ὅτι] οὐσίαν σημαίνει, τὸ δ´ ὃτι ποιών, τὸ δ´ ὃτι poios, καὶ τὰς ἄλλας δῆ κατηγορίας), poioν οὐν τὰ ὄντα πάντα ἐν, εἰ μὴ τὸ μὴ ὧν ἐσται; Πότερον αἰ οὐσία, ἡ τὰ πάθη καὶ τὰ ἄλλα δῆ ὁμοίως, ἡ πάντα, καὶ ἐσται ἐν τὸ τὸδε καὶ τὸ τοιόνδε καὶ τὸ τοσόνδε καὶ τὰ ἄλλα ὄντα ἐν τι σημαίνει; Ἀλλ´ ἀτοπον, μᾶλλον δὲ ἀδύνατον, τὸ μίαν φύσιν τινὰ γενομένην αἰτίαν εἰναι τοῦ τοῦ ὄντος τὸ μὲν τὸδε εἰναι τὸ δὲ τοιόνδε τὸ δὲ τοσόνδε τὸ δὲ ποι. However, in the first place, if the being has several senses (for some of it means substance, some quality, sometimes quantity and the other categories), what sort of one thing will all the existing things be, unless not-being is allowed to exist? Will the substances be one thing or the affections and the other categories similarly, or everything be one thing, so that the “this” and the “such” and the “so much” and everything else indicating each some one thing will all be one? But it is absurd, or rather impossible for some single nature to become the cause of one part of being’s being this, another part being such, another part being so much, another part being somewhere.

| 1089a14-19 | Ἐπείτα ἐκ ποίου μὴ ὄντος καὶ ὄντος τὰ ὄντα; Πολλαχῶς γὰρ καὶ τὸ μὴ ὧν, ἐπειδή καὶ τὸ ὄν· καὶ τὸ μὲν μὴ ἀνθρωπον <εἶναι> σημαίνει τὸ μὴ εἶναι τοδί, τὸ δὲ μὴ εὐθὺ τὸ μὴ εἶναι τοιονδι, τὸ δὲ μὴ τρίπηχυ τὸ μὴ εἶναι τοσονδι. Εκ ποίου οὖν ὄντος καὶ μὴ ὄντος πολλὰ τὰ Further, from what sort of not-being and being will the existing things come? For, not-being has many senses too, since being also has many; and not being a human being means not being a this, and not being straight means not being such, and not being three cubits means not being so much. So, from what kind of being and not-being will the
| 1089a19-31 | Βούλεται μὲν δὴ τὸ ψεύδος καὶ ταύτην τὴν φύσιν λέγειν τὸ οὐκ ὁν, ἐξ οὗ καὶ τοῦ ὄντος πολλὰ τὰ ὄντα, διὸ καὶ ἐλέγετο ὅτι δεὶ ψεύδος τι υποθέσθαι, ὡσπερ καὶ οἱ γεωμέτραι τὸ ποδιαίαν εἶναι τὴν μὴ ποδιαίαν· ἀδύνατον δὲ ταύθ᾽ οὖν ὃς ἕχει, οὔτε γὰρ οἱ γεωμέτραι ψεύδος οὐθέν υποτίθενται (οὐ γὰρ ἐν τῷ συλλογισμῷ ἢ πρότασις), οὔτε ἐκ τοῦ οὖν μὴ ὄντος τὰ ὄντα γίγνεται οὔδε φθείρεται. Ἀλλ᾽ ἐπειδὴ τὸ μὲν κατὰ τὰς πτώσεις μὴ ὁν ἰσαχώς ταῖς κατηγορίαις λέγεται, παρὰ τούτῳ δὲ τὸ ψεύδος λέγεται [τὸ] μὴ ὁν καὶ τὸ κατὰ δύναμιν, ἐκ τούτου ἢ γένεσις ἐστίν, ἐκ τοῦ μὴ ἀνθρώπου δυνάμει δὲ ἀνθρώπου ἀνθρώπος, καὶ ἐκ τοῦ μὴ λευκοῦ δυνάμει δὲ λευκοῦ λευκόν, ὁμοίως ἐὰν τε ἐν τι γίγνηται εὰν τε πολλά. | The false tends to signify also this nature, i.e. not-being” from which, together with being, comes a plurality of existing things. That is why it used to be said that you have to assume something false, just as the geometers assume a line which is not a foot long to be a foot long. But it is impossible for these things to be thus. For neither do the geometers assume anything that is false (since the proposition is not a part of their syllogism), nor are the existing things generated from or destroyed into not-being in this sense. However, since not-being in its different cases has as many senses as there are categories, and in addition to this the false and the potential are called not-being, it is from the latter that generation takes place, a man comes into being from that which is not a man, but is potentially a man, and white from that which is not white, but is potentially white, and similarly whether it is one thing that is generated or many. |
| 1089a31-1089b3 | Φαίνεται δὲ ἡ ζήτησις πῶς πολλὰ τὸ ὅν τὸ κατὰ τὰς οὐσίας λεγόμενον· ἀριθμοὶ γὰρ καὶ μῆκος καὶ σώματα τὰ γεννώμενα ἐστίν. Αὐτοποι δὴ τὸ ὅπως μὲν πολλὰ τὸ existing things come to be many? | It appears that the inquiry is about how the being which is spoken of in the sense of substances is many; for it is numbers and lengths and bodies the things that are generated. Now it is absurd to inquire how |
being in the sense of “what is” is many and not how being in the sense of either qualities or quantities is many. For, neither the Indefinite Dyad nor the great and small are a cause of there being two sorts of white or many colours or flavors or shapes; for then these would also be numbers and units. But, if they had pursued these inquiries, they would have seen the cause in the former case, too; for, the cause is the same thing, or something analogous.

This aberration is also the reason why in searching for the contradictory of the being and the One, from which together with the being and the One the existing things will be, they posited the relative, i.e. the unequal, which is neither the contrary nor the negation of these, but is one sort of being, just like the “what” or the quality.

They should also have searched for this, i.e. how the relatives are many, but not one as things stand, they ask how there are many units besides the first One, but not also how there are many unequals besides the unequal. However, they make use of them and talk about great and small, many and few (from which the numbers come), long and short (from which the length comes), broad and narrow (from
| 249 | ταπεινόν, ἐξ ὁν οἱ ὄγκοι· καὶ ἐτι δή πλεῖω εἴδη λέγουσιν τоῦ πρὸς τι· τούτοις δή τι αἶτιον τοῦ πολλὰ εἶναι; | which the plane comes), deep and shallow (from which the masses come); and they mention even more kinds of relative. But what is cause of these things being many? |
| 1089b16-24 | Ἀνάγκη μὲν οὖν, ὡσπερ λέγομεν, ύποθέναι τὸ δυνάμει ὃν ἐκάστῳ (τούτῳ δὲ προσαπεφήνατο ὁ ταύτα λέγων, τί τὸ δυνάμει τόδε καὶ οὐσία, μή ὃν δὲ καθ᾽ αὐτό, ὁτί τὸ πρός τι, ὡσπερ εἰ έτε τὸ ποιόν, ὁ οὔτε δυνάμει ἐστι τὸ ἐν ἤ τὸ ὃν οὔτε ἀπόφασις τοῦ ἑνὸς οὐδὲ τοῦ ὄντος ἀλλ’ ἐν τι τῶν ὄντων), πολὺ τε μᾶλλον, ὡσπερ ἐλέχθη, εἰ ἐξήτει πῶς πολλά τὰ ὄντα, μὴ τὰ ἐν τῇ αὐτῇ κατηγορίᾳ ἐξίτειν, πῶς πολλαὶ οὐσίαι ἢ πολλὰ ποιά, ἀλλὰ πῶς πολλά τὰ ὄντα· τὰ μὲν γὰρ οὐσίαι τὰ δὲ πάθη τὰ δὲ πρός τι. | It is indeed necessary, as we say, to presuppose for each existing thing something which potentially is that thing; but the holder of this theory further declared what it is that is potentially a “this”, i.e. a substance, but it is not an existing thing in itself, and he said that it is the relative -as if he had said that it is the quality- which is potentially neither the One nor the being nor the negation of either the One or the being, but it is just one particular kind of being; and it was even more necessary, as we have said, if he was inquiring how the existing things are many, not to confine his inquiry to things belonging to the same category, asking how there are many substances or many qualities, but how the existing things in general are many; for, some of them are substances, some affections and some relatives. |
| 1089b24-29 | Ἐπὶ μὲν οὖν τῶν ἀλλῶν κατηγορίων ἔχει τινά καὶ ἄλλην ἐπίστασιν πῶς πολλά (διὰ γὰρ τὸ μὴ χωριστὰ εἶναι τῷ τὸ ύποκείμενον πολλὰ γίγνεσθαι καὶ εἶναι | In the case of the other categories, there is a difficulty as to how there are many; for since they are not separable because their underlying subject is and becomes many it is necessary that qualities and |
quantities are many. Yet there has to be a type of matter for each category, only it it is impossible for it to be separated from the substances.

However, in the case of things that are a particular “this”, there is a reason as to how a particular “this” is many, unless there is to be something which is both a particular “this” and some such nature. The real difficulty arising from these facts is rather how there can be many substances in actuality and not one.

But, again, if the “this” and the quantity are not the same, then it is not explained how and why the existing things are many, but how the quantities are many. For all number means a particular quantity; and the unit, if it does not mean a measure and means that the quantity is indivisible. If, then, the quantity and the “what is” are different, we are not told whence nor how the “what is” is many; and, if they are the same, the holder of this view has to face many contrarieties.

And someone might turn their inquiry also on this question about the numbers, from where we should derive the belief that they exist. For him who postulates Forms they provide some kind of cause for the
existing things, since each number is some Form and the Form is, in some way or the other, the cause of the being for the other things; for let us grant them this supposition. But as for the person who does not hold this sort of view, because he sees the difficulties inherent in the theory of the Forms, so that he does not assume that there are numbers for this reason, but who assumes that there is the mathematical number, whence should we derive the belief that this sort of number exists, and of what use is it to the other things? For neither he who maintains that it exists says that it is the cause of anything, but he rather holds that it is a kind of nature in itself, nor is number evidently the cause of anything; for all the theorems of the arithmeticians will apply equally to perceptible things, as has been said.
OVERVIEW:

In N.2 Aristotle continues objecting to Platonists' material principle and addresses his criticism to the theory that numbers exist separately and act as causes.

In the beginning of N.2, Aristotle formulates the question of whether an eternal thing can be composed of elements. The answer is that nothing can be eternal insofar as it is composed of matter, which is potentially this thing and therefore it is capable of not being this thing. A serious difficulty is in what sense matter is referred to here and whether Aristotle once more ascribes his sense of matter to the Platonists or not.

Then, Aristotle unexpectedly returns to the subject of the difficulties entailed by the introduction of the Unequal or the Indefinite Dyad as a material principle by the Platonists. He says that there are some Platonists who treat the Indefinite Dyad as the material principle and avoid calling it "Unequal". However, by avoiding calling the material principle "Unequal" they only solve the problems resulting from making the relative term "Unequal" an element. It is a matter of question what are the problems that are solved and what are the problems that remain unsolved.

Aristotle claims next that the main reason why the Platonists introduced such principles is that they stated the problem in a primitive way. They thought that all the existing things would be one, i.e. Being, if they did not oppose Parmenides' dictum and try to prove that Not-Being is. So, they assumed that, given that the existing things are many in number, they should have been generated by Being and something else. It is a matter of question here to what extent Aristotle's criticism is fair, for there is no secure evidence that the Platonists had identified their contrary first principles with Being and Not-Being.

According to Aristotle, it seemed necessary to the Platonists to prove that Not-Being is, because, given that Being has as many senses as there are categories (substance, quality, quantity etc.), it is very difficult to say what sort of unity all
the existing things will form, unless the Not-Being is. But, again, it is impossible for one thing, i.e. Not-Being, to be responsible for the diversity between the different categories of things. Moreover, given that Not-Being can also have as many senses as there are categories, it is a matter of question of what sort Not-Being is. One problematic point is that Being and Not-Being are treated here as the components that make the universe up, while in fact they correspond to the states of “being something” and “not being something” respectively.

By “Not-Being” Plato is said by Aristotle to mean the “false”. Furthermore, it is claimed that something false must be presupposed, just as geometers assume the line which is not a foot long to be a foot long. However, a not-being of such a sort will not account for the generation or the destruction of anything. The existing things are generated by the Not-Being that is characterized by potentiality of being and not by falsity. Once more, it should be clarified here to what extent Aristotle’s criticism of the Platonic treatment of the material principle is fair.

Aristotle continues saying that what the Platonists derive from their material principle are numbers, lines and magnitudes. The enquiry is how there are many substances, but also how there are many qualities or quantities. The Indefinite Dyad cannot account for the plurality of substances, qualities and quantities, for then substances, qualities and quantities would have been numbers, lines and magnitudes, i.e. mathematical objects. It seems that Aristotle’s approach here presupposes that the Platonists recognized his distinction of Form and Matter, which is doubtful. This is a general problem that applies equally to A.6, A.9, N.1, N.2 and elsewhere.

Nevertheless, there must be a cause of the plurality of the substances also, which is at least analogous to the cause of the plurality of the mathematical objects. If the Platonists had taken this fact into account, they would have avoided the error of considering the material principle to be a relative term, i.e. the Unequal. Moreover, the Unequal is not contrary either to Being or to the One. In fact, the Unequal is one particular kind of being.
Afterwards, Aristotle claims that the Platonists ought to have explained how the relatives are many. Even though they ask how there are many units apart from the One, they do not ask how there are many particular unequals (great and small, many and few, long and short etc.) apart from the Unequal in the universal sense.

It is necessary that the relative, as a material principle, is potentially a substance. But, the relative is a particular kind of being, in the same way in which quality is a kind of being. Moreover, it is necessary that the material principle is not confined to the generation of substances or to qualities. On the contrary, it must be a principle of plurality in general. For Aristotle's criticism to be fair, it is necessary that the terms “Indefinite Dyad”, “Unequal” and “Great and Small” are used as relative terms by the Platonists, which is doubtful.

In the categories apart from substance there is another problem as to how the plurality of their items is to be explained. There must be a matter for each category. The problem is that qualities, quantities etc. do not exist apart from substances. Therefore, the matter of each category cannot exist apart from the matter of substances.

In the case of substance, we can explain how the individual is many things, but not how there are many actually existing substances, i.e. individual things that are natures of some specific sort.

Given that the things which the Platonists derive from their material principle are numbers and units -which are quantities- it is not explained by them how the existing things in general are many.

The final problem that Aristotle raises in N.2 is how we are to be convinced that numbers exist. For the thinkers who believe in Forms, i.e. the Platonists, each number is a Form and therefore it acts as a cause. But, for him who posits the mathematical number, i.e. Speusippus, the number does not seem to be the cause of anything. As far as the theorems of mathematics are concerned, they do not imply the separate existence of numbers.
The fact that this passage from N.2 deals with the same subject as M.1 and M.3 has led Ross\(^\text{155}\) to the conclusion that M and N were originally independent essays. However, as Annas\(^\text{156}\) remarks, the present passage from N.2 deals with an enquiry which is different from the one that is dealt with in M. At 1090a15, “καθάπερ ἔλεξθη” makes a backward reference to M.3 (1077b17-19: “ὡσπερ γὰρ καὶ τὰ καθόλου ἐν τοῖς μαθήμασι οὐ περὶ κεχωρισμένων ἐστὶ παρὰ τὰ μεγέθη καὶ τοὺς ἀριθμοὺς, ἀλλὰ περὶ τούτων”). It is also noteworthy that this backward reference to M acts as a further evidence that M.3 is presupposed for the understanding of the present passage from N.2.

\(^{155}\) Ross (1924), v. 2, 479

\(^{156}\) Annas (1976), 207
COMMENTARY:

1088b14-28: In this section, Aristotle points out that the Platonists are confronted with a serious difficulty as regards the eternality of numbers. The reason is that they consider the numbers to be things that are composed of elements. Aristotle expresses here his opposition to the Platonic doctrine that numbers are composed of elements and eternal at the same time:

a) Any object that is composed of elements is a compound.

b) Any compound object has matter.

c) Any compound object, whether it exists eternally or is generated, must necessarily be created out of the elements of which it is composed, namely it must be created out of matter.

d) Any object that is composed of elements comes into existence out of what has the possibility of being this object. For, it could not come into existence out of what has the impossibility of being this object. The matter has the possibility of being, since it is potentially the object of which it is a constituent part, in the sense that it has the capacity to develop into this object. Therefore, any object comes into existence out of its matter.

e) However, the fact that the matter has the possibility of being entails that it also has the opposite possibility, i.e. that of not being. The matter is potentially the object of which it is a constituent part. What is potentially something may or may not be actually this thing. Hence, the matter, which has the possibility of being the object, also has the possibility of not being the object.

f) Moreover, what is potentially something cannot remain forever a “potentially something”. It has to be actually this thing at some time. Anything that involves the possibility of not being cannot be eternal. For, this capacity has to be actualized and the very moment this happens the object ceases to exist. So, any object that is composed of matter -and thus involves the possibility of not being- cannot be eternal.

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g) In conclusion, any object that is a compound of elements is not eternal. This conclusion applies to the numbers that are said by the Platonists to be composed of elements. Consequently, according to Aristotle, the numbers as understood by the Platonists cannot be eternal.

There are two ways in which we could understand the assumption that any object which is composed of elements is not eternal. Firstly, from the perspective of any object that is generated, it could mean that the matter is potentially the object and, the moment at which it actualizes its capacity of being, is the time at which the object comes into existence. So, there must be a particular point in time at which the object comes into existence and before which it had no actual being. But, anything which starts to exist at some specific point in time and does not exist earlier than that is not eternal.

Secondly, from the perspective of both the objects that are generated and the objects that are non-generated, being composed of matter involves both the possibility of being and the possibility of not being. However, any object that does not involve being as a possibility. Matter's capacity of being so and so has been actualized, since the object that is so and so actually exists. On the contrary, it still involves the possibility of not being, in the sense that it is capable of ceasing to be. However, there is no object that is both eternal and capable of not being. For, any capacity can be thought of as such, only if it is to be actualized at some time. Nothing can be potentially something ad infinitum. Aristotle has already argued so ἐν ἄλλος λόγος, which probably indicates Θ.8 of the *Metaphysics* (1050b7: “ἐστι δ’ οὐδὲν ὀννίμει ἄῤῥιον”) and also A.12 of the *De Caelo*.

Furthermore, we should pay attention to the way in which Aristotle seems to use the concept of ὑλη (matter). At 1088b15 (“ὁλην γὰρ ἐξει”/they will have matter), he claims that anything which is composed of elements necessarily contains matter. Aristotle seems to imply that, if any object is composed of elements, the matter is necessarily one of the elements out of which it is composed. He has already argued that some thinkers, including Plato, make the one of the contraries matter (1087b4-5: “οὗ δὲ τὸ ἐτερον τῶν ἐναντίων ὑλην ποιοῦσιν”). However, none of these previous
philosophers explicitly calls this principle “matter”. Instead, some of them call it “Indefinite Dyad” or “Great and Small”, while some others call it “Plurality”. Aristotle, though, believes in the existence of two principles of which the one is called Form and the other is called Matter. So, he could be accused of unfairly ascribing his own distinction of first principles to the Platonists in order to prove the inadequacy of their theory\textsuperscript{157}.

However, at 1088b27 (“τὰ δὲ στοιχεῖα ὑλῆ τῆς οὐσίας”/ the elements are the matter of substance), Aristotle seems to use the term “στοιχεῖα” in order to refer to the constituents of the Infinite Dyad of the Great and the Small rather than to the One and the Dyad. Matter and form as constituents of a hylomorphic compound can be said to be its elements. But matter can also be said to be composed of elements, i.e. the four primary bodies.

1088b28-35: It seems a little peculiar that in this section Aristotle chooses to return to the subject that was discussed in Chapter One, i.e. the difficulties arising from the introduction of the Unequal or the Indefinite Dyad as a material principle. However, I would agree with Ross\textsuperscript{158} in that it is quite reasonable and appropriate that, after having referred to the difficulties resulting from the derivation of number from a material element in general, Aristotle goes on to discuss the difficulties resulting from the derivation of number from the Platonists’ material element in particular that is the Unequal or the Indefinite Dyad.

Aristotle makes here a distinction between the two sorts of number that are derived from the Indefinite Dyad, i.e. the Form Number and the Mathematical Number. It seems that he aims at focusing on the first principles as the elements which generate primarily the numbers. Thus, Aristotle seems to describe the Unequal or the Indefinite Dyad as especially the element of numbers, which makes it impossible for them to be eternal.

\textsuperscript{157} Annas (1976), 200
\textsuperscript{158} Ross (1904), v. 2, 474
Moreover, in Aristotle’s view the Indefinite Dyad is more preferable than the Unequal as a material element, for it shuns the difficulties that are caused by the introduction of the Unequal.

a) Due to certain difficulties, some thinkers abandoned the name “the Unequal” and maintained the name “the Indefinite Dyad” to indicate the material element.

b) The thinkers who object to the Unequal and retain the Indefinite Dyad as one of the two first principles avoid the difficulties that arise from the theory which involves the Unequal.

c) The Indefinite Dyad differs from the other two variants, i.e. the Unequal and the Great and Small, in that it is not a relative term.

d) Therefore, it does not involve the difficulties that are entailed by the relational nature of the Unequal and the Great and Small.

e) However, all the other general difficulties continue to exist.

f) One of these inevitable difficulties is that the element in question, whether it is called “Unequal” or “Indefinite Dyad”, acts as matter that is potentially the being. Therefore, it involves both being and not being as a possibility, which entails the inability of comprising anything that is eternal, whether it is Form Number or Mathematical Number.

1088b35-1089a6: In this section, Aristotle discusses the Platonists’ inclination to pursue Parmenides’ primitive way of thinking and focuses on the difficulties that result from the treatment of the elements of all the existing things as Being and Not-Being by the Platonists.

a) According to the Platonists, all the existing things will turn out to be one, i.e. the Being itself, unless they partly refute Parmenides’ theory about the Being and the Not-Being.

b) Parmenides’ claim was that either Being is or Being is not, in a black-or-white fashion. However, to say that “Being is not” is a contradiction, since Being is that which is, hence to say that Being is not is to say that which is is not.
Given that it is impossible to say that Being is not, the only possible option is to say that it is. Therefore, Being is. But, if Being is, then Not-Being is not. But, if Not-Being is not, then Being must be one. For, if Being is not one, something must separate the one being from the other being. Now, if this separating "something" is a being, it will be part of the original one Being. But, if this separating "something" is a not-being, it must be nothing since we have accepted that Not-Being is not. It turns out that there is only one genuine Being that can be spoken or thought of and it cannot be divided, either internally or externally, from anything else. So, Parmenides reaches the conclusion that Being is one. This monism seems very difficult to deny, unless Parmenides' saying is partly refuted.

c) To overcome this difficulty and also be able to account for the existence of the numerous different existing things of our experience, the Platonists assumed that one of Parmenides' premises is wrong. They claimed that Not-Being in some way is. If Not-Being has a being in some way, it follows that it is something that can be thought of and spoken of.

d) The Platonists used this sophisticated notion of "the Not-Being that is" as a principle which together with the Being, which is the other principle, might produce a plurality of things.\footnote{Plato does that in the Sophist where the initial opposition of Being and Not-Being turns out to be a distinction of the five so-called "highest genera", i.e. Being, Sameness, Difference, Rest and Motion. Not-Being is thus translated into or interpreted as Difference and Motion, a translation or interpretation which allows for both a plurality of things and all kinds of change. Significantly, it is a Stranger or Guest from Elea, the hometown of Parmenides, who does the refutation and thus commits a sort of parricide to "father" Parmenides.} This rehabilitation of the Not-Being makes the plurality, the movement and the change possible\footnote{Annas (1976), 202}. Therefore, the Platonists assumed that existing things originate from Being and Not-Being. This means that they put the problem of the first principles in a primitive bipolar way, considering them to be contraries.

Aristotle criticizes Plato for identifying his principles with Being and Not-Being. It is not certain to what extent Aristotle's criticism is fair. It is true that the treatment of Being and Not-Being which is ascribed to Plato here reminds of the way Being and
Not-Being are treated by the Eleatic Stranger in the *Sophist* (237a, 256e). It is also true that Plato’s two principles are contraries and can in some way be thought of as something that is and something that is not. The one of the two elements, i.e. the One, could perhaps be thought of as something that is the One, in the sense that it has the being of the One, while the other element, i.e. the Indefinite Dyad, might be thought of as something that is not the One, in the sense that it has a being which is different from that of the One. However, if this is true, nothing could prevent us from treating also the Indefinite Dyad as something that is and the One as something that is not. For, in such a case, the Indefinite Dyad could be thought of as something that is the Indefinite Dyad and the One could be thought of as something that is not, i.e. is different from, the Indefinite Dyad. Therefore, both the One and the Indefinite Dyad could be thought of things that are what they are and as things that are not what they are not. I would agree with Annas\(^{161}\) in that even though Plato’s two opposite principles have been identified with wider oppositions, such as that of good and bad, there is no secure evidence that Plato had identified his two opposite principles with Being and Not-Being.\(^{162}\)

\(^{1089a6-14}\): In this section, Aristotle discusses the difficulties resulting from the fact that Being applies to many different categories of existing things. These difficulties demand that Not-Being is. But, even if Not-Being is, the diversity of the categories of the existing things cannot be explained.

First, we should inquire of what sort of thing Being is. For, the Being has as many senses as it has applications in different categories.

Given that “Being” means either “being a substance” or “being an affection” and, in general, “being any one of the categories”, the serious difficulty arising is of what nature (substance, quality, quantity etc.) Being as a first principle will be.

\(^{161}\) Annas (1976), 201

\(^{162}\) In a relative sense he may have done so. Cf. the being ascribed to Space in the *Timaeus*, a very peculiar sort of being which can only be grasped, if at all, by a “bastard reasoning”. If the Indefinite Dyad were indeed called Not-Being the name would indicate that it is nothing specific, no thing in particular but, rather, the entire range of possibilities which can only come into actual being if this principle is fertilized by the One.
The unity of Being may either be thought of as a unity of substance or as a unity of quality etc. separately from any other category or be thought of as a unity embracing all the categories. The following passage from the *Physics*, is relevant (A.2 185a20-30): “Ἀρχὴ δὲ οἰκειοτάτη πασῶν, ἐπειδὴ πολλαχῶς λέγεται τὸ ὅν, πῶς λέγοντος οἱ λέγοντες εἶναι ἐν τὰ πάντα, πόσερον ὁσιαν τὰ πάντα ἢ ποιά ἢ ποιά, καὶ πάλιν πόσερον ὁσιαν μιᾶν τὰ πάντα... ἢ ποιόν ἐν... ἢ τῶν ἄλλων τι τῶν τοιουτῶν... εἰ μὲν γὰρ ἔσται καὶ ὁσία καὶ ποιόν καὶ ποσόν, καὶ ταύτα εἰτ᾿ ἀπολελυμένα ἀπ᾿ ἄλληλων εἰτε μη, πολλὰ τὰ ὅντα· εἰ δὲ πάντα ποιόν ἢ ποσόν, εἰτ᾿ ὁσίης ὁσιας εἰτε μη ὁσίης ἀτοπον, εἰ δὴ ἀτοπον λέγειν τὸ ἄδύνατον.”). Although Plato is not mentioned by name in the quoted passage, it seems that Aristotle implies that Plato did not pay serious attention to the sort of unity that was entailed by Parmenides’ denial of Not-Being. Was it merely a unity of substance or a unity of quality etc.? Or was it a unity of substance together with the quality and all the other categories, namely a unity of everything?

It is a matter of question what the “unity of substance” means. It could mean jointly the unity of all substances into an overarching Substance. Perhaps it would seem that, if all the substances were one substance, then the universe would look like a uniform mass lacking order and organization. However, it is more reasonable to think of the universe at large as an overarching Substance of which all particular substances are parts. Thus, the universe as a whole is treated as a living being. The “unity of substance” could also mean severally the unity of each substance with itself. Nevertheless, it is not what is meant by “τὰ ὅντα πάντα ἐν” in the current passage of the *Metaphysics*, since it is not the sort of unity that makes the introduction of Not-Being necessary.

If the unity of Being concerns the unity of one particular category of existing things, then reasonably there have to be many such “unities”, provided that there are many different categories of existing things. In such a case, it is not necessary that Not-Being is, but it is necessary that Being is not one.

If the unity of Being concerns the unity of all the categories of existing things, then reasonably there has to be one factor that can explain the plurality of the categories of
existing things. In such a case, it is necessary that Not-Being is. But, even if one accepts that Not-Being is, it seems totally absurd that Not-Being, which is one single principle of one single nature, is responsible for the generation of a plurality of categories of existing things. Aristotle seems to point out here that τὸ μὴ ὄν might divide the world into some random distinct parts, but it could not divide it into a particular number of classes of things having specific characteristics, like items that belong in different categories. Hence, Not-Being would not save Plato from the consequences of the unity of all the categories together.

In conclusion, it turns out that it is impossible both to say that Not-Being is and to say the Not-Being is not. For, as Alexander states, even if Not-Being is, the plurality of the categories of existing things cannot be explained, but also, even if Not-Being is not, we can by no means argue in favour of the unity of the Being.

It is noteworthy in this section that Being and Not-Being are treated as the components of the universe. However, Being and Not-Being are neither substances nor qualities nor quantities. They indicate in fact the events of being or not being a substance or a quality or a quantity etc.

1089a15-19: In this section, Aristotle, after having referred to the many different senses which the Being can have, continues to refer to the many different senses which Not-being also can have. It has already been accepted that Being has as many different senses as the categories to which it applies. Aristotle’s criticism focuses on the fact that the Platonists needed also to explain the many senses in which the Not-Being can be considered.

a) Provided that Being has many senses and also that Not-Being is the opposite of Being, Not-Being must have many senses, too. In particular, Not-Being must have as many senses as Being has. Not-Being must have as many senses as there are categories to which it applies. For instance, “not being a human being” has the sense of “not being this sort of substance”. Similarly, “not being straight” has the sense of “not being this sort of quality” and “not being

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163 Annas (1976), 202
164 [Alex.], 785.13-14
three cubits” means “not being this sort of quantity”. So, there have to be many kinds of not-being, in the same way in which there have to be many kinds of being.

b) The question is from what kind of “not-being” the plurality of the categories of existing things comes.

Not-Being is treated as meaning “not being something” and not merely “not being” in the sense of “not existing”. Aristotle says elsewhere that Not-Being has many senses (cf. *Metaphysics*, K.11 1067b25 and *Physics*, E.1, 225a20: “τὸ μὴ ὁν \( \lambda \)έγεται πλεοναχὺς”) and distinguishes “not being” from “not being something” (cf. *Physics*, A.3 187a5-6: “οὐθὲν... κωλυει μὴ ἂπλως εἶναι, ἀλλὰ μὴ ὁν τι εἶναι τὸ μὴ ὄν.”). It seems that the latter sense, i.e. “not being something” is more relevant to the present context. For the former sense, i.e. “not being” in the sense of “not existing” cannot apply to the categories to which Being applies. Moreover, both Being and Not-Being are to be recognizable as principles that exist. Granted that any object comes into being from Not-Being, this means that Not-Being is potentially something. Not-Being, as a principle, cannot be treated as not existing. Therefore, the mere sense of “not existing” seems unsuitable for the Not-Being here.

There is again the difficulty of Being and Not-Being’s treatment as principles here, while they describe the events of “being something” and of “not being something” respectively. Crubellier\(^{165}\) insightfully claims that “Not-Being” could be thought of as “being the other/something else”. For instance, “not being blue” can be considered as an equivalent to “being red”, just as “not being Pollux” can be considered as an equivalent to “being Castor”. Nevertheless, such a “Not-Being” cannot be treated as a principle in the first philosophy. In fact, only the Forms of “Not-Being” that are sufficiently universal -and not “Not-Being” in general- could act as principles.

1089a19-31: In this section, Aristotle’s criticism focuses on the false nature that is ascribed to Not-Being by the Platonists.

\(^{165}\) Crubellier (1994), 447
a) Plato means the false and identifies it with Not-Being. A similar—but not identical—view is formulated by Plato at *Sophist* (237 A.3-4: “Τετόλμηκεν ὁ λόγος οὗτος ὑποθέσαι τὸ μὴ ὅν εἶναι· ψευδὸς γάρ οὐκ ἂν ἄλλως ἐγίγνετο ὃν”), where he claims that the presence of Not-Being makes the falsity possible and that a false statement puts things in a way that is different from the way things really are.

b) According to Aristotle, the Platonists argue that the geometers formulate false assumptions in order to prove truths. When, for instance, they assume that the line they draw is one foot long, while it is not, the geometers set up an initial postulate and a proof depending on this, which however does not itself assert the truth of the postulate. It also seems that, according to the Platonists, given that the geometers draw true conclusions, even if they start from false facts, there is no reason why the existing things, too, could not be derived from false principles in the same way. In the case of geometry, the line which is not one foot long is an instance of “not-being (something)”. In the case of the principles, Not-Being, which is not Being, is also an instance of “not-being (something)”. Philosophers have to assume what is not something or else what is false in order to explain what something is or else what is true. Therefore, in both cases, “not-being” is necessary for “being” or else the false assumption is necessary for the true outcome.

c) Aristotle addresses his strong criticism to both cases. As far as the case of the geometry is concerned, he points out that the geometers do not assume false premises, given that the false assumption of, say, a length which is not the length of the line actually drawn on board does not constitute part of the syllogism and does therefore affect the truth of the conclusion. So, the false in geometry does not occur in the proof, but in the setting down of some particular data for the sake of sense-perception, as Alexander 166 also clarifies. Similarly, as far as the case of principles is concerned, the beings can neither be generated nor be destroyed due to Not-Being. For, just as the false facts do

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166 [Alex.], 785.33
not form parts of the proof, in the same way Not-Being cannot form part of beings.

d) The senses of the Not-Being are as many as there are categories. Furthermore, the false and also the potential are called “not-being”. The false is “not-being” in the sense of “not being true” and the potential is “not-being” in the sense of “not being yet”. The potential has the capacity of developing into something that it is not yet. The beings, whether they are substances, for example a human being, or qualities, for example white, or one or many can be generated from something which is not yet what they are. Therefore, as far as the cases that involve coming into being are concerned, Not-Being has not the sense of “not being true”, i.e. being false, but the sense of “not being actual”, i.e. being potential. If an object X comes into being from what is not an object X, this is not relevant to falsity, but to potentiality.\(^{167}\)

Firstly, we should pay attention at the meaning of the term πρότασις at line 1089a24. Πρότασις, in its ordinary sense, means “proposition” or “premise”. Aristotle claims that the proposition is not a part of the geometers’ syllogism. The propositions or premises lay down the facts which are necessary for the deduction to be drawn. Given that in any syllogism we cannot reach the conclusion without the data of the premises, the premises seem to be essential parts of the syllogism. According to Crubellier\(^{168}\), we had better interpret “τὸ ψεύδος” (the false) as a really existing term that could be described as “the false” and not treat it as “a false premise”. Crubellier’s proposal seems reasonable, if the term πρότασις is understood in its ordinary sense (i.e. “proposition/premise”) and provided that the false is said not to exist in the premises.

Nevertheless, we should take into account another sense in which the term πρότασις could be understood. Aristotle’s claim that the πρότασις is not a part of the geometers’ syllogism might mean that the πρότασις, in the sense of the process of putting forward the particular facts, as is the process of drawing a line which is not a foot long, is not a part of the syllogism, in which the datum is that the line is a foot

\(^{167}\) Annas (1976), 203
\(^{168}\) Crubellier (1994), 449-450
long. It does not matter that the drawn line is not really a foot long, for this drawing is constructed for the sake of sense-perception and does not form a part of the proof. But, the geometers’ assumption that the line is a foot long is not false, since any line can truly be perceived by the mind as being a foot long. It is this line, i.e. that which is perceived by the mind as being a foot long, which forms part of the proof and there is no falsity in it as there is in the line which is drawn. As Ross\textsuperscript{169} points out, the falsity does not occur in the general enunciation, but in the setting down of the particular data.

Aristotle could perhaps be accused of unfairly criticizing the Platonic theory about the way in which they treat their material principle. Syrianus claims that the Platonists believed in the existence of Not-Being both in the intelligible realm, in the form of Otherness, and in the sense-world, in the form of Matter\textsuperscript{170}. If there are two Not-Beings, Otherness and Matter, they must be somehow related, perhaps as copy to original. The material substrate must then be an extended copy of essentially not-spatial Otherness. However, a copy is something distinct from the original and derivative from it. It can therefore be perhaps compared with a drawn line which is not as long as the mental assumption would like it to be.

1089a31-1089b3: In this section Aristotle goes on to refer to the principle that makes plurality possible, and his criticism focuses on the fact that the Platonists’ material principle does not seem suitable for any category of objects and cannot be thought of as a principle of plurality in general.

a) It seems that the Platonists try to explain how the beings that are substances can be many.

b) For, the individual beings that are generated are numbers, lengths and bodies, which are treated as substances by the Platonists.

c) However, it is absurd that they try to explain only the plurality of the individual substances and not the plurality of the qualities or the quantities or of the items of any other category.

\textsuperscript{169} Ross (1924), v. 2, 476
\textsuperscript{170} Syr., 172.5-8
d) The reason is that the Indefinite Dyad -or else the Great and Small- is the generative principle of the numbers.

e) Therefore, the Indefinite Dyad -or the Great and Small- cannot be thought of as a principle of the qualities, such as the colors, the flavors and the shapes, given that it can exclusively produce units and numbers (which are thought of as pluralities made up of units).

f) However, in the case of the qualities too, there should be some material element that is the same as or analogous to that of numbers and units. So, if the Platonists had pursued this inquiry, they would have sought for the cause which ascribes distinctness and multiplicity to the qualities, the quantities etc., i.e. the matter of each category that is peculiar to it and makes its items many in number.

Aristotle again takes it for granted that the Platonists seem to think of the Indefinite Dyad as the material principle that is the source of the plurality in general. However, according to Aristotle, the Indefinite Dyad can account for the plurality only as regards the individuals of a certain kind sharing all the same sort of matter and not as regards the items of different categories (substances, qualities, quantities etc.). Provided that the Indefinite Dyad acts as matter, it seems that it serves as a factor that is capable of individuating the items belonging to the same category of beings. For, each and every different category is to have its own matter. The matter in all categories cannot be the same, but only by analogy (cf. *Metaphysics*, Λ.4 1070b16-18: “τούτων μὲν οὖν ταύτα στοιχεῖα καὶ ἀρχαί, ἄλλων δ᾿ ἄλλα, πάντων δὲ οὕτω μὲν εἰπεὶν οὐκ ἐστιν, τῷ ἀνάλογῳ δὲ”). The matter’s being “analogically” the same in all categories means that, even though the matter in each category is different, the role it plays and its relation to the Form remain always the same. But, this approach presupposes that the Platonists recognized Aristotle’s distinction of Form and Matter, which is doubtful. Moreover, even if the Platonists had formulated such a theory about the existence of a different sort of matter for each category which is only by analogy the same as the matter of any other category, probably Aristotle
would again have strongly criticized it\textsuperscript{171}. For, how would it be acceptable that one separate principle which is the prime matter is responsible for the existence of the many different sorts of matter applying to the different categories of existing things? Therefore it seems unlikely that one and only material principle can be used to explain plurality in general.

\textbf{1089b3-7}: In this section Aristotle refers to the fact that the principle of plurality is treated by the Platonists as a relative. His criticism focuses on the Platonists’ belief that the Unequal is the opposite principle of the One.

The Platonists’ deviation from providing a principle of plurality for colours, flavours, shapes etc. has led them to assume that the Unequal is the principle that is contrary to the other principle, i.e. Being and the One.

a) The Unequal is neither the opposite nor the negation of the One.

b) Moreover, the Unequal is a relative, i.e. part of just one category of existing things, just as other things are parts of other categories such as substance or quality.

c) Aristotle seems to imply here that the Platonists’ failure to distinguish the material principle of the one category of beings from the material principle of another category of beings has led them to the point of deriving the items of one category from a principle which belongs to another category.

What is added here about the material principle is that it should be the contrary both of Being and of the One. After the reference to Parmenides’ dictum and to the way in which the Platonists maintain and treat the concept of Not-Being, Aristotle now turns to the treatment of the One, to which the Platonists are said to ascribe the role played by Being. Aristotle uses both the term “Being” and the term “One” here, probably because he wants to emphasize even more the weakness of the Platonists’ treatment of the principles. The use of both terms highlights even more their failure

\textsuperscript{171} Annas (1976), 204
to do what they wish, i.e. to introduce a material principle that is contrary to the other first principle.

1089b8-15: In this section, Aristotle refers to the need of explaining the plurality of relatives. His criticism focuses on the Platonist’s failure to provide a principle of plurality for the relatives.

a) The principle which is not the One, i.e. the material principle, is (especially under the descriptions of the “Unequal” and the “Great and Small”) a relative.

b) The Unequal is treated by the Platonists as the material principle which is responsible for the plurality of individual substances. This means that a principle which is a relative explains the plurality of substances.

c) Even if somebody was to accept the fact that a relative acts as a principle of substances, they could by no means also accept that this relative acts as a principle also of relatives.

d) Furthermore, the Platonists try to explain the existence of many different units apart from the first One. However, they do not try to also explain the existence of many different unequal things apart from the Unequal.

e) Although the Platonists seem to assume that there are many different versions of the Unequal, i.e. the many and the few, the long and the short, the broad and the narrow and the deep and the shallow, they have not tried to account for this fact, i.e. how there can be many different relatives.

f) Therefore, it is true that there is a plurality of pairs of unequal things that are relatives. Nevertheless, this plurality is not explained by the Platonists, as it happens with the units in the case of numbers.

1089b16-24: In this section Aristotle makes reference to the potential nature of the relative as a principle, addressing his criticism to the Platonists’ view that the relative as a principle is potentially substance:
a) We must necessarily assume that for each existing thing there is something which is potentially that thing. In other words, it is necessary that there is a material substrate for all the existing things.

b) The Platonists assumed that a relative principle is potentially a “this”, i.e. a substance, but it is not a substance in itself. Given that the material principle is potentially a being, the relative which is a principle is potentially the One and Being.

c) Saying that the relative is potentially substance is as bad as if we had said that quality is potentially substance. The reason for this is that both relative and quality are, each, merely one kind of being. How could one sort of being be potentially another sort of being?

d) Moreover, it is a matter of question how the categories of existing things can be many. ὅσπερ ἔλέχθη makes a reference to the lines 1089a33-34 (“ὅπως μὲν πολλά τὸ ὑπὲρ τὸ τι ἐστι ζητῆσαι”).

e) Aristotle’s claim is that a principle which is itself an existing thing of some sort cannot provide any explanation of the existence of numerous categories of existing things, such as substances, affections / attributes and relatives.

f) If the second principle is an existing thing of a certain kind, then it belongs to a certain category.

g) Thus, whoever wants to answer a “how many?” question, needs to be aware in which category he is raising the question.

h) However, this is absurd, for the “how many?” question should be answered with reference to the beings in general and not to the beings of one specific category. We can observe that the point concerning the need for a principle of plurality in general is repeated here.

i) Therefore, in case the principle which is not the One is itself an existing thing, it cannot be treated either as the contrary of the One or as something which is potentially an existing thing or as an element explaining the plurality of beings.
The phrase ὁ ταῦτα λέγων at line 1089b17 indicates Plato, who is said by Aristotle to have used something relative as a principle of plurality. In all likelihood, Aristotle does not imply here that Plato used the word “relative” as a name for the second principle. However, though he did not explicitly use the word “relative”, Plato is said to have introduced the Unequal-or else the Great and Small- which is a relative in Aristotle’s terminology, as the second principle. Aristotle points out that Plato in fact committed the same mistake as if he had taken a quality to be the second principle. For, neither a relative nor a quality nor anything else belonging to any category of existing things could be considered to be potentially the One or Being or the negation of either of them. Anything falling within relatives or quantities or any other category is thought of as merely a particular sort of being. So, it cannot be thought of as a principle which explains the plurality of beings in general. Nevertheless, for Aristotle’s criticism to be fair it is necessary that the terms “Indefinite Dyad”, “Unequal” and “Great and Small” are really assimilated by the Platonists to a relative and not used just to display the irregularity and the disruptive force of the principle of plurality, which is what Syrianus172 claims.

1089b24-29: In this section Aristotle’s criticism focuses on the Platonists’ failure not only to explain the plurality of qualities and the plurality of quantities, but also to argue in favour of the existence of a different kind of matter for each of these categories.

a) It has already been mentioned by Aristotle (cf. 1089a31-1089b4) that it is absurd if one is to investigate how there are many of substances, but not how there are many of qualities or quantities. However, it seems quite difficult to explain the plurality of the items belonging to the other categories.

b) Aristotle adds one further argument so as to reinforce the claim that there are many qualities and quantities. He says that because qualities and quantities are not separate from the subjects to which they apply and also because the

172 Syr., 174.11-17
subjects to which qualities and quantities apply are many, this entails that
the qualities and the quantities are necessarily many.

c) Moreover, there has to be a distinct type of matter for each category. The
matter in each category is only by analogy the same, as it has already been
said at line 1089b3 ("τὸ γὰρ αὐτὸ καὶ τὸ ἀνάλογον αἴτω"). So, the existence
of many different classes of existing things presupposes the existence of a
different type of matter for each category.

d) However, in the case of the categories other than substance, such as quality
and quantity, we cannot assign to any of them a matter which shall make a
plurality of separable existing things possible. For, neither quality nor
quantity is separable from the substance.

e) This entails that non-substantial matter is inseparable from the substantial
matter. But, in such a case, every substance should consist of two or more
sorts of matter, which is absurd.

f) Hence, it seems that there is no sort of matter especially for other categories,
given that the kind of matter in question ought to be inseparable from that of
substances. As far as colours are concerned, the matter of colour cannot be
separate from the matter of the corresponding substance. For instance, if we
say that the red as a quality must have a matter which is different yet
inseparable from that of the apple as a substance, what sort of matter can this
be and in what way is it distinguishable from that of the apple?

An interesting point that deserves our consideration is in what sense there can be
a plurality of individual qualities or individual quantities. An individual quality can
firstly be thought of as an item that is unique to the individual substance in which it
exists and perishes when this particular substance perishes. For example the green of
a particular tree perishes when this tree does. Secondly, an individual quality can be
thought of as an item that can exist in all substances belonging to the same sort of
substance. For example, a tree shares its green with the other trees. Unfortunately,
Aristotle does not clarify in which of the two senses he speaks of individual
properties. Perhaps, individual properties indicate properties that are proper to particulars, not one-over-many common universals such as a particular shade of green.

1089b29-32: In this section, Aristotle claims that it is a more reasonable inquiry to pursue how the sensible individuals are many than how the individuals that are also natures of a specific sort are many. The material principle can be used to explain the “how many” of any existing thing that is a “this”, i.e. an individual object. Aristotle’s criticism focuses on the Platonists’ failure to account for the plurality of the actually existing substances.

a) The material principle cannot be thought of as an element that can be used to explain the plurality of any existing thing that is both an individual object and “some such nature”, meaning a particular essence.

b) Therefore, it is a matter of discussion how the material principle can explain that there are actually many substances, for instance, that there are actually many human beings. What Aristotle seems to imply in this passage -that is rather hard to understand- is that the material principle can only explain the plurality of the individual things, but not the plurality of the individual things having a certain identity which includes them in the category of substances. In other words, Aristotle’s view is that the material principle explains how there are many individual substances sharing the same essence but not how there are many different essences.

c) Therefore, the material principle proves once more inadequate as regards the explanation of the plurality of existing things.

After closely reading this passage and also taking into account the passage coming earlier as well as the one coming later, we can notice that by “this (something)” Aristotle seems to mean any individual substance of our experience, for example a human being, a tree etc. Nothing prevents many individual things, for example many human beings or many trees, from existing. However, by saying that there is a

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173 Annas (1976), 204
plurality of human beings or a plurality of trees we do not mean that there is a plurality of merely individual objects, but that there is a plurality of individual objects that are also a nature of specific kind, i.e. human beings and trees respectively. Hence, the difficult question arising is how we can explain the plurality not of individual objects, but of substances.

The plurality of individuals is to be explained by the material principle which, as we are about to see, can only serve to explain the quantity of a “this”, but not how and why there can be many “thises” of the same kind. In the case of ten trees, for instance, the material principle can be used to explain how there are ten individual entities, but not how and why there are ten trees. Things seem to be rather complicated due to the fact that the “this” seems capable of referring not only to individuals, but also to types of thing. Therefore, there is a serious difficulty left over, even if it is admitted that the material principle can explain the plurality of the individual beings.

One interesting point that deserves our consideration is Alexander’s interpretation of line 1089b30 (“πῶς πολλὰ τὸ τόδε τι”). He chooses to interpret this passage in the following way: an individual being, for example Socrates, can be thought of both as one, in the sense that it is one individual entity, and as many, in the sense that it is both a substance (i.e. Socrates) and a quality (i.e. white) and a quantity (i.e. two cubits long). Alexander’s interpretation seems reasonable.

1089b32-1090a2: In this section, Aristotle emphasizes that the Platonists’ principles can only explain quantities. His criticism focuses on the Platonists’ confusing mathematical objects with physical bodies.

   a) If it is agreed that substances and quantities are not the same, then it remains to be explained how substances can be many.

   b) In fact, this inquiry has been considered by the Platonists only with regard to quantities.

\[174 \text{[Alex.], 790.31-33}\]
c) For, it is quantities that are meant by numbers. And the unit also, if it is not considered to be a measure, means a quantity which is indivisible.

d) If this is so, then, it is not substances but quantities that are explained as to their plurality by derivation from the One and the Indefinite Dyad. As far as substances are concerned, it is not explained either from what they originate or in what way there is a plurality of them.

e) If, though, substances and quantities are to be the same, then the person who postulates such principles as the One and the Indefinite Dyad has to face many difficulties.

Alexander\textsuperscript{175} mentions two serious difficulties: (1) If substance is a quantity, then substance will be considered as an accident. (2) A substance as substance must act as a substrate for properties, while it as quantity must itself be in a substrate.

If we compare this passage with the passage from lines 1089a31-35, we can observe that, while in the previous passage Aristotle implies that the Platonists confuse the mathematical objects derived from the One and the Indefinite Dyad with substances, in this passage he clearly distinguishes mathematical objects -which are quantities- from physical bodies, i.e. from substances. It seems that, according to Aristotle, the Platonists are obliged either to identify mathematical magnitudes with physical bodies -which is absurd- or to accept that their principles produce only magnitudes and are irrelevant to the plurality of bodies\textsuperscript{176}.

1090a3-15: In this section, Aristotle turns to the subject of the existence of the numbers. It is true that this subject has already been dealt with in M. However, while in M Aristotle’s account focuses on how the objects of mathematics are to be understood and how their existence is to be explained, in the present passage Aristotle focuses on the question of whether the mathematical objects can be the causes of existing things. Therefore, this passage belongs to the “third inquiry” that is announced in the M.1, i.e. whether the Forms and the Numbers can be causes.

\textsuperscript{175} [Alex.], 791.17-20
\textsuperscript{176} Annas (1976), 205
It is a matter of question on what grounds we are entitled to posit the numbers as separate entities.

a) It is true that the introduction of numbers is helpful to the thinkers who posit the Forms, given that, according to them, each number is a Form of some sort and the Form is the cause of the being for the other existing things. So, he who believes in the existence of Forms, i.e. Plato, treats the numbers as the causes of the existing things, given that each number is a Form.

b) There are also thinkers (e.g. Speusippus) who postulate the existence of the mathematical number, without considering the numbers to be Forms. However, it is impossible to explain how such a number has come into existence and of what use it is for the other existing things. Hence, Speusippus is said not to treat number as a cause for anything, but as a self-subsistent entity. This means that the number neither acts as a first principle or a cause of existence for the beings nor is in general needed for the explanation of the being of substances. What is then to be the causal usefulness of making numbers self-subsisting entities?

c) Finally, the mathematical theorems are true of the perceptible things, and do not imply self-subsistent mathematical numbers. This means that we cannot treat numbers as separable and thus individual entities due to our ability of treating the mathematical theories as abstracted from the perceptible things to which they apply.

Aristotle seems to claim that nothing can be said to exist unless we can explain from what principles it has originated and of what use it is for the other existing things (teleology). This is reasonable, because anything that is must be either principle which generates or existing thing which is generated. Nothing exists without either been generated from something or generating something. In other words, nothing exists without being connected with something else through a process of (active or passive) generation.
**N.3 (1090a16-1091a22)**

"Criticism against the status of the Form number, the mathematical and the Pythagorean number"

<table>
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<th>SECTION</th>
<th>ANCIENT GREEK TEXT</th>
<th>TRANSLATION</th>
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<tbody>
<tr>
<td>1090a16-20</td>
<td>Οἱ μὲν οὖν τιθέμενοι τὰς ἱδέας εἶναι, καὶ ἀριθμούς αὐτὰς εἶναι, &lt;τῷ&gt; κατὰ τὴν ἐκθεσιν ἐκάστου παρὰ τὰ πολλὰ λαμβάνειν [τῷ] ἐν τῷ ἐκαστὸν πειρόμενας χρήσει πως διὰ τί ἐστιν, οὐ μήν ἄλλα ἐπεὶ οὕτως ἀναγκαία οὕτως δυνατὰ ταῦτα, οὐδὲ τὸν ἀριθμὸν διὰ γε ταῦτα εἶναι λεκτέων·</td>
<td>The people who have supposed that the Forms exist and that they are numbers, by assuming -in their exposition of each thing over and above the many- that there is some one being, at least try to explain somehow why each particular being exists. However, since these considerations are neither necessary nor indeed possible, one must not assert that the number exists, at least due to these.</td>
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<tr>
<td>1090a20-30</td>
<td>Οἱ δὲ Πυθαγόρειοι διὰ τὸ ὅραν πολλὰ τῶν ἀριθμῶν πάθη ὑπάρχοντα τοίς αἰσθητοῖς σώμασιν, εἶναι μὲν ἀριθμοὺς ἐποίησαν τὰ ἄντα, οὐ χαριστοῦς δὲ, ἀλλ᾿ ἔξω ἀριθμῶν τὰ ἄντα διὰ τί δὲ; Ότι τὰ πάθη τὰ τῶν ἀριθμῶν ἐν ἀρμονίᾳ ὑπάρχει καὶ ἐν τῷ σωφατῷ καὶ ἐν πολλοῖς ἄλλοις. Τοὺς δὲ τὸν μαθηματικὸν μόνον λέγονταν εἶναι ἀριθμὸν οὔθεν τοιοῦτον ἐνδέχεται λέγειν κατὰ τὰς υποθέσεις, ἀλλ᾿ οτι οὐκ ἔσονται αὐτῶν αἱ ἐπιστήμημα ἑλέγετο. Ἡμεῖς δὲ φαμεν εἶναι, καθάπερ</td>
<td>But the Pythagoreans, by reason of observing that many attributes of numbers apply to perceptible bodies, made existing things be numbers, not separate though but in the sense that existing things consist of numbers; but why? Because the attributes of the numbers are present in the due arrangements both in the heavens and in many other things. But as for those who assert that only the mathematical number exists, it is not possible for them to say anything like this in accordance with their assumptions, but it used to be said that without assuming the existence of mathematical</td>
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είπομεν πρότερον. Καὶ δήλον ὅτι οὐ κεχώρισται τὰ μαθηματικά· οὐ γὰρ ἂν κεχωρισμένων τὰ πάθη ύπήρχεν ἐν τοῖς σώμασιν.

<p>| 1090a30-1090b5 | Οἱ μὲν οὖν Πυθαγόρειοι κατὰ μὲν τὸ τοιούτον οὐθεν ἐνοχοὶ εἰσιν, κατὰ μέντοι τὸ ποιεῖν εἰς ἄριθμῶν τὰ φυσικὰ σώματα, ἐκ μὴ ἐχόντων βάρος μηδὲ κουφότητα ἐχόντα κουφότητα καὶ βάρος, ἐσώκαι περὶ ἄλλου οὐφανοῦ λέγειν καὶ σωμάτων ἀλλὰ οὐ τῶν αἰσθητῶν· οἱ δὲ χωριστὸν ποιοῦντες, ὅτι ἐπὶ τῶν αἰσθητῶν οὐκ ἔσται τὰ ἀξιώματα, ἀλλήθη δὲ τὰ λεγόμενα καὶ σαίνει τὴν ψυχὴν, εἶναι τε ὑπολαμβάνουσι καὶ χωριστά εἰναι ὅμοιας δὲ καὶ τὰ μεγέθη τὰ μαθηματικά. Δήλον οὖν ὅτι καὶ ὁ ἐναντιούμενος λόγος τάναντι ἐρεί, καὶ ὁ ἄρτι ἴησον ἐν τοῖς αἰσθητοῖς ὑπαρχόντων τὰ πάθη ύπάρχει τοῖς αἰσθητοῖς. |
| The Pythagoreans in so far as this point is concerned are not guilty of anything; but in constructing the natural bodies out of numbers, namely things which have lightness and weight out of things which have neither lightness nor weight, they seem to be speaking about a different universe and different bodies, but not about the perceptible ones. But those who make the number separate suppose that that numbers exist and are separate, because the mathematical propositions will not turn out to be true of perceptible things, while the mathematical statements are true and appeal to the soul; and similarly with the mathematical magnitudes. It is clear then both that the contrary theory will say the opposite and that the difficulty we just raised -i.e., if numbers are in no way present in perceptible things, how are their attributes | number there will be no branches of knowledge having them as objects. But we maintain that there are, as we have said before. And it is clear that the mathematical objects do not exist in separation; for, if they were separate, their attributes would not be present in bodies. |
| 1090b5-13 | There are some people who, due to the fact that the point is the limit and the extreme of the line, and the line of the plane, and the plane of the solid, think that it is necessary that there are natures of this sort. We must therefore examine this argument, too, and see whether it is not extremely weak. For the extremes are not substances, but rather they are all limits, since even walking and movement in general has some limit. So, this will be a “this something” and a substance. But this is absurd. Moreover, even if they are substances, they will all be substances of these perceptible things; for the argument has applied to these. Why, then, should they be separate? |
| 1090b13-20 | Further, if one is not too easily satisfied, regarding all the numbers and the objects of mathematics, one might raise questions about the ‘fact’ that they make no contribution to one another, i.e. the prior to the posterior; for, if numbers did not exist, then the magnitudes would none the less exist for those who maintain that only the present in perceptible things?- has to be solved by those who hold this theory. |</p>
<table>
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<tr>
<th>1090b20-31</th>
<th>Those who postulate the Forms fail to notice this; for they construct the magnitudes out of matter and number, the lengths out of the twoness, the planes out of the threeness, presumably, and the solids out of the fourness, or out of other numbers, too; for it makes no difference. But, will these magnitudes be Forms, or what is their manner of existence and what do they contribute to existing things? For these contribute nothing; just like the mathematical objects, they too contribute nothing. Moreover, no mathematical theorem applies to them, unless one wants to change mathematics and invent theories of one’s own. It is not difficult to assume any random hypotheses and spin out a long string of conclusions. These thinkers, then, are wrong in this way, in striving to unite the objects of mathematics with the Forms.</th>
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<tbody>
<tr>
<td>1090b31</td>
<td>The first people who posited two kinds of number (the Form</td>
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1091a12 eidōn kai ton mathematikon, ou’ti eisēkasin ou’ti ’echein an eitein pois kai ek tinos esto o mathematikos. Poiousoi gar auton metaxu tou eisidikou kai tou aïsthetou. Ei men gar ek tou megallon kai mikrou, o autos ekeinou esto tò twn ideon (ex allon de tinos mikrou kai megallon tà [gar] megēthi poiei). ei de eteiron ti érei, pleio tà stoicheia érei kai ei en ti ekatérou h arxh, koivon ti épí touton esto tò en, zetetéon te pós kai tauta polla to en kai ama tôn aristomôn genvesthai allas hè ex énous kai dunameis armistou adbunatow kat’ ekeinon. Panta de tauta álloga, kai màxetai kai auta éautois kai tois eulógous, kai eikov en autois einai o Simwunidou makroû lógos- gíngnetai gar o makroû lógos ósper o tòs doullon otaan mèthen úgieis lêgasin. Fainetai de kai auta tà stoicheia to méga kai to mikron boan ás elkómena ou dynatai gár oudamaìs gennhsei tón aristomôn all’ hè tòn ar’ énous ditplasiaszómenon.

number and the mathematical) in no way have said nor could they say how and whence the mathematical number is to exist. For they make it intermediate between the Form number and the perceptible number. For, if it is composed out of the great and small, it will be the same as that number, i.e. the Form number (however, from some other small and great he produces the magnitudes); and if he happens to mention some other beginning, he will make more elements (than those assumed in the first place); and, if the principle of each of the two kinds of number is some one thing, then the One will be something common to them. And we should inquire how the One can be these many things, while at the same time, according to him, number cannot be generated otherwise than from One and the Indefinite Dyad. All these views are irrational, and they conflict both with one another and with the reasonable views, and we seem to see in them Simonides’ “long story”; for the long story comes into being, like the stories of the slaves, when they say nothing sound. And the very elements, the great and the small, seem to cry out as though they were being
dragged in; for they cannot in any way generate umbers except the numbers coming from One by doubling.

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<th>1091a12-22</th>
<th>Ατοπον δὲ και γένεσιν ποιεῖν αἰδίων ὑντων, μᾶλλον δ᾽ ἐν τι τῶν ἀδυνάτων. Οἱ μὲν οὖν Πυθαγόρειοι πότερον οὐ ποιοῦσιν ἢ ποιοῦσι γένεσιν οὐδὲν δεὶ διστάζειν: φανερῶς γὰρ λέγουσιν ὡς τοῦ ἕνου συσταθέντος, εἰτ᾽ ἔξ ἐπιπέδῳν εἰτ᾽ ἐκ χρόνου εἰτ᾽ ἐκ σπέρματος εἰτ᾽ εἰς ἄν ἀποροῦσιν εἰπεῖν, εὐθὺς τὸ ἐγγίστα τοῦ ἀπείρου ὅτι εἰλκετο καὶ ἐπεραίνετο ὑπὸ τοῦ πέρατος. Αλλ᾽ ἐπειδῆ κοσμοποιοῦσι καὶ φυσικῶς βοῦλονται λέγειν, δίκαιον αὐτοὺς ἔξετάζειν τι περὶ φύσεως, ἐκ δὲ τῆς νῦν ἀφείναι μεθόδου· τὰς γὰρ ἐν τοῖς ἀκινήτοις ἐπηκύμεν ἀρχὰς, ὡστε καὶ τῶν ἀριθμῶν τῶν τοιούτων ἐπισκεπτέον τὴν γένεσιν.</th>
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Moreover, it is absurd to introduce generation in the case of the things that are eternal, or rather this is one of the things that are impossible. There is no need at all to hesitate about whether or not the Pythagoreans introduce generation; for they clearly say that when the one was composed, whether out of planes or out of surface or out of seed or out of something that they are at a loss to specify, that immediately the nearest part of the Unlimited was drawn in and limited by the Limit. But since they are putting forward a world-formation and wish to speak in the terms of physics, it is fair to make some examination of their account of nature, and to exempt them from the present inquiry; for we are searching for the principles inside unchangeable things, so it is this kind of numbers whose generation we must study.
OVERVIEW:

In N.3 from Metaphysics, Aristotle addresses his criticism to the Platonists and the Pythagoreans, as regards the way in which they derive the universe from the numbers and also the way in which they treat the mathematical number as a separate entity.

At the beginning of Chapter Three, Aristotle attacks the Platonists' theory, according to which the Forms are numbers. According to Aristotle, the Platonists do not succeed in proving that there are Forms and thus in proving that there is Form number.

Next, Aristotle deals with the Pythagoreans' theory, according to which the beings are numbers. Having seen many attributes of the numbers belonging to the beings, the Pythagoreans assumed that the perceptible things must be created from numbers.

Then, Aristotle refers to those thinkers who believe that the mathematical number is the only number that exists. Speusippus—who is probably the thinker that Aristotle has in mind here—is said to treat the number as an entity that exists separately from the beings. However, if the mathematical objects are to exist separately, then their attributes would not be found in the physical objects.

Therefore, according to Aristotle, the Pythagoreans are right in finding the attributes of the mathematical objects in the beings, but they are wrong in assuming that the physical objects were constructed out of non-physical objects. And, those—who treat the number as self-subsistent and claim that the objects of the sciences could not be the perceptible things, since the axioms cannot apply to them, are open to a harder criticism than the Pythagoreans.

Furthermore, there are some thinkers according to whom the point, the line and the plane, which are limits, are separate entities. Nevertheless, Aristotle points out that if this was true, then all the limits, including those of events, should be treated as separate entities, which is absurd. It should be noted here that the point, the line and the plane constitute in fact a part of the object of which they are
limits. Therefore, there seems to be no way in which the point, the line and the plane could be treated as separate entities.

After that, Aristotle says that somebody—probably again Speusippus—might claim that the number does not contribute anything to the magnitudes and also that the magnitudes do not contribute to the soul and the perceptible things. However, nature is not episodic, according to Aristotle.

Those who believe in the Forms—namely the Platonists and especially Xenocrates—claim that the magnitudes are constructed out of matter and number. Nevertheless, Aristotle says that, even though they avoid the difficulties entailed in the case of Speusippus’ theory, their magnitudes still do not contribute anything to the perceptible things. In addition, no mathematical theorem is valid in the case of magnitudes that are thus constructed (i.e. out of matter).

As for those who believe both in the Form number and in the mathematical number, i.e. Plato, they fail to explain how the mathematical number exists. According to them, the mathematical number exists between the Form number and the sensible number. If the mathematical number is derived from the same principles as the Form number, then it must be identified with the Form number. If it is derived from different principles, then the elements should be rather numerous. And, if it is derived from the One from which the Form number is derived, then it needs to be explained how one and the same principle takes two different forms.

Finally, Aristotle returns to the Pythagorean theory, in order to criticize the way in which they explain the generation of the universe from the One. In particular, they claim that, when the One had been composed, the nearest part of the unlimited began to be drawn in and limited by the limit. But, according to Aristotle, such a discussion is more appropriate to physics and it is not a secure method of looking for the principles of the eternal things.
COMMENTARY:

1090a16-20: In this section, Aristotle addresses his criticism to the thinkers who believe in the existence of Forms and claim that the Forms are numbers, i.e. Plato and his followers. According to Aristotle:

(a) These thinkers consider Forms to be numbers.
(b) They also try to specify the mode and the reason of the existence of numbers.
(c) The method that is used by them is setting each kind apart from the particulars.
(d) Thus, they try to prove the existence of numbers based on the way in which universal beings (i.e. Forms) are said to exist apart from particular beings.
(e) However, it is neither necessary nor possible that Forms exist.
(f) Therefore, the existence of Form numbers cannot be proven.

Aristotle uses the term ἐκθεσις, when he describes the method which is adopted by some thinkers, in order to prove the existence of Form numbers. The term ἐκθεσις is also used elsewhere in Metaphysics (cf. A.9, 992b10: “τῇ γὰρ ἐκθέσει” & Z.6, 1031b21: “κατὰ τὴν ἐκθεσιν”). “Ἐκθεσις” indicates the process of setting forth. Alexander\textsuperscript{177} claims that the object of ἐκθεσις is not each Form, but each perceptible being, i.e. each particular instance. However, there is no way in which the formulation “setting out each particular being apart from the many other particular beings” could work here, since there is no real reason for pointing out a distinction between different particular beings, as there is for pointing out a distinction between a universal being and a particular one. According to Ross\textsuperscript{178} and Annas\textsuperscript{179}, in this case, what is set forth is each general term (or else each Form), which is set out over and above the many particular instances of a single kind. The procedure during which a Form, for example the Horse, is set apart from the many particular instances of the same kind, i.e. the many horses, acting as a model for them, is based on this method of setting out each Form apart from its particular instances. I agree with Ross and

\begin{itemize}
\item \textsuperscript{177} [Alex.], 792.19
\item \textsuperscript{178} Ross (1924), v. 2, 480
\item \textsuperscript{179} Annas (1976), 208
\end{itemize}
Annas, for it seems that “ἑκάστου” should be thought of as meaning each Form number. Given that Plato thought of only some Forms as numbers, “ἑκάστου” would refer generally to each Form, provided that Aristotle has Plato in mind here. The thinkers whom Aristotle criticizes are accused of trying to explain the reason why and the way in which each number exists by means of the same process which is used to describe the reason why and the way in which the Forms exist. This means that they are said by Aristotle to set forth each Form-number, for example the ideal Three, distinguishing it from the many particular instances of the same number, i.e. the many particular threes. The ideal Three, which is a Form number, acts as a model for the many particular threes that exist. It is also noteworthy that “ἑκάστον” in the next line (1090a18) needs to be interpreted as “each particular being”. Therefore, provided that “ἑκάστου” refers to each Form number as a species, it seems that by “ἑκαστον”, which is said to be set forth (ἐκθεσις) in line 1090a17, Aristotle means each particular instance of the same species.

There is a serious difficulty as regards the construction of lines 1090a17-18 (“κατὰ τὴν ἐκθεσιν ἑκάστου παρὰ τὰ πολλὰ λαμβάνειν τὸ ἐν τὶ ἑκαστὸν πειρόνται γε λέγειν πως διὰ τὶ ἐστιν”). The passage lying in the aforementioned lines has two verbs (“πειρόνται” and “ἐστιν”). Ἐστιν is the verb of the interrogative sentence, which is introduced with διὰ τὶ. Πειρόνται is the verb of the rest of the passage and it is constructed with the infinitive λέγειν, which is its object. The other infinitive, i.e. λαμβάνειν, cannot be constructed with same verb, i.e. πειρόνται, for it is not linked to λέγειν by any conjunction. In fact, τῷ λαμβάνειν indicates the process by means of which the Platonists try (πειρόνται) to formulate their theory about the existence of the Form-numbers. Hence, it might be thought of as a modifier of means. Ross proposes that, in line 1090a17, we need to add a τῷ before the phrase “κατὰ τὴν ἐκθεσιν” and, in line 1090b18, we need to omit τό. Τῷ is the article which accompanies λαμβάνειν introducing a modifier of means. If τῷ is added, the whole passage can be translated as follows: “by means of assuming -in virtue of the method of setting out each Form number apart from its particular instances- that some one

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180 Ross (1924), v. 2, 80-481
thing exists, they (i.e. the Platonists) try to say somehow why each number exists”. Aristotle’s point here is that Plato tries to explain the existence of numbers by giving prominence to the unity of each Form number that acts as a general term for many particular instances.

1090a20-25: In this section, Aristotle describes the Pythagorean theory, according to which the beings are numbers:

(a) The Pythagoreans are said by Aristotle to have noticed that the attributes of the numbers, such as the ratios and the proportions, exist in the universe.
(b) So, the Pythagoreans assumed that the beings are numbers.
(c) By saying that, they did not claim that the numbers exist separately from the beings.
(d) They just claimed that the beings are composed of numbers.

The Pythagoreans are said to have been impressed by the way in which the attributes of numbers are true of the existing things in the universe. The Pythagoreans claim that the universe behaves in a way which reminds us of the mathematical proportions. For instance, the numerical ratios can be detected in the musical intervals of the octave, the fourth and the fifth. Having made this notice, the Pythagoreans are said to have been led to the assumption that the beings are composed of numbers. The fact that each being is composed of numbers means that the essence of each being is number. Therefore, according to Aristotle, the Pythagoreans treat beings’ composition of numbers as equivalent to beings’ identification with numbers. This could have two different meanings. It could mean either that each being is identified with a specific number or that each being is considered to be of numerical nature, in the sense that its structure is built in accordance with numerical proportions. The second alternative could save Pythagoreans from some significant absurdities to which the first alternative leads. The difficulties deriving from the first alternative are presented afterwards by Aristotle.
In this section, Aristotle talks about the thinkers who claim that the only number which exists is the mathematical one. It is probably Speusippus that Aristotle has in mind here. These thinkers are said to believe that the numbers are substantial entities and independent of the physical objects:

a) According to the aforementioned thinkers, the mathematical numbers are not identified either with the beings or with their principles.

b) Moreover, they are said to have stated that there will not be any scientific knowledge.

c) However, Aristotle has already argued that there is scientific knowledge about the numbers.

d) Moreover, he disagrees with these thinkers’ belief that the mathematical numbers exist separately from the beings.

e) For, if they existed separately from the beings, then their attributes could not be true of the sensible bodies.

Τοις δὲ probably refers to Speusippus and his followers. Aristotle criticizes here these thinkers for committing an error. In particular, he accuses them of arguing that there is no scientific knowledge about numbers and also of treating numbers as separate entities. Aristotle points out that, as he has already said, there is scientific knowledge about numbers. As Alexander 181 explains, the thinkers who posit only the mathematical number believe that, if numbers are to be identified with sensible beings (which is what Pythagoreans claim), then there could not be any mathematical science. The reason is that, if numbers are to be identified with sensible beings, there could not be any object of knowledge that would be purely number. Therefore, there could not be any scientific knowledge that would particularly focus on numbers and that would be able to be called “mathematical science”. However, Aristotle argues both that the mathematical objects cannot exist separately from the physical ones and that there can be scientific knowledge about numbers. It is noteworthy that, according to Aristotle, the sensible beings are the objects of the mathematical sciences, yet not insofar as they are sensible, but insofar as they are

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181 [Alex.], 793.13-17
lengths, planes, solids etc. (cf. M.3 1078a2-5: “οὐ τῶν αἰσθητῶν ἐσονται αἱ μαθηματικαὶ ἐπιστήμαι, οὐ μέντοι οὐδὲ παρὰ ταῦτα ἄλλων κεχωρισμένων”).

Sensible things exhibit properties of mathematical objects, as the Pythagoreans claimed. Therefore mathematical objects -especially numbers- must be not separate but, rather, present in sensible things.

1090a30-1090b5: In this section, Aristotle discusses and plays the Pythagoreans’ view against the Academics’ beliefs concerning the way in which numbers exist in relation to the beings. He uses what is sound in each stance as a refutation of the opposite stance, and vice versa:

a) On the one hand, the Pythagoreans are accused by Aristotle of claiming that the physical objects, which have weight or lightness, come from the numbers, which do not have weight or lightness.

b) On the other hand, the Academics are said by Aristotle to claim that the mathematical objects have a separate existence. Nevertheless, they seem unable to explain the existence of the attributes of numbers inside the physical objects.

c) Therefore, both the Pythagorean and the Academic convictions can be easily attacked and negated.

The Pythagoreans are accused of identifying the numbers with the beings, without paying any attention to the fact that they identify something immaterial with something material. However, this serious difficulty makes their theory sound -at least partly- absurd. The Academics avoid this absurdity by regarding the numbers as existing separately from the physical bodies. Nevertheless, they fail to explain how the properties of numbers apply to the physical objects. It is noteworthy that the Pythagoreans treat numbers as entities having material nature and thus they are wrong as regards the Aristotelian and, in general, the reasonable treatment of numbers. Nevertheless, they are not wrong as regards the way in which they probably treated numbers. For, if it is true that the Pythagoreans did not identify each being with a specific number, but they identified beings with numbers, in the
sense that each being has a structure which is based on numerical relations, then the difficulty is avoided. In such a case, numbers are not treated as material objects having weight. Therefore, their identification with the beings does not entail the absurdities Aristotle may have in his mind in case each being was identified with a particular number.

1090b5-13: In this section, Aristotle addresses his criticism towards some thinkers according to whom the point, the line and the plane, which are limits, are separate entities:

a) These thinkers treat the point, the line and the plane as separate entities, just because they are limits.

b) However, limits cannot be substances.

c) If limits were to be substances, then this would be true of any limit in any case. For example, the limit of walking and, in general, moving, would be a separate entity, which is absurd. So, it seems that limits cannot be substances.

d) But, even if limits really were substances, they would not also be separate entities, since they belong to the objects of which they are limits.

At 1090b5, Aristotle refers to τινὲς (i.e. some thinkers), who, according to Ross¹⁸², are probably some Pythagoreans. The thinkers in discussion could be Pythagoreans, given that they are said to have formulated a theory as far as limits are concerned and this theory could be reminiscent of Philolaus’ theory about the first principles, i.e. the Limit and the Unlimited. These thinkers argue that: 1) the point is the limit of line, 2) the line is the limit of plane and 3) the plane is the limit of the solid. At 1090b7, by τοιαύτας φύσεις Aristotle probably means substances. In fact, it is the infinitive ("εἶναι") that has the strong meaning of being, in the sense of being in the mode of substance. The thinkers in discussion are said to believe that the point, the line and the plane are separate entities, i.e. substances. Therefore, according to these thinkers, the limits are substances.

¹⁸² Ross (1924), v. 2, 481
These thinkers’ argument is feeble, according to Aristotle. For, there is also a limit in the case of walking and, in general, moving. However, to treat the cessation of movement as a substance is even more absurd than to treat movement as a substance. For, if movement is not a substance, a fortiori an attribute of movement (i.e. cessation) cannot be a substance. Furthermore, even if the limits are really substantial things, there is no reason why they must necessarily exist separately. On the contrary, they need to be parts of the sensible things of which they are limits. But, if the plane, the line and the point are treated both as limits and as separate entities, then they must be limits that are capable of independent existence. However, the limit of something cannot exist separately from it, for then there will necessarily be another limit of this being, and, if this too can exist separately, there will be another limit, and so on. Therefore, which one among all these limits will be truly capable of existing both as a limit and as an independent substance? If they are to be all independent substances, it means that any substance having limits contains other substances in it, which is absurd.

Moreover, the line cannot be divided into points, just as the plane cannot be divided into lines and the solid cannot be divided into planes. For, if this was the case, then the continuum of the line, the plane, the solid and the magnitudes, in general, would be interrupted. The existence of point is independent from the existence of line and the existence of line is independent from the existence of plane etc. In conclusion, although a line is limited by a point, the point could not be thought of as part of the line.

The argument that the limits are incapable of independent existence suffices for disproving the Pythagorean beliefs. Therefore, Aristotle does not need to have recourse to the superfluous argument that the limit of movement will also have to be treated as a substance. It is noteworthy that this argument involves the sequence of dimensions, which is of great significance for the following argument.

183 If we draw a line and want to take a particular length of it, we shall draw two points at the appropriate distance from each other and so take the limited part of the line that we wanted.
184 Annas (1976), 209
In this section, Aristotle makes a reference to those thinkers who believe in the existence of only the mathematical number, i.e. mainly Speusippus:

a) They are said by Aristotle to claim that, in case the mathematical numbers did not exist, this would not entail that the mathematical magnitudes would not exist either.

b) And, if the mathematical magnitudes did not exist, this would not prevent the sensible bodies from existing.

c) However, according to Aristotle, this way of thinking about the universe entails that the nature of the universe is episodic. Ἐπεισοδιώδης means that numbers do not contribute anything to the existence of magnitudes and magnitudes do not contribute anything to the existence of bodies\textsuperscript{185}. This is absurd, since there has to be a rational way in which point must contribute to line, line to plane, plane to magnitude and magnitude to body\textsuperscript{186}. This rational account is what Aristotle demands here, for it is not reasonable that every being has an autonomous existence and also the Academy has not said anything definite on how the production of mathematical objects is connected to numbers.

Aristotle here strongly opposes the Speusippean belief that, even if number did not exist, magnitudes could exist and, even if magnitudes did not exist, soul and perceptible bodies could be capable of existing. Aristotle believes that, judging by what we can observe, the natural system is coherent and not constructed in episodes, like a bad drama. So, the posterior things or principles have to be generated by the prior ones. The Aristotelian belief seems reasonable. However, the number, the magnitudes, the soul and the sensible bodies are entities of totally different nature. Hence, in what rational way could number contribute to the existence of magnitudes or magnitudes to the existence of perceptible existing objects? The lines cannot be just made up out of the points. For, if this was true, then it would not be necessary to produce lines separately out of a different version of the second principle\textsuperscript{187}. The only

\textsuperscript{185} [Alex.], 794.28-29
\textsuperscript{186} Annas (1976), 209
\textsuperscript{187} Annas (1976), 209
way in which number could be related to magnitudes and magnitudes to sensible bodies is by means of considering magnitudes as mathematical objects having dimensions that are measurable by number and sensible bodies as perceptible objects that have three dimensions like magnitudes. Nevertheless, this does not entail that magnitudes could not exist without number or that bodies could not exist without magnitudes, but perhaps that the existence of magnitudes could not be explained without number and bodies could not be explained without magnitudes. Consequently, the relation between numbers and magnitudes as well as between magnitudes and sensible bodies seems not ontological, but only explanatory.

1090b20-32: In this section, Aristotle addresses his criticism to Plato and his followers, who believe in the existence of the Forms:

a) Plato is said to claim that the magnitudes come from the matter and the numbers.

b) In particular, according to the Platonic theory, either the lengths come from the number Two, the planes come from the number Three and the solids come either from the number Four or they come from other numbers.

c) It is a matter of question whether the line, the plane and the solid will be Forms or not and also what their contribution to the composition of the beings is to be.

d) However, even if the mathematical objects are Forms, as Xenocrates argued, they cannot be used to explain the sensible world, anymore than the mathematical objects that are not Forms—which is what Speusippus claimed—can be used for the same purpose.

In the case of Platonists and the Academic thinker Xenocrates, one further difficulty is discussed by Aristotle. At first, the way in which Plato and his disciples generate lengths from twoness, planes from threeness and solids from fourness does not really differ from the Speusippean way of deriving magnitudes from numbers. They only produce the objects of different dimensions from different numbers. Therefore, Xenocrates’ assumption of mathematical objects which were Forms does
as little to explain the sensible world as Speusippus’ assumption of mathematical objects which were not Forms\textsuperscript{188}. However, in the case of Platonists and Xenocrates as well, one further difficulty remains to be dealt with: are all these, i.e. all mathematical objects, Forms or not? Aristotle’s answer is that no matter whether mathematical objects are Forms or not, there is no way in which they could contribute to the existence of beings. Moreover, Aristotle’s argument here is that, since Platonists probably identify mathematical objects with Forms -which are entities that do not exist-, they describe mathematical objects as entities of some sort that does not exist, which is absurd and entails a great deal of difficulty.

Moreover, he points out that no theorem can be true of Forms, in the same way as it can be true of mathematical objects. No theorem can be true of the Form of circle or the Form of pyramid. It is a matter of question what is the reason for this inadequacy of the Forms. Alexander claims that the mathematical objects would have been confuted, if they were truly incompatible with theorems\textsuperscript{189}. However, as Syrianus\textsuperscript{190} points out, what Platonists could really have in mind is that theorems and demonstrations concern mathematical objects -not Forms- and the mathematical objects about which the scientific knowledge is ready for looking upon the ideal paradigms, i.e. Forms. This may be true from a Platonist’s perspective but it is not something that might convince Aristotle who does not believe in the existence of Forms in the first place. In my opinion, if the mathematical objects are identified with Forms, there seems to be no way in which mathematical theorems could apply to them, for two reasons: firstly, as Annas and Crubellier\textsuperscript{191} also accept, each Form is probably thought of as a unique entity, while the existence of theorems requires the existence of many similar instances of the same thing on which the theorems will apply. Secondly, if we take into account the use of the almost parallel phrase “for all the theorems of the arithmeticians will apply equally to perceptible things” at N.2, 1090a13-15, it could be assumed that the theorems result from the empirical

\textsuperscript{188} Ross (1924), v. 2, 482
\textsuperscript{189} [Alex.], 795.29-796.1
\textsuperscript{190} Syr., 179.28-180.5
\textsuperscript{191} Crubellier (1994), 501
observation of the properties of particular perceptible things around us, which often leads us to the formulation of universal rules. The universal nature is a common feature of Forms and theorems. Therefore, it seems that Forms and theorems would serve the same purpose, which is the representation of the characteristics of a genus of things. This renders the generation of theorems for Form numbers meaningless and thus impossible.

1090b32-1091a5: In this section, Aristotle attacks the Platonists’ intermediate number, which is the mathematical one:

a) Aristotle claims that the Platonists cannot explain the existence of this intermediate number - which is said to exist between the Form number and the sensible number - unless they assume that there are plenty of first principles and therefore that there are more than one formal principles.

b) However, this is absurd, if we take into account that the formal principle according to the Platonists is the One. For, how can the One be more than one principle?

It is a matter of question why Aristotle chooses to address his criticism to the intermediate number at this point of his discussion. The belief in intermediate number is ascribed to those thinkers who posit Forms, especially Plato and his disciples. Firstly, it seems that, according to Aristotle, Platonists’ mathematical number should be identified either with their Form number or with their sensible number. This is true, because, if the intermediate number comes from the same elements with the Form number, then the intermediate number has to be identified with the Form number. Otherwise, its existence cannot be explained. For, if the intermediate number comes from different elements, then there will be rather a lot of primitive elements, which is contrary to rules of simplification and unification. Secondly, if the Form number and the mathematical number come from different elements, then there will be different types of the first principle and different types of the second principle. However, the first principle is one. So, how can there be two or

\[\text{Annas (1976), 211}\]
more principles that are One? In such a case, plurality, which is said to be generated only by the union of the One with an Indefinite Dyad, appears even in the One. A possible answer to Aristotle’s argument here could be that the elements of mathematical number are only by analogy the same to those of Form number\(^\text{193}\). Nevertheless, such an answer would not really disprove the need of existence of a plurality of first principles and the difficulty that results from it.

1091a6-12: In this section, Aristotle argues that the Great and Small cannot produce any number apart from the Two and its multiples and powers, since it can only double something:

a) In \(\text{Ἀτάκτοι}\), Simonides of Ceos presents the sort of story that a slave will tell in order to cover his failure in some duty. The Platonists are accused of using the same sort of story for formulating their theories.

b) Furthermore, the Platonists are accused of “tormenting” the elements Great and Small.

c) For, the Great and Small cannot generate Form number, but they can only duplicate One and thus create number two and its multiples and powers.

Platonists are said by Aristotle to frequently discuss in length about the composition and structure of numbers\(^\text{194}\) without success, for they often make false and controversial assertions. One of their faults is that they derive number from the Great and Small, while the Great and Small -which is the Indefinite Dyad- cannot produce any number except for number two and its powers. In order to produce the rest of numbers, Platonists would have to use addition and multiplication\(^\text{195}\).

1091a13-22: In this section, Aristotle claims that it is absurd to derive eternal beings from principles, as Pythagoreans do:

a) The Pythagoreans maintain that, when the One had been put together, the nearest part of the Unlimited began to be drawn and limited by the Limit.

\(^{193}\) Syr., 180.20

\(^{194}\) [Alex.], 797.15-16

\(^{195}\) Ross (1924), v. 2, 483
b) However, according to Aristotle, such a discussion is more appropriate to physics.

c) What Aristotle is in pursuit of here are the principles of unmovable things. Aristotle here returns to the claim that the mathematical objects are generated from the principles. He says that it is impossible for eternal numbers, i.e. Form numbers, to be generated in a way that is similar to that in which the Pythagoreans generate eternal things. The process by means of which the Pythagoreans derive their eternal things is a temporal process. The universe becomes ordered and begins to have qualities after the Unlimited, which is the material principle, is drawn and limited by the Limit, which is the formal principle\textsuperscript{196}. The reference to the Pythagoreans here is made so that Aristotle claims that the Academics are really committed to such a temporal process of derivation of number, without admitting it\textsuperscript{197}. Hence, their principles remain an unanswered question in this section.

\textsuperscript{196} [Alex.], 798.10-11 & Syr., 181.15-16
\textsuperscript{197} Annas (1976), 211
### N.4 (1091a22-1092a8)

"Are the first principles good and beautiful or not?"

<table>
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<tr>
<th>SECTION</th>
<th>ANCIENT GREEK TEXT</th>
<th>TRANSLATION</th>
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<tr>
<td>1091a22-29</td>
<td>Τοῦ τοῦ μὲν οὖν περιττοῦ γένεσιν οὐ φασιν, ὡς δηλοντί τοῦ ἀρτίου οὐσίας γενέσεως· τὸν δ᾽ ἀρτίον πρῶτον ἐξ ἀνίσων τινές κατασκευάζουσι τοῦ μεγάλου καὶ μικροῦ ἰσαθέντων. Ανάγκη οὖν πρότερον ύπάρχειν τὴν ἀνισότητα αὐτοῖς τοῦ ἰσαθῆναι· εἰ δ᾽ ἂει ἦσαν ἰσαμένα, οὐκ ἂν ἦσαν ἀνίσαι πρότερον (τοῦ γὰρ ἂει οὐκ ἔστι πρότερον οὐθὲν), ἀπὸ φανερὸν ὅτι οὐ τοῦ θεωρήσας ἐνεκεν ποιούσι τὴν γένεσιν τῶν ἀριθμῶν.</td>
<td>These thinkers omitted to say that there is generation of the odd numbers, which clearly implies that there is generation only of the even numbers; and some thinkers produce the even numbers as the first thing to come from unequal parts, namely the great and small, after having been equalized. So, the inequality must apply to them before their being equalized. If they had been always equalized, they would not have been unequal before; for there is nothing which is prior to always. Therefore, it is clear that they do not introduce the generation of the numbers for the sake of a theoretical understanding.</td>
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<tr>
<td>1091a30-1091b3</td>
<td>Ἐχει δ᾽ ἀποφαίνει καὶ εὐπορήσαντι ἐπιτίμησιν πῶς ἔχει πρὸς τὸ ἀγαθὸν καὶ τὸ καλὸν τὰ στοιχεῖα καὶ αἱ ἀρχαι ἀποφαίνει μὲν ταύτῃ, πῶτερον ἐστὶ τὶ ἐκείνων οἰόν βουλόμεθα λέγειν αὐτὸ τὸ ἀγαθὸν καὶ τὸ ἀριστον, ἢ οὐ, ἀλλ᾽ ὑπερογενή; Παρὰ μὲν γὰρ τῶν θεολόγων ἐοικεν ὑμολογεῖσθαι τῶν νῦν τισίν, οἷ οὐ</td>
<td>It is though a perplexity and a criticism to anyone who has his doubts resolved, how the elements and the principles are related to the good and the beautiful; and the difficulty is this, whether any of the elements is of such a sort as we mean when we speak of the good itself and the best, or whether this is not so, but these are produced later. Theologian poets seem to agree with some of the present-day thinkers, who say it is</td>
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not so, but that the good and the beautiful make their appearance after
the nature of existing things has made some progress. They do this to
avoid a real difficulty, which confronts those who say, as some people
do, that the One is a first principle. However, the difficulty arises not
due to ascribing the good to the first principle as an attribute, but due to
making the One a principle - and a principle in the sense of an element-
and due to generating the number from the One.

And the old poets agree with this inasmuch they say that not the first in
time, for instance Night and Heaven or Chaos and Ocean, reign and
rule, but Zeus.

But still, they are led to saying such things due to changing the rulers of
the beings, for, those of them who combine two characters, as they do
not describe everything using a mythical language, for instance
Pherecydes and some other thinkers, say that the first generating
element is the best and so do the Magi and some of the later sages, like
Empedocles and Anaxagoras, when the former makes Love an element
and the latter makes Mind a (first) principle.
Of those who hold that the unchangeable substances exist, some say that the One-itself is the Good-itself, since they thought that primarily the One is the essence of the good. So, this is the problem: in which of the two ways we must speak. It would be strange, if that which is primary and eternal and most self-sufficient did not possess this very quality -self-sufficiency and self-maintainance- in a primary way and as something good. But indeed it is not indestructible for any other cause than because it is in a good state. So, it is probably true to say that the first principle is of such a nature; but it is impossible that this first principle be the One or, if not that, an element as well as an element of numbers. For many difficulties follow, to avoid which some people have rejected the theory, i.e. those who agree that the One is a first principle and element, but of the mathematical number.

For every unit becomes something that is good in itself, so that there is rather a great profusion of goods. Further, if the Forms are numbers, then all the Forms are something that is good in itself. But, let someone assume that there are Forms of anything he pleases. If there are Forms
only of good things, then the Forms will not be substances. And, if there are Forms of substances also, then all the animals and the plants and everything participating in them will be good.

Indeed, these absurdities follow and it also follows that the contrary element, whether it is plurality or the unequal, i.e. the great and small, will be the bad itself. For this reason, one thinker avoided attaching the good to the One, because it would necessarily follow, given that the generation is from contraries, that the nature of the plurality is the bad. Other thinkers hold that the unequal is the nature of the bad. It follows that (a) all the existing things partake of the bad except for one thing, i.e. the One itself, (b) the numbers participate in something more undiluted than spatial magnitudes (b) the numbers participate in a more untempered bad than magnitudes do and (c) the bad is the space of the good and partakes of and desires that which tends to destroy it. For the contrary tends to destroy the contrary. And if, as we said, the matter is that which potentially is each thing e.g. the matter of actual fire is that which is potentially fire, then the bad will be precisely the potentially good.
<table>
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<tr>
<th>1092a5-8</th>
<th>Ταύτα δὴ πάντα συμβαίνει, τὸ μὲν ὅτι ἄρχην πᾶσαν στοιχεῖον ποιοῦσιν, τὸ δ᾽ ὅτι τάναντια ἄρχας, τὸ δ᾽ ὅτι τὸ ἐν ἄρχην, τὸ δ᾽ ὅτι τοὺς ἀριθμοὺς τὰς πρώτας οὐσίας καὶ χωριστὰ καὶ εἰδῆ.</th>
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<td>Indeed, all these absurdities follow, (a) because they make every principle an element, (b) because they make the contraries principles, (c) because they make the One a principle and (d) because they treat the numbers as the primary substances and separate and as Forms.</td>
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OVERVIEW:

In N.4 from Metaphysics, Aristotle addresses his criticism to the Platonists, as regards their belief that the first principle is the One and is also good.

At the beginning of Chapter Four, Aristotle refers to the Platonists’ belief, according to which the even numbers are generated, while the odd ones are not. According to Aristotle, the Platonists believe in a chronological process of generation and not in a logical process for the sake of their teachings.

Then, Aristotle poses the question whether the good and the beautiful are among the first principles or are later than them.

Firstly, Aristotle refers to cosmologists who claim that the good and the beautiful should not be thought of as principles and he explains that the real absurdity does not lie in treating the first principle as something that is good, but in treating the One as a first principle.

Secondly, he talks about the way in which ancient poets and previous philosophers tried to avoid the difficulty that resulted from treating the One as a first principle.

Afterwards, Aristotle accepts that goodness is a quality which belongs to the first principle. However, he denies that the One is the first principle.

In addition, he explains the reason why the One cannot be a first principle, provided that the first principle is also something good.

Finally, Aristotle refers to all the absurdities that follow, if we accept that the one principle is good and the other is evil.
COMMENTARY:

1091a22-29: In this section, Aristotle discusses the Platonic belief that the even numbers are generated, while the odd ones are not. In particular:

a) Aristotle claims that “they” (i.e. the Platonists) do not talk about the generation of odd numbers, but they only describe the generation of even numbers.

b) Moreover, some Platonists are said to think that the even numbers were generated from the Great and Small after having been equalized by the One.

c) There must have been some time at which the Great and Small were unequal, before their being equalized. Otherwise, the Great and Small would be eternally equal. But, this is impossible, given that the Platonic principle Great and Small has to be unequal, as it is also indicated by its name. So, it would be absurd, if one stated that there was no time at which the Great and Small were unequal.

d) From the aforementioned points, Aristotle draws the conclusion that the Platonists truly believe in the generation of numbers and did not invent it for the sake of theorizing.

Aristotle claims that the Platonists considered the equalization of the Great and Small to be a process that occurred at some particular time. In other words, he accuses them of treating the origination of things from principles as a chronological - and not just logical- process. Therefore, according to Aristotle, the Platonists treated equalization as a literally temporal process and did not talk figuratively for the sake of formulating a theory for their teachings.

Alexander\textsuperscript{198} as well as Syrianus\textsuperscript{199} tries to defend the Platonic Theory of Forms. In particular, Alexander cites Xenocrates, who also defended Plato. According to Xenocrates, if, for the sake of teaching, the Forms are to have been generated, then there has to be something from which they were generated. This generating principle

\textsuperscript{198} [Alex.], 799.5-12
\textsuperscript{199} Syr., 181.30
is the Great and Small. Therefore, the inequality existed before Forms did. However, this entails that Forms cannot be thought of as eternal. But, Xenocrates says that the equalization of the Great and Small by the One was not in fact an event which occurred at some specific time, but it was an eternal state. Aristotle here attacks Xenocrates’ interpretation of Platonic account as merely logical and not chronological and points out that the Theory of Forms is absurd, for it becomes evident that the Forms are erroneously presented as eternal entities. He attacks Xenocrates’ interpretation of Plato and considers this necessity of the pre-existence of inequality as an evidence of Plato’s belief in a temporal process of generation, even though he did not admit it. Syrianus explains that the Platonists were speaking symbolically. It is obvious that Xenocrates’ and Syrianus’ purpose is to argue in favor of Plato’s theory by making comments that could save it from possible absurdities.

It should also be noted that, although Aristotle insists on a literal interpretation, elsewhere he claims that Plato’s theory about the creation of the universe can be interpreted in a non-literal manner (cf. De Caelo, A.10 279b32-280a11 and Physics, Θ.1 251b17-18). Therefore, speaking in a non-literal sense seems to be something usual in philosophical accounts. However, Aristotle insists on ascribing the plainest and most literal sense to Plato’s words and this insistence could be integrated in an Aristotelian plan of constant reaction to Plato and his followers and defenders.

Moreover, it seems peculiar that the Platonists do not give any account of the generation of odd numbers. According to Ross, it is hard to believe that the Platonists really denied the generation of odd numbers. It is thus probable that Syrianus is right in saying that the Platonists were speaking “symbolically”. According to Syrianus’ explanation, the Platonists derived both even and odd numbers from the same principles. Nevertheless, they claimed that odd numbers were ungenerated by analogy to the One, which is the formal principle that does not change, and to the gods, while even numbers were generated, by analogy to the

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200 [Alex.], 798.32-799.1
201 Ross (1924), v. 2, 484
202 Syr., 181.20-25
Indefinite Dyad, which is the material principle that changes, and to the existing things that are tangible unlike gods.

In my opinion, the Aristotelian conclusion that the Platonists state that even numbers were literally generated does not necessarily result from what he has said before. The theory about the generation of even numbers from the Great and Small after their having been equalized by the One could have been symbolically formulated. The Platonists are not bound by such a theory to believe that the generation of even numbers was a literally temporal process, since this theory could have been formulated for the sake of explaining the nature of numbers.

1091a30-33: In this section, Aristotle poses the question whether the good and the beautiful are among the first principles or are later than them. Especially:

a) Aristotle wonders what the relation of the elements and the principles to the good and the beautiful could be.

b) Then, he clarifies the type of relation he has in mind, claiming that the good and the beautiful could be either principles of some sort or entities that are posterior to the principles.

One point that needs to be discussed is that the beautiful is referred to only at 1091a31 and then Aristotle no more talks about “the good and the beautiful”, but turns to “the good and the best” (cf. 1091a33). Aristotle has already discussed the good and the beautiful at M.3 1078a31, where he draws a distinction between them. Annas claims that the beautiful is treated by the Academy as a part of the general discussion concerning the good. However, Aristotle could have chosen to concentrate on the good, in order to make a transition to the discussion concerning the goodness of the One. It seems normal to lay emphasis on the good, given that the question which is discussed afterwards is what the relation between the good (“ἀγαθόν”) and the first principle really is.

203 Annas (1976), 212
Furthermore, we should pay attention at the fact that the Good and the Beautiful seem to be treated in this section as entities and not as qualities (“αὐτὸ τὸ ἄγαθον”).

1091a34-1091b3: In this section, Aristotle refers to cosmologists who claim that the good and the beautiful should not be thought of as principles. According to Aristotle:

a) Some cosmologists consider that the good and the beautiful become evident in the existing things, while their nature progresses.

b) However, these cosmologists are said by Aristotle to think so, because they want to avoid the absurdities deriving from Plato’s theory that the One is a first principle.

c) Aristotle explains that the real absurdity does not lie in treating the first principle as something that is good, but in treating the One as a first principle and the first principle as an element and the number as a product of the One.

Aristotle uses the term “theologists (i.e. θεολόγοι)” in order to refer to the poets who have spoken mythically of gods. Θεολόγοι are contrasted to the φυσιολόγοι or φυσικοί, namely to the proper natural philosophers who have tried to give a rational-rather than mythical-account of reality. Homer and Hesiod (cf. Β.4 1000a9-12) could be thought of as theologists, while the natural philosophers are the pre-Socratic philosophers (cf. Λ.6 1071b21).

It is a matter of question to whom does Aristotle refer when he talks about “some of the present cosmologists (τῶν νῦν τισίν)”. Aristotle probably refers here to Speusippus who seems to have rejected the idea that the good and the beautiful are the ultimate principles (cf. Λ.7 1072b31). Speusippus is said to have claimed that the good and the beautiful did not apply to mathematical objects, but only to items produced “later” than them. Aristotle does not say it clearly, but perhaps he means that Speusippus has claimed so, because he wished to avoid the absurdities deriving from Plato’s identifying the good with the being or the One. Moreover, Aristotle may also have Pythagoreans in mind here, as they are said to have claimed that the good and the beautiful do not belong to the first principles (cf. Λ.7 1072b31).
Aristotle says that the reason why some present cosmologists, such as Speusippus, avoid to identify the good and the beautiful with the principles is that they are afraid of the difficulties that result from treating the One as a principle and also claiming that the One is good and beautiful. He does not specify what the difficulties were which Speusippus and Pythagoreans tried to avoid. What Aristotle probably means is that Speusippus was conscious of the difficulties in which Plato fell due to treating the One as first principle and regarding it as good (cf. A 6 988a14-15). However, Speusippus erroneously tried to avoid the difficulties by ceasing to treat the first principle as good instead of ceasing to treat the One as a first principle\footnote{Ross (1924), v. 2, 486}. Therefore, according to Aristotle, the real problem lies in regarding the One as a first principle and not in regarding the first principle as something good.

I think that what should be paid attention to here is the difference between “element (i.e. στοιχεῖον)” and “first principle (i.e. ἀρχή)” (cf. Metaphysics, Δ 1, 3, Z 1041b31 and Λ 1070b25). Ἀρχή means beginning, while στοιχεῖον means (roughly) element. Aristotle describes “first principle” as (a) the starting point, (b) the point from which each thing may best come into being, (c) the thing as a result of whose presence something first comes into being and (d) the thing from which -although not present in it- a thing first comes into being (cf. Metaphysics, Δ.1, 1012b34-1013a10). According to Aristotle, the “element” is the primary thing of which the beings are composed and which cannot be divided into formally different parts (cf. Metaphysics, Δ.3, 1014a26-27). For example, water is an element, because it acts as a primary constituent part and it is indivisible into parts of different form, i.e. any part of water is just water. Therefore, judging from the meaning that is ascribed to “element” and “first principle” by Aristotle, it could be concluded that element is the primary constituent part of beings, while first principle is the starting point from which the beings’ existence begins.

However, the constituent part may also be the origin of something. If something is made of some pre-existing elements, we can say that these elements are the beginnings of the thing since they pre-existed. According to Aristotle, the One is
considered by Plato to be a principle in the sense that it acts as the constituent part of which the number is composed. The mistake of Plato seems to be that he treats the One as a sort of principle that is an element, i.e. an actual constituent part of its product, namely number. If the good and the beautiful were to be identified with the principles in the same way in which the One does, then the good and the beautiful should be thought of as elements that would be part of each and every being. But, this is absurd. So, the difficulties do not result from granting the qualities of good and beautiful to the One, but from treating the One as principle. Syrianus disagrees with Aristotle in that the One was treated by Plato as an element of numbers and he claims that for Plato the One and the good is one and the same thing which has not the material nature of a substance.

1091b4-1091b15: In this section, Aristotle talks about the way in which ancient poets and previous philosophers tried to avoid the difficulty that resulted from treating the One as a first principle. The problem addressed here is whether in the universe there is an evolution from lower to superior forms or, alternatively, the good and the beautiful, however conceived (as Zeus, as Sphaerus, as Nous or whatever). So, Aristotle argues that:

a) Firstly, ancient poets dealt with this difficulty by saying that the sovereign entities were not Night and Heaven or Chaos and Oceanus, but Zeus. Those "mixed theologians", like Pherecydes, who thought that Zeus is not a newcomer in the world (as Hesiod thought, who made Zeus the son of Kronus who was the son of Ouranus who was the son of Gaia) but existed from the very beginning are interpreted by Aristotle as saying that the good and the beautiful, personalized in the supreme deity, were there all along.

b) Secondly, people who theorize about the creation of the universe, but are only partly based on myths, such as Pherecydes, say that the first principle has the quality of goodness to the greatest extent. The same conviction is

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205 Ross (1924), v. 2, 486
206 Syr., 182.5-7
shared by Magi and pre-Socratic philosophers like Empedocles and Anaxagoras.

c) Thirdly, among the thinkers who believe in the existence of unchanging substances, there are some who identify the One itself with the good itself. These thinkers treat the One as the essence of the good.

It is a matter of question to which ancient poets Aristotle refers at 1091b4. It seems that it is primarily Hesiod and Homer who are referred to here. Also some Orphic poets are referred to; for it is in Orphic theogonies that Night assumed the role that Gaia has in Hesiod. Homer also made Zeus to be afraid of the Night, which some later interpreters took as an allusion to the fact that the Night (Nyx) was anterior to Zeus. Aristotle has already said (cf. A.4 985a27) that Hesiod considered Chaos to be the first principle. As for Homer, he refers to Oceanus and Tethys (cf. *Iliad*, xiv, 201, 302) as the entities from which the universe originated. The references to Night and Heaven have been thought of as Orphic convictions, but they could just be later elaborations on Homer and Hesiod.

The aforementioned ancient poets tried to reconcile their belief in the primary and generative entities, i.e. Night, Heaven, Chaos and Oceanus, with their belief in the ruler of the world, i.e. Zeus, who is characterized by justice and goodness. Thus, they ended up in claiming that the good did not exist in the first principles, i.e. Night and Heaven or Chaos and Oceanus, but in the sovereign entity of the universe, i.e. Zeus. For, if the first principles were to be identified with the good, then Zeus would be the ruler who overturned his good predecessors. So, they treated the good as something that is posterior to the principles, given that they believed in a succession of rulers of the universe. Of course, this is a different reason to avoid the identification of the first principles with the good, if we compare it with the reason for which Speusippus tried to avoid this identification, according to Aristotle. So, Aristotle's choice to refer to ancient

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207 Annas (1976), 213
208 [Alex.], 800.20-22
poets here could be justified, given that his purpose is to continue and reinforce his polemic against Plato.

Pherecydes of Syros began his work by putting “Zas” (i.e. Zeus) at the beginning of the universe. So, he is presented by Aristotle as a person who thought of the generating principle as the entity that has the quality of goodness to the greatest extent. Therefore, Pherecydes acts as a counter-example to the narrative of ancient poets who place Night, Heaven, Chaos or Oceanus at the beginning of the world, but they consider Zeus to be the ruler of it.

The Magi, who were a Persian caste of priests, treated the good and the evil as fundamental principles. In particular, Aristotle identified Magi’s two principles, Ormuzd and Ahriman, with Zeus and Hades. As far as Empedocles and Anaxagoras are concerned, they are probably referred to by Aristotle as two thinkers who considered good things, i.e. Love and Mind, to be first principles. According to Aristotle (cf. *Metaphysics*, A.4 985a5-10), the good is a first principle for Empedocles, given that Love is the source of good things. Although, Empedocles and Anaxagoras nowhere ascribe explicitly the quality of goodness to their first principles, the things they say about Mind and Love clearly imply that they are good things. Anaxagoras says of Mind that it is superior in power to everything and also that it is unmixed with everything (strength and purity). Empedocles says that in the reign of Aphrodite (Love) there was no war, strife, antagonism, struggle, and he wished he had lived earlier. The goodness of these principles may indeed be Aristotle’s inference but it is an inference that has a sound basis on the extant fragments of both Anaxagoras and Empedocles. So, it seems that Aristotle includes Empedocles and Anaxagoras in the group of people who treated the good as a first principle, because he interprets their first principles as generative agents bearing the quality of goodness and producing beings that bear the same quality.

Finally, Aristotle refers to the thinkers who believe in the existence of unchanging substances. He probably means the Pythagoreans and the Platonists.

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209 Annas (1976), 213

210 Ross (1924), v. 2, 488
here. In particular, οἱ μὲν are perhaps Plato and the thinkers who agree with Plato in that the One is the good itself and the One is the essence of the good.

1091b15-1091b22: In this section, Aristotle accepts that goodness is a quality that belongs to the first principle. However, he denies that the One is the first principle. According to him:

   a) There are two ways in which we can theorize about the first principle, but only one of them is the right way.
   b) We can say either that the first principle is good or that it is not.
   c) The qualities of self-sufficiency and being eternal are good qualities. Therefore, these qualities belong to the first principle as something that is good. Thus, goodness belongs to the first principle. This is reasonable, according to Aristotle.
   d) However, according to Aristotle, it is unacceptable to say that the One is the first principle or an element and especially an element of numbers.

Self-sufficiency and eternality seem to be good-making qualities, since they are presented as one basic reason for treating the first principle as something good. But, also, the first principle could not have the qualities of self-sufficiency and eternality, if it was not in itself good. Therefore, the goodness of the first principle is for Aristotle a precondition for its possessing the good qualities of self-sufficiency and eternality. However, according to Aristotle, this good first principle cannot be the One or, in any case, an element and especially an element of numbers.

1091b22-1091b35: In this section, Aristotle explains the reason why the One cannot be a first principle, provided that the first principle is also something good. He says that:

   a) If the first principle is something good and the One is the first principle, then all monads must be thought of as good things and the result will be the existence of too many good things.
b) Moreover, if we accept that the Forms are numbers, then the Forms must be thought of as good things, given that the One (which is something good) is the element of numbers.

c) However, if the Forms are to be thought of only as good things, then they cannot be thought of as substances. For, then, all living beings and plants which imitate the Forms should be thought of as good things too, which is not reasonable.

d) Finally, if the One is good, then the opposite principle, i.e. the many or the unequal or the great and small, should be thought of as something bad.

e) For all the aforementioned reasons, one thinker is said to have avoided arguing that the One is good, because then he would have to argue also that the opposite principle is bad, which is absurd.

If the first principle is thought of as a good thing, then the One should be considered to be a good thing. Moreover, each number that is generated from the One is a unified entity, namely it constitutes one specific unit and thus it also constitutes one specific good thing, like the One.

If it is accepted that the Forms are numbers, then Forms must be thought of as good things. Nevertheless, if there are to be only “good” Forms, then there must be Forms only of beings that are good things. According to Annas, if there are to be Forms only of good things, then there will be Forms only of virtues, such as bravery and generosity. In such a case, Forms cannot be thought of as independent substances, i.e. self-sufficient entities. For, virtues are qualities of the beings and thus the Forms in which they participate have to be dependent on the beings of which they are predicated.

If there are to be Forms not only of virtues, but of physical objects and beings (e.g. man, horse), too, then the universal Man and the universal Horse must be thought of as good things and also the individual men and the individual horses must be thought of as good things, since Forms are models of good things. Within the

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211 Annas (1976), 215

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framework of the Aristotelian biological teleology, each being reaches its complete form through the possession of certain “good” features, such as health, symmetric development, and harmonious relation with the environment etc. In this sense, the beings that participate in Forms could be characterized as good things.212 However, Aristotle aims at showing the absurdity of the “goodness” of beings.

It is a matter of question if, provided that we accept the goodness of Forms, we should also accept that the beings which participate in them are also good, due to the participation in things which possess the quality of goodness. Aristotle has already claimed at M.4 1079a28-30 that, if a thing participates in the Form of Doubleness, then it participates in something eternal. However, it participates in something eternal only accidentally, because it is an accident of the Form of Doubleness to be eternal. In the same way, it could be assumed that the beings which participate in Forms are participants in things that are good. Though, each being participates in something that is good only accidentally, for it is an accident of Form to be good. Nevertheless, there seems to be no way, according to Aristotle, in which, Platonically speaking, we could make a distinction between the beings that participate in Forms qua Forms of certain concepts and the beings that participate in Forms qua eternal or good things.

One further point that deserves discussion is that, if the One is the good principle, then the other principle has to be the evil one, so that the first principles are contraries. At Λ.10 1075a34-36, Aristotle also refers to the second principle as evil. Further down, he explains the difficulties which follow, if we accept that the One principle is good and the other is bad.

1091b35-1092a7: In this section, Aristotle refers to all the absurdities that follow, if we accept that the one principle is good and the other is evil. According to him:

a) The first difficulty is that all the beings have to participate in a principle which is evil.

b) The second difficulty is that the evil acts as space which hosts the good.

212 Crubellier (1994), 533
c) The third difficulty is that the good is considered to be something which destroys the evil and also the evil is thought of as something that destroys the good.

d) The fourth difficulty is that, given that the material principle is potentially each being, the evil will have to be potentially the good.

Aristotle presents here a whole group of absurdities which result from treating the one principle as good and the other principle as evil. We could add to the aforementioned difficulties that, if the first principles are to be identified with the elements of beings, then the good and the evil, which are contraries, will have to coexist as constituent parts of the same being.
### N.5 (1092a9-1092b25)

"The generation of numbers from the first principles"

<table>
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<th>SECTION</th>
<th>ANCIENT GREEK TEXT</th>
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<tr>
<td>1092a9-21</td>
<td>Εἰ οὖν καὶ τὸ μῆ τιθέναι τὸ ἀγαθὸν ἐν ταῖς ἀρχαῖς καὶ τὸ τιθέναι οὕτως ἀδύνατον, δήλον ὅτι αἱ ἀρχαὶ οὐκ ὀρθῶς ἀποδίδονται οὐδὲ αἱ πρώταις οὐσίαι. Οὐκ ὀρθῶς δ᾿ ὑπολαμβάνει οὐδ᾿ εἰ τις παρεικάζει τὰς τοῦ ὅλου ἀρχὰς τῇ τῶν ἔρων καὶ φυτῶν, ὅτι εὐ ἀρχοίστων ἀπέλαν τε αἱ τὰ τελειότερα, διὸ καὶ ἐπὶ τῶν πρώτων οὕτως ἔχειν φησίν, ὡστε μηδὲ ὅν τι εἶναι τὸ ἐν αὐτῷ. Εἰς γὰρ καὶ ἑνταῦθα τέλεια αἱ ἀρχαὶ εὐ ὁ ταύτα ἀνθρωπος γὰρ ἀνθρωπον γεννᾶ, καὶ οὐκ ἔστι τὸ σπέρμα πρῶτον. Αἰτοῦν δὲ καὶ τὸ τόπον ἃμα τοῖς στερεοῖς τοῖς μαθηματικοῖς ποιῆσαι (ὁ μὲν γὰρ τόπος τῶν καθ’ ἐκαστὸν ἰδίος, διὸ χωριστὰ τόπω, τὰ δὲ μαθηματικὰ οὐ ποῦ), καὶ τὸ εἰπεῖν μὲν ὅτι ποῦ ἔσται, τί δὲ ἐστιν ὁ τόπος μῆ.</td>
<td>So, if it is in this way impossible both not to place the good among the principles and to place it there, then it is evident that neither the principles nor the primary substances have been correctly described. Nor is someone correct in his assumption, if he compares the principles of the universe to that of animals and plants, on the ground that the more complete things always come from indefinite and incomplete things, and this is what leads him to say that this is also true of the first principles, so that the One itself is not even a being of some sort. For, even in this case, the principles from which these things come are complete; for it is a human being that produces a human being and the seed is not first. It is also absurd to generate the place simultaneously with the mathematical solids (for the place is peculiar to particular things, and this is why they are separate in place, while the mathematical objects do not occupy any place), and to say on the one hand that they must have a position, yet not to say on the other hand</td>
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The thinkers who say that the existing things come from elements and that of beings the primary ones are the numbers, ought to have first distinguished the ways in which the one thing comes from another, and then said in which way the number comes from the principles. Is it by mixture? But not everything is capable of being mixed and what is produced is different; and the One will neither be separate nor a distinct nature; however, they want it to be so. Is it by composition, like a syllable? But then it is necessary that there is a position, and, when one thinker considers the One and plurality, he will think of them separately. So, the number will be this, namely a unit and plurality, or the One plus the unequal.

Also, since being derived from certain things means in one sense that these are still inherent in it and in another sense that they are not, in which of the two senses is the number derived? For, derivation from things which inhere in the final product is only possible for things which are generated. But is it derived as from seed? However, it is not possible for anything to come from that which is indivisible. But is it
| 1092b3-8 | Ετι τιδὴ ποτε τὰ μὲν ἄλλα ὅσα ἐξ ἐναντίων ἤ οἷς ἐστὶν ἐναντία φθείρεται κἀ̂ν ἐκ παντὸς ἢ, ὁ δὲ ἁριθμὸς οὗ; Περὶ τούτου γὰρ οὐθὲν λέγεται. Καίτοι καὶ ἐνυπάρχον καὶ μὴ ἐνυπάρχον φθείρει τὸ ἐναντίον, οἷον τὸ νεῖκος τὸ μίγμα (καίτοι γε οὐκ ἐδεῖ οὐ γὰρ ἐκεῖνω γε ἐναντίον). Further, why on earth do the rest of things which come from contraries or have contraries perish, even if they came from all the contrary, while the number does not? Nothing is said about this. However, whether inherent or not, a contrary destroys, like the Strife destroys the Mixture (however, it shouldn’t; for it is not the contrary of that). |
| 1092b8-25 | Οὐθὲν δὲ διώρισται οὐδὲ ὑποτέρως οἱ ἁριθμοὶ αἰτίων τῶν ὑσιῶν καὶ τοῦ εἶναι, ποτέρον ἢς ὁροι (οἷον αἱ στιγμαὶ τῶν μεγεθῶν, καὶ ὡς Εὐρύτος ἐταττε τις ἁριθμὸς τίνος, οἷον ὡδὶ μὲν ἁνθρώπων ὡδὶ δὲ ἵππου, ὡσπερ οἱ τοὺς ἁριθμοὺς ἄγοντες εἰς Nor has it in any sense been determined in which of the two ways are the numbers causes of the substances and of the being, whether as boundaries, like the points (are boundaries) of the spatial magnitudes, and as Eurytus determined (by asking) “which is the number of what?”, for example, this is the number of human being and that is the number |
of horse, like those who arranged the numbers into the shapes of a triangle or a square, imitating in this way the forms of plants using pebbles. Or is it because harmony is a ratio of numbers, and so is human being and everything else? But, in what way are the attributes - the white and sweet and the hot-numbers? It is clear that the numbers are not the essence of existing things nor are they causes of the form; for the ratio is the essence, while the number is the matter. For instance, the essence of the flesh or the bone is number only in this way that is three parts of fire to two parts of earth. And the number, whatever it is, is always a number of certain things, either of fire or of earth or of units; however, the essence of the things is being so much of one thing to so much of another thing in the mixture; but this is no longer a number, but a ratio of mixture of numbers whether these are corporeal or of any other kind. So, the number -whether number in general or number composed of abstract units- is neither a cause by producing things, nor as matter, nor as ratio and form of the existing things. Nor, again, is that for the sake of which.

| τὰ σχήματα τρίγωνον καὶ τετράγωνον, οὔτως ἀφομοιών ταῖς ψήφοις τὰς μορφὰς τῶν φυτῶν, | of horse, like those who arranged the numbers into the shapes of a triangle or a square, imitating in this way the forms of plants using pebbles. Or is it because harmony is a ratio of numbers, and so is human being and everything else? But, in what way are the attributes - the white and sweet and the hot-numbers? It is clear that the numbers are not the essence of existing things nor are they causes of the form; for the ratio is the essence, while the number is the matter. For instance, the essence of the flesh or the bone is number only in this way that is three parts of fire to two parts of earth. And the number, whatever it is, is always a number of certain things, either of fire or of earth or of units; however, the essence of the things is being so much of one thing to so much of another thing in the mixture; but this is no longer a number, but a ratio of mixture of numbers whether these are corporeal or of any other kind. So, the number -whether number in general or number composed of abstract units- is neither a cause by producing things, nor as matter, nor as ratio and form of the existing things. Nor, again, is that for the sake of which. |
| ἢ ὅτι [ὁ] λόγος ἡ συμφωνία ἀριθμῶν, ὑμιῶς δὲ καὶ ἀνθρώπως καὶ τῶν ἄλλων ἔκαστων; Τὰ δὲ δὴ πάθη πῶς ἀριθμοὶ, τὸ λευκὸν καὶ γλυκὸ καὶ τὸ θερμὸν; Ὅτι δὲ οἷς οἱ ἀριθμοὶ οὐσία οὐδὲ τῆς μορφῆς αἴτιοι, δῆλον· ὁ γὰρ λόγος ἡ οὐσία, ὁ δ᾽ ἀριθμὸς ὄλη. Οἷον σαφκός ἢ ὅστοι ἀριθμῶς ἡ οὐσία οὕτω, τρία πυρὸς γῆς δὲ δύο· καὶ ἄει ὁ ἀριθμὸς ὡς ἢ τινῶν ἐστιν, ἢ πῦριν ὡς ἢ γῆϊν ἢ μοναδικός, ἄλλ᾽ ἡ οὐσία τὸ τοσόνδ᾽ εἶναι πρὸς τοσόνδε κατὰ τὴν μιξίν· τούτῳ δ᾽ οὐκέτι ἀριθμὸς ἄλλα λόγος μίξεως ἀριθμῶν συμμετοχῶν ἢ ὅποιωνοιν. Οὔτε οὖν τῷ ποιῆσαι αἴτιον ο ἀριθμός, οὔτε ὅλως ὁ ἀριθμός οὔτε ὁ μοναδικός, οὔτε ἴλη οὔτε λόγος καὶ εἴδος τῶν πραγμάτων. Ἀλλὰ μὴν οὐδ᾽ ἃς τὸ οὐ ἕνεκα. |
OVERVIEW:

In N.5 from Metaphysics, Aristotle discusses the ways in which numbers can be said to be produced from the first principles. Moreover, he argues that numbers cannot be thought of as the causes of things.

At the beginning of Chapter Five, Aristotle raises the following point: although it seems that the first principles must naturally be indeterminate and imperfect, we should keep in mind that the undeveloped seed must come from a fully developed being. Therefore, not all the beginnings should be treated as undeveloped, for, even though the beginnings of everything are in a truly undeveloped state, the beginnings of living beings are in fact fully developed parents.

There is also an objection to the view that place is generated together with the mathematical solids.

Then, Aristotle raises the question of whether number originates from the first principles through a process of mixture or through a process of composition. He points out that number can neither be produced from the first principles by mixture nor by composition, since in both cases there are serious difficulties. Furthermore, Aristotle claims that number cannot come from elements that remain intact in it nor as from seed. Finally, given that the Platonists claim that number comes from contraries, Aristotle states that in this case a substratum which persists is necessary.

Afterwards, Aristotle attacks the Platonic theory about the generation of number from contraries by saying that the things which are constituted of contraries must be destroyable. However, according to the Platonists, this is not true in the case of number, for number is indestructible. Hence, number cannot be said to come from contraries.
At the end of Chapter Five, Aristotle challenges the assertion that numbers are the causes of substances by pointing out the difficulties that result from such a view.
COMMENTARY:

1092a9-21: In this section, Aristotle claims that, although one might claim that the principles of the whole universe are undeveloped by likening them to the beginnings of animals and plants, i.e., to seeds, we should keep in mind that the undeveloped seed must come from a fully developed being. In particular:

a) Aristotle says that, if it is impossible either to place the good among the first principles or not to place it there, it becomes evident that neither the first principles nor the primary substances (i.e. the mathematical objects) are described in the right way. This is a backward reference to what has been argued for in N.4.

b) Moreover, although the first principles of everything are considered to be undeveloped in the way the seed of a plant or an animal is undeveloped, it is not true that the principles of each and every plant or animal must be undeveloped, too. For, the undeveloped seed which produces a plant or an animal comes from a fully developed parent.

c) Finally, according to Aristotle, it is absurd to claim that place is generated simultaneously with geometrical objects, such as solids (for they are not located in any specific place). But also, it is absurd to claim that they are located in some place without saying what their place is.

Based on what has already been said previously in N.4, Aristotle claims that it proves impossible both to place the good among the first principles and not to place it there. Firstly, the reason why it is impossible to place the good among the first principles is that the principles are contraries. Therefore, if the one principle was to be considered as something good, then the other one should be thought of as something bad, which is impossible. Secondly, the reason why it is impossible not to place the good among the first principles is that the principles naturally have the quality of goodness, given that they are primary, eternal and self-sufficient entities.
According to Alexander\textsuperscript{213}, Aristotle claims that the first principles are not represented in the right way, because it seems unlikely both that they are good and that they are not good. Everything that exists is either good or not good, depending on the degree to which it has the qualities of the species to which it belongs. Therefore, the existence of neutral principles that are neither good nor not good seems impossible.

At 1092a12, Aristotle uses τις to refer to Speusippus. According to Speusippus, the One is the beginning of all things. Furthermore, Speusippus claims that the most beautiful and the best do not exist in the beginnings of everything, but he also argues that the qualities of the uttermost beauty and goodness do not exist in the beginnings of each and every distinct being. For instance, the beginnings of animals and plants cannot be treated as having the greatest beauty and goodness in them. For, the perfect animal and the perfect plant are derived from beginnings which are imperfect. However, according to Aristotle, Speusippus’ reasoning is not right, since the undeveloped seed of animals and plants comes from fully developed entities and thus what comes first is the perfect and fully developed and not the imperfect and undeveloped (cf. *Metaphysics*, Α.7 1072b31-1073a1).

As far as the discussion about the beginnings of living beings is concerned, Syrianus\textsuperscript{214} notices that Aristotle has not taken into consideration cases such as the insects which are generated from decaying matter or the trees, bushes and grass growing of themselves. In the aforementioned cases, according to Syrianus, the generating cause preexists in beings’ nature. This is meant to be cases of spontaneous generation (i.e., generation without a seed). This is also the case of a branch (a part of a fully developed plant) which, when put in water or appropriate soil, generates an entire plant out of it. Syrianus seems to think that the generating cause is present in an undeveloped form in all parts of plants.

Moreover, as regards the first principles of everything, Aristotle draws a conclusion from Speusippus’ aforementioned views, which probably was not what Speusippus really had in his mind. According to Aristotle, if we accept that the One

\textsuperscript{213} [Alex.], 802.28-803.2
\textsuperscript{214} Syr., 186.3-5
is the first principle of everything and that all first principles are undeveloped and
also that what is undeveloped cannot be thought of as an existing thing, then we
have to face the consequence that the One, which is first principle and therefore
undeveloped, cannot be said really to be.

I think that Aristotle's approach here seems a little exaggerated, because it does
not seem reasonable to assume that what is imperfect does not exist at all. It would
seem more reasonable to claim that the thing that is imperfect exists, but without
having the fully developed nature of a substance. As Annas215 notes, this could mean
that the thing that is imperfect is only potentially -and not actually- the perfect
nature into which it will develop. However, this does not mean that the imperfect
cannot be said to exist at all. So, Aristotle probably once more aims at pointing out
the difficulties resulting from the views he attacks by intensifying their negative
consequences, in order to negate the Speusippean claim that all beginnings are
imperfect.

At 1092a17-21, Aristotle accuses those who generate the place of mathematical
solids simultaneously with mathematical solids and also those who assign a place to
mathematical solids without explaining what this place is. It cannot be said with
certainty to whom does Aristotle refer here. He could have the Academy in his mind.
In particular, according to Alexander216, he refers to Plato. Nevertheless, as Ross217
rightly points out, Plato's χώρα to which Aristotle often refers as τόπος (cf. Physics,
Δ.1 208b7 & 209a8-15), was considered by the Platonists to be eternal and not
generated at some specific time. Therefore, a Platonic discussion about a temporally
defined generation of mathematical objects' place seems highly implausible.

The place of ideal mathematical solids cannot be of the same kind as the place of
sensible mathematical solids. In fact, it seems wrong to assign a place to ideal
mathematical objects at all, because these objects do not have the sensible three-
dimensional nature that would make it necessary for them to exist in a particular

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215 Annas (1976), 216
216 [Alex.], 803.16-20
217 Ross (1924), v. 2, 489
place so as to be distinct and identifiable. Syrianus\textsuperscript{218} points out that Plato has ascribed one type of place to natural bodies, another to forms involved in matter, another to mathematical bodies and another to immaterial ratios. Therefore, Plato, in all likelihood, did not mean literally the place of ideal mathematical objects. It could again be Speusippus that is referred to here by Aristotle.

It is a matter of question what is the link between the discussion about the principles and the present discussion about the place of mathematical solids at 1092a17-21. Perhaps Aristotle aims at addressing one further accusation against the Platonic thought concerning the generation of mathematical objects, so as to reinforce his criticism against the Platonists.

1092a22-1092b3: In this section, Aristotle discusses the possible senses in which the number is said to come from the first principles. In particular:

a) He says that the thinkers who derive the beings from the first principles and claim that of beings the primary ones are the numbers had to explain the process in which number is generated from the first principles or else in which sense number is derived from them.

b) Firstly, according to Aristotle, number could have come from the first principles through a process of mixture. However, not everything is capable of being mixed. Moreover, the product of mixture is something different from the elements from which it is produced. Finally, if number is the outcome of a process of mixture of the first principles, then the One (which is a principle) can neither exist in separation nor be different in nature from the beings, since it is part of their nature.

c) Then, Aristotle examines the case of generation of number from the first principles through a process of composition. However, such a process requires the existence of some place into which the composition happens. Furthermore, if number is the result of composition, then the principles will be thought of as entities that exist next to each other. Therefore, the number

\textsuperscript{218} Syr., 186.22-28
will be composed from two distinct and separately existing things, i.e. the unit and the plurality, which do not become one homogeneous thing.

d) According to Aristotle, number cannot be produced by derivation from things which inhere, i.e. remain intact, in the final product. In addition, number cannot be derived from principles as from seed, for the One is indivisible.

e) Finally, Aristotle claims that -given that number is derived by the Platonists from contraries- number should be thought of as generated from the contrary of plurality, i.e. the One, and something else that acts as a substratum which underlies the change.

Two reasons are given by Aristotle why the One cannot be mixed with the other principle, i.e. the Indefinite Dyad or Great-and-Small. Firstly, according to Aristotle, not everything can be mixed. In particular, as he explains in his treatise *On Generation and Corruption* at 327b21, the things that can be mixed must necessarily be capable of existing separately. However, the great-and-small is quality of things and it cannot exist separately from them. Therefore, number cannot be produced from the first principles through a process of mixture.

Secondly, according to Aristotle, the thing that is the outcome of a process of mixture is something different from the elements which have been mixed with each other to produce it. As Aristotle claims at 328b20 from *On Generation and Corruption*, mixture is the union of the things that can be mixed after their having altered (ἡ δὲ μίξις τῶν μικτῶν ἀλλοτριών ἐνωσις). Therefore, if the first principles were to be mixed, then the One would no longer be able to keep its unique nature\(^{219}\). However, the One must necessarily be capable of keeping its nature.

Finally Aristotle claims that, if number is produced by mixture, then the One cannot exist in separation. Perhaps, Aristotle says that the One cannot have separate existence, because, if it is mixed with the plurality, then the two principles are brought together to constitute one homogeneous mixture and thus they no longer

\(^{219}\) Ross (1924), v. 2, 490
can be thought of as distinct natures which exist separately from one another and keep their pure form.

Afterwards, Aristotle explains that number cannot come from the first principles through a process of composition, since such a process demands the existence of some position into which composition takes place. However, the One and the plurality cannot be thought of as extending in space, at least in the same sense in which the things of our experience extend in space and occupy a specific place. One further difficulty, according to Aristotle, is that, if the first principles are composed, they must be thought of as capable of existing in separation, given that each of them will occupy a specific place. However, as it has already been shown at 1092a13-15, Aristotle would deny that the One and the plurality can be considered to have an autonomous being given that they are undeveloped natures.

It is interesting to examine the difference between μίξις and σύνθεσις here. According to Aristotle (cf. 328a3-17 from On Generation and Corruption), σύνθεσις is a process the product of which is not homogeneous as it is in the case of μίξις. In other words, σύνθεσις is a sort of mechanical combination, i.e. the one element is juxtaposed with the other. On the other hand, μίξις is a sort of chemical combination, i.e. the one element is combined thoroughly with the other.

There seems to be an inconsistency in Aristotle’s reasoning, for he claims that there would be a problem both if the first principles proved to be incapable of existing in separation (1092a25-26: “οὐκ ἔσται τε χωριστὸν τὸ ἐν οὐδ’ ἐτέρα φύσις”) and if they were considered to exist separately (1092a27-28: “καὶ χωρίς ὁ νοῦν νοησεῖ τὸ ἐν καὶ τὸ πληθος, ἤ τὸ ἐν καὶ τὸ ἀνισον”). As we have seen, Aristotle would deny that the first principles are capable of existing separately and being autonomous. Therefore, if the first principles were to be thought of as having an independent being, Aristotle would treat it as a serious absurdity. There is an important detail at which we should pay attention: at 1092a25-26 Aristotle says that, if number comes from the principles by mixture, then the One will neither be separate nor a distinct nature, as “they” (i.e. Platonists) want it to be. Hence, in fact it is the Platonists here -

\[\text{[Alex.], 804.10-15}\]
\[\text{Ross (1924), v. 2, 490}\]
and not Aristotle- who will face difficulty by accepting a view that is inconsistent with their teaching about the first principles. On the other hand, it is Aristotle who will face difficulty, if number comes from the principles by composition, since in this case the first principles will be thought of as separately existing entities. So, probably Aristotle here does not use contradictory arguments, but he aims at pointing out as many difficulties as possible, so that he rules out both the possibility of generation of number by mixture and the possibility of generation of number by composition.

Then, Aristotle raises the question whether number comes from elements which inhere in it or it comes from elements which do not inhere in it. At first, Aristotle denies that the elements which produce number inhere in the final product explaining that, if the elements were to inhere in number, then it would mean that number should belong to the class of things to which generation (in the strict sense of coming-into-being rather than the metaphorical sense of something being derivative though, strictly speaking, ungenerated) applies, which is not true.

It is a matter of discussion, if ἐξ ἐνυπαρχόντων should be treated as subdivided into μιξίς and σύνθεσις. It is true that both in the case of mixture and in the case of composition, the elements from which the thing is produced continue to be present in it either having undergone alteration (in the case of μιξίς) or remaining intact (in the case of σύνθεσις). Aristotle could be drawing a conclusion here concerning both μιξίς and σύνθεσις by using the more general term ἐξ ἐνυπαρχόντων. What Aristotle probably means to lay emphasis on is that both in the case of mixture and in the case of composition a process of generation is required. For, if the elements from which something is produced inhere in the final product, then these elements must have been combined at some specific time in order to produce it. However, number is thought of as an eternal entity. Therefore, it could not have been generated from its constituent parts at some specific time. As a result, number could not have been produced by elements which inhere in it. Moreover, as Alexander points out, the elements inhere in things that have been artificially produced. For example, stone and wood inhere in the house that was built from them.

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222 Ross (1924), v. 2, 490-491
223 [Alex.], 804.13-15
Then, Aristotle discusses the case of number production as from seed. However, the One is indivisible. According to Alexander\textsuperscript{224}, the One is ἀφθαρτον, i.e. incorruptible, and ἀτρεπτον, i.e. unchangeable. Thus, it cannot produce anything like a seed. Number production as from seed is a process of generation. However, it is not a process of artificial generation like mixture or composition. In fact, it is a process of natural generation. Those thinkers who posited the One as first principle and generated the numbers out of it would answer the question about how the numbers can be generated out of the One with the reply “the One is the seed of numbers”. But, there is a serious difficulty: the seed is not something indivisible, while the One is. Hence the One cannot be like a seed; for from something indivisible nothing can come out (as an oak-tree does comes out of an acorn which is not indivisible).

Finally, Aristotle passes to the case of number production from an element that does not inhere in it. This element is, according to Aristotle, a contrary. The element which does not persist is contrary to another element that persists. Number is said by the Platonists to have been produced from two principles that are contraries. The pair of contraries is either the One and the plurality or the One (which is equal) and the unequal. It is a matter of question to whom Aristotle refers at 1092a35-36 by ὁ μὲν and ὁ δὲ. It seems that he has in mind two particular thinkers. The first one is perhaps Speusippus. That Speusippus regarded the first principles to be the One and Plurality is quite certain. About the identity of the thinker who considered the first principles to be the Equal and the Unequal there might be a controversy. Ross thinks that the latter is Plato. There must be some substratum from which together with one contrary number is produced. It is true that the production of number from the One and something else that acts as a substratum is something different from the Platonic theory concerning the production of number from two principles which are contraries\textsuperscript{225}. Ross claims that Aristotle probably identifies number with plurality and thus thinks that the Platonists should have derived number from its contrary, i.e. the One, and something else which can be the substratum at one time of the One and at

\textsuperscript{224} [Alex.], 804,21-22

\textsuperscript{225} Ross (1924), v. 2, 492
another of the plurality. For, as Alexander\textsuperscript{226} rightly points out, plurality cannot act as substratum for the One, since it is contrary to the One.

Syrianus\textsuperscript{227} reasonably claims that Aristotle in all likelihood talks about Form numbers and not mathematical ones. So, according to him, Aristotle should not discuss at all the cases of mixture, juxtaposition, corruption and privation, since the Form number is treated by the Platonists as self-appearing (αὐτοφανῆς) and self-produced (αὐτογονος).

\textbf{1092b3-1092b8}: In this section, Aristotle attacks the Platonic theory about the generation of number from contraries by saying that the things which are constituted of contraries must be destroyable. In particular:

a) Aristotle wonders why all the other things which are produced from contraries can perish, while number cannot.

b) For, the contrary of a thing (whether it inheres or not) has the capacity of destroying the final product.

c) Aristotle uses the example of Empedocles’ system, according to which the Strife destroys the unity of the mixture, even though, as Aristotle himself admits, the Strife cannot be thought of as the contrary of the mixture.

Aristotle again makes a reference to those things which come from contraries. He could mean either that each thing is produced from its contrary or that each thing is produced from two elements which are contrary to each other. It is likely that the meaning of Aristotle's words is the former one, especially if we take into consideration the phrase κἂν ἐκ παντῶς ἦ (“even if even if it is from all contraries”).

This phrase shows that the contrary to a thing is the element which produces it. The contrary can be used up in producing a thing and thus inhere in it. However, the contrary to a thing can play no part in the production of this thing and exists outside it as non-inhering in the final product. According to Aristotle, both in the case of a

\footnotesize{\textsuperscript{226} [Alex.], 804.30-31 \\
\textsuperscript{227} Syr., 187.1-13}
contrary that inheres and in the case of a contrary that does not inhere the result is the same: the contrary is capable of destroying the thing of which it is contrary. It should be noted here that the analogy from Empedocles’ system does not reinforce effectively Aristotle’s argumentation, as Aristotle himself also admits. Alexander explains the way in which Empedocles’ system could operate in favor of Aristotle’s argument here. In Empedocles’ system Love and Strife are the contraries. Similarly, in the case of Form numbers the One and the plurality are the contraries. Just as, according to Empedocles, Strife enters the mixture that is called Sphairos and destroys its unity, in the same way, according to Aristotle, the plurality fights against the One and leads to the corruption of Form number. Syrianus points out that Empedocles’ Strife has not destructive but creative power, since it generates plurality and difference. However, this is highly problematic, since Empedocles himself seems to consider Strife as something bad and disruptive.

1092b8-25: In this section, Aristotle raises the question how numbers are the causes of substances and he points out the difficulties resulting from such a view. In particular:

a) Aristotle claims that numbers could be the causes of substances either as boundaries or as ratios.
b) If numbers are to be thought of as boundaries, then it means either that they are points of spatial magnitudes or that they act as the outline of each thing, according to Eurytus’ way of thinking.
c) If numbers are to be thought of as ratios, then it means that harmony, man and everything else is a ratio of numbers.
d) However, it is a matter of question how can attributes, like white, be numbers.
e) Moreover, the ratio is the essence of the things. The essence of the things is being so much of one thing to so much of another thing in the mixture; but

228 Ross (1924), vol. 2, 492
229 [Alex.], 805.8-15
230 Syr., 187.15-24
this is no longer a number, but a ratio of mixture of numbers whether these are corporeal or of any other kind.

f) Thus, number is neither efficient nor material nor formal nor final cause of things.

Aristotle finally passes from the topic of generation of numbers to the topic of generation of things from numbers. Judging from Aristotle’s references here, it seems that he has in mind mostly the Pythagoreans. In particular, at 1092b10-11, Aristotle refers to Eurytus and his system of deciding what the number of each thing is by counting the pebbles he used in order to form the outline of it.

It is a matter of question to what degree Eurytus’ method (which is based on the view that as point acts as the limit of line in the same way Form acts as the limit of substance) is consistent with the original Pythagorean views. Eurytus was a disciple of Philolaus and, according to Alexander, his method was to draw the outline of each being using coloured pebbles. Then, Eurytus claimed that the number of pebbles he had used in order to sketch the outline of the being was the number of this specific being. For instance, 250 was considered to be the number of man.

Therefore, it seems that Aristotle uses the term φυτόν at 1092b13 in its general meaning of “living being”, which is found in Plato (cf. Sophist 233e: “ἐμὲ καὶ σὲ καὶ τὰλλα φυτά”). However, as Ross rightly points out, Eurytus’ method seems to be a bad imitation of the method of limits, according to which the early Pythagoreans used the number two to represent the line (for the line is determined by two points) and also the number three to represent the plane and the number four to represent the solid. Finally, it seems to be the same process that was followed when the numbers were reduced to the shapes of triangle and square. For, three points were required in order to mark the boundaries of the triangle and four points were needed so as to delimit the square and so on.

Then, Aristotle objects to Eurytus’ method of ascribing a number to each thing. In particular, he claims that, although one can sketch the outline of a man or a horse,
there is no way in which they can sketch the outline of a quality, such as whiteness, sweetness and hotness. This is a serious weakness of Eurytus’ method.

Immediately after, Aristotle objects to the second Pythagorean mode of treating numbers as causes of things. The Pythagoreans had observed that the intervals of octave, fourth and fifth can be expressed as simple numerical ratios. So, they assumed that all the other natural phenomena were also expressible as numerical ratios. However, Aristotle claims that, if harmony is a ratio of numbers, then numbers are the matter and the ratio is the essence of things. Therefore, numbers cannot be the essence of things, since a ratio is not a number (cf. *Metaphysics*, A.991b2-21). According to Syrianus\textsuperscript{234}, a ratio—which is a harmony consisting of numbers—displays how nature fits together beings through natural numbers. Aristotle turns to an example which is framed on the analogy of Empedocles’ analysis of bone that is referred to in *De Anima* (cf. 410a4-6 “ἡ δὲ χθόν ἐπίης ἐν ἐνότερος χοάνοισιν/ τῷ δῷ τῶν ὀκτὼ μερέων λάχε νησίδας αὐγλη/ τέσσαρα δ’ Ἡραίοσω τὰ δ’ ὀστεα λευκά γένοντο”). According to this analysis, two parts of earth, two parts of water and four parts of fire lead to the generation of bones.

It is worth examining the different senses of number that are used by Aristotle in this section. Firstly, at 1092b20, Aristotle talks about the number that is μοναδικός, i.e. made up of units. In this sense, number is not the matter of any being, but only acts as the measure of its constituents. Secondly, at 1092b22, Aristotle talks about the number that is σωματικός, i.e. corporeal. In this sense, number acts as matter of a being that is numerically determined\textsuperscript{235}. According to Alexander\textsuperscript{236}, number is the quantity and the measure of matter.

The conclusion at which Aristotle eventually arrives is that number can in no reasonable way be considered as the cause of things.

\textsuperscript{234} Syr., 188.5
\textsuperscript{235} Ross (1924), v. 2, 495
\textsuperscript{236} [Alex.], 806.25-26
Somebody could raise the question what is the good that the things get from the numbers, because their mixture is expressible by a number, whether one which is easily calculable or an odd number. For, as things stand, honey-water can be no more wholesome, if it has been mixed in the proportion of three times three, but it could be more beneficial, if it was in no particular ratio, but diluted, than it would, if it was expressed in number, but not diluted.

Besides, the ratios of mixtures are expressed by the adding of numbers, but not merely by numbers, for instance “three to two”, not “three times two”. For, in the multiplications, the factors must be of the same kind; therefore the product of 1x2x3 must be measured by 1, and the product of 4x5x7 must be measured by 4; so, all the products which involve the same factor must be measurable by that factor. Hence, the number of fire will not be 2x5x3x7 and at the same time that of water 2x3. If it is necessary that everything shares in number, then it must
follow that many things are the same, i.e. that the same number belongs to one thing and also to another.

Therefore, is number a cause and is it due to number that the thing exists, or is it unclear? For example, there is a certain number of the motions of sun, and again of the moon, and indeed of the life and maturity of each living thing; what prevents then some of these numbers being square numbers, some others being cubes, and some equal, while others double? There is nothing to prevent it, but it is necessary that they fall within these types, if everything shared in number, and it was possible that different things fall under the same number. Therefore, if the same number happened to belong to certain things, then those would be the same as each other, because they would have the same kind of number, for example the sun and the moon would be the same.

But, why are these numbers causes? There are seven vowels, the scale has seven strings, there are seven Pleiads, the animals lose their teeth at seven (some of them at least, while some others do not) and there were seven heroes against Thebes. Was it due to this, namely that the number
is of such a kind, that there were seven heroes or that the Pleiad consists of seven stars? Surely the heroes against Thebes were seven because of the gates or due to some other reason and, while we count the Pleiad as seven and the Bear as twelve, other people count more stars in them.

They also assert that the Ξ, Ψ and Ζ are concords and, since there are three concords, there also are three double consonants. They do not care about the fact that there might be thousands of such letters; for there might be one symbol for Γ and Ρ together. But, if they say that each of these letters is equal to two of any of the others, while no other is so, the cause being that there are three areas in the mouth and in each area one letter is combined with Σ, it is for this reason that they are only three of them, but not due to the fact that there are three concords, because the concords are more; however, the double consonants can be no more. These people are like the old Homeric scholars, who see the small similarities, but overlook the important ones. Some still say that there are many such cases, for instance the middle strings are the one nine and the other eight, and the epic verse has seventeen syllables, which is equal to these in number; and the scansion is in the right part of the line.
They also say that the distance in letters from A to Ω is equal to that from the lowest to the highest note of the flute, of which the number is equal to the whole system of the universe. However, we must observe that no one would find any difficulty either in stating such analogies or at finding them in the eternals, because they occur also among the perishable things. But the celebrated characteristics of the numbers and the contraries to these, and mathematical properties in general, as some thinkers describe them and make them causes of the nature, seem, if we examine them in this light, to escape us; for, none of them is a cause in any of the ways we have distinguished with reference to the first principles.

In the sense however in which they proceed, it becomes clear that the good exists, and that the odd, the straight, the equal multiplied by equal, and the powers of certain numbers belong to the series of the beautiful; for the seasons and a certain kind of number go together; and the other examples which they derive from the mathematical theorems have all this force. Hence, they are like coincidences. For they are

In the sense however in which they proceed, it becomes clear that the good exists, and that the odd, the straight, the equal multiplied by equal, and the powers of certain numbers belong to the series of the beautiful; for the seasons and a certain kind of number go together; and the other examples which they derive from the mathematical theorems have all this force. Hence, they are like coincidences. For they are
ἔστι γὰρ συμβεβηκότα μὲν, ἀλλ᾽ οἰκεία ἀλλήλοις τάντα, ἐν δὲ τῷ ἀνάλογῳ· ἐν ἑκάστῃ γὰρ τοῦ ὅντος κατηγορίᾳ ἦστι τὸ ἀνάλογον, ὡς εὐθὺ ἐν μήκει οὕτως ἐν πλάτει τὸ ὁμαλὸν, ἴσως ἐν ἀριθμῷ τὸ περιττόν, ἐν δὲ χροῖ ὑ τὸ λευκὸν.

Further, it is not the numbers among the Forms that are responsible for the harmonic relations and the like; for those differ from each other in form, even though they are equal; for the units also differ. Therefore, for these reasons at least, we should not make numbers Forms. So, these are the consequences and even more might be brought together. But, the fact that they find so much trouble with the generation of Form numbers and can in no way bring them into harmony, would seem to indicate that the objects of mathematics are not separate from sensible things, as some say, and that they are not the first principles.
OVERVIEW:

In *Metaphysics* N.6 Aristotle argues against the belief that numbers are causes of other things and draws the final conclusions to books M and N as a whole.

At the beginning of N.6, Aristotle wonders what good comes from the fact that a mixture can be expressed in numbers, and he uses the example of the mixture of honey and water. The point of this example is that, as in a recipe for a particular food, it is not the abstract formula of the ratio (say, three parts of water to one part of honey, or 3:1) but the precise fluidity and strength of the mixture that is important. This depends on the quality of honey (and of water) so that in some cases one may need to add more honey (or water) than the formula prescribes.

Moreover, Aristotle points out that the ratios expressing mixtures come not merely from a process of combining numbers in any possible way, but especially from numbers that are combined to each other according to the pattern “x” of one thing to “y” of another thing”. The point is that the mixture is not a matter of multiplication but of addition according to a formula. Presumably, people spoke of the ratios of mixture with a vocabulary derived from multiplication (i.e. twice three, or twice five etc.).

Then, Aristotle claims that, if all things share in number, it turns out that many different things have the same number. If this is true, then many things of different sort must necessarily be the same, which is absurd.

Furthermore, he argues that, if we accept that each thing falls under a certain number, there is no reason why we should treat this number as the cause of the thing’s having this specific number.

Refuting the parallel derivation of parts of speech and of parts of the musical scale, Aristotle uses the example of the three double consonants, i.e. Ξ, Ψ and Ζ, and the three concords, i.e. the fourth, the fifth and the octave. He tries to show that in both cases the number is arbitrary.

The theorems and properties of numbers are not causes of things in any of the senses in which we could talk about “cause”.

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It is noteworthy that Aristotle, making a reference to the Pythagorean classification of things into two categories of “good” and “evil”, seems to accept that there is a sort of correspondence between numbers and other things. For instance, there is an analogy between the odd in number and the straight in length, since both of them belong to the category of “good” things. However, this correspondence between numbers and other things cannot be thought of as a cause and effect relation.

Finally, Aristotle explains that the Form numbers cannot be treated as causes due to their being different in kind. So, there is no reason why one should believe in the existence of Forms.

All the aforementioned difficulties show that the objects of mathematics cannot be considered as separate from perceptible objects. Therefore, the theory which treats the mathematical objects as first principles is highly refutable.
COMMENTARY:

1092b26-30: In this section, Aristotle wonders what good comes from the fact that a mixture can be expressed in numbers. In particular:

a) He asks what the point of being expressible in number of any sort is.

b) Then, he uses the example of a mixture consisting of honey and water and he claims that this mixture is not efficient and good for health, if it can be expressed in any particular ratio, but it is not diluted. On the contrary, the mixture is more beneficial, if it is efficiently diluted.

Firstly, we should pay attention at the distinction drawn at 1092b27-28 between the “easily calculable” (i.e. ἐν ὑλογίστῳ) and the “odd” (i.e. περ ιττῷ) number. The distinction that we would normally expect to be drawn here is between the even and the odd number. Therefore, as both Alexander\[^{237}\] and Syrianus\[^{238}\] claim, by “ἐν ἐν ὑλογίστῳ” Aristotle could mean “ἐν ἀρτίῳ”. The question which arises is in what sense the even number is said to be easily calculable, while the odd one is not. The answer could be that the even number or the ratio which consists of even numbers has symmetry and thus it is more easily computed than the odd number or the ratio which consists of odd numbers (for example 3/2 or 3/4), that is uneven and lacks symmetry.

However, in De Sensu (439b27-32), Aristotle uses again the term “ἐν ὑλογίστοις” in order to refer to ratios like 3/2 and ¾ and, in general, to ratios consisting of numbers of any sort. It seems that, in the aforementioned passage from De Sensu, Aristotle uses “ἐν ὑλογίστοις” not to refer exclusively to even numbers, but to draw a distinction between being based on particular mathematical proportions and being based on some indefinite excess or defect, but not on a certain mathematical proportion.

Therefore, it is likely that by “ἐν ὑλογίστῳ” Aristotle means an easily reckoned ratio, of which both terms are rational numbers and are involved in the series 1-10\[^{239}\].

\[^{237}\] [Alex.], 808.23
\[^{238}\] Syr., 189.7
\[^{239}\] Ross (1924), v. 2, 495
However, if the term “εὐλογίστῳ” is used by Aristotle with the aim of describing an integer instead of a fraction, then it is undoubtedly a small integer or the outcome of some simple factors that is perhaps a power of a small integer.  

The question that remains to be answered is what the odd is. Ross suggests that “περίττῳ” possibly is used with reference to a ratio that has the form n/n+1, e.g. 2/3.  

Then, Aristotle claims that, in the case of a mixture like honey and water, what matters is that the honey is mixed with enough water so that the final product is diluted enough. But it is not equally important that the two constituents of the mixture are in a particular ratio. By saying so, Aristotle does not mean that the right quantities of honey and water cannot be expressed in number at all, but that it is not necessary that there is strictly one particular ratio which describes them. We could draw a parallel between this process and the process of cooking. The good-tasting quality and the perfect texture of food are not necessarily the outcome of an accurately measured mix of its ingredients. On the contrary, what is most important is the harmonious combination of all the constituent parts, where none of them keeps its own special nature, but all the different natures change into a new one, i.e. that of the final mixture. This must necessarily happen, even if the quantities of the constituents have to diverge a little from some predetermined numerical proportions.  

1092b30-1093a1: In this section, Aristotle claims that the ratios which describe mixtures come from numbers that are combined to each other according to a specific pattern. In particular:  

a) The numbers that are involved in ratios of mixtures are numbers of different things which are added to one another based on a pattern like “three of one thing to two of another thing”.  

b) Therefore, the ratio by which a mixture is expressed is not in fact one number, but a combination of numbers that are different in genus.  

240 Crubellier (1994), 578  
241 Ross (1924), v. 2, 495
c) On the contrary, if the numbers were to be multiplied, the genus of numbers should be the same. Therefore, the numbers that are involved in mixtures cannot be numbers that are combined according to a pattern like “thrice two”.

d) For, if numbers of a mixture were to be multiplied, then the number of the elementary substance A would be considered as a measure of the outcome \( \text{ABG} \). However, this would be impossible, given that A, B and G would be numbers of things that are different in genus.

e) Moreover, if fire was to be expressible by the number \( \text{BEIZ} \) and water was to be expressible by the number “thrice two”, then a great deal of difficulty would occur. For, it would turn out that many things are described by the same number and, thus, different beings would necessarily be identified with each other.

Aristotle claims that a mixture cannot be produced by multiplication. Granted that the mixture can be produced according to a multiplying formula like \( \tau\rho\zeta\delta\upsilon\omega \) (“three times two”), then it has to be accepted that it is created through the multiplication of, say three parts of an ingredient by two parts of another. For, a mixture must necessarily be composed of at least two different ingredients. However, “three times two” means in fact “taking three times two parts of the same constituent”. If the mixture consists of parts that are only of one and the same nature, then it cannot be called a “mixture”. The only way in which a mixture can be produced is by addition, in the sense of taking ingredients of two or more different sorts in a specific numerical proportion, like “three of one thing to two of the other”, i.e. by ratio\(^{242}\).

Any perceptible being can be thought of as a body that is a mixture of different constituent parts. Therefore, any physical body -considered as a mixture- cannot be the outcome of multiplication of one and the same elementary substance.

If a being was to be the outcome of a process of multiplication, then it would be described by a number like \( \text{ABG} \). Ross\(^{243}\) explains that, in case a being was described

\(^{242}\) Crubellier ([1994], 581), clarifies the sense in which a ratio can be thought of as an adding formula using the example of three parts of fire and two parts of earth (or else \(3/2\)) giving five parts of bone.

\(^{243}\) Ross (1924), v. 2, 496
by ABΓ, it should be measured by 1x2x3 or else 1x6, which means that one unit of a
certain elementary substance would be taken six times for the being to be generated.
Similarly, the being described by ΔΕΖ, i.e. 4x5x7 or else 4x35, should be measured by
four portions of an elementary substance that would be taken thirty five times.
However, according to Aristotle, if it was true that any being is a mixture expressible
in a certain number that is got by the multiplication of units of one particular
elementary substance then it would follow that the number of fire could not be
BEΓΖ, i.e. 2x5x3x7, if at the same time the number of water was 2x3, i.e. ΒΓ. For, the
two basic elements would be measured by the same factor, i.e. B that is number two.
If we accepted that the number of fire and the number of water have a common
factor, then we should also accept that the substance which is expressed by this
factor is constituent part of both elements244. However, this is impossible. Moreover,
if we take into consideration that 2x5x3x7 can be expressed as 6x35 and that 2x3
equals 6, then it would turn out that fire would consist of thirty five unitary amounts
of water, which is also absurd245.

Alexander246 reasonably claims that earth, water and fire cannot be measured by
one and the same factor. On the contrary, if there were ten fires, they would be
measured by one factor that would be of the same genus as them, i.e. “one fire”. And, if there were twenty airs, they would be measured by “one air” and so on.

Nevertheless, it is a matter of discussion whether the first factor in a series of
multiplied numbers should be thought of as different in kind from the other factors
in the same series. In particular, Ross and Annas seem to consider that the first factor
in each of the multiplication series to which Aristotle refers is one (A) or four (Δ) or
two (Β) portions of a certain elementary substance, which, when multiplied with the
other factors of the same series (B and Γ in case of series ABΓ, E and Z in case of
series ΔΕΖ, and E, Γ and Z in case of series BEΓΖ), give the numbers of specific
existing things.

244 Annas (1976), 218
245 Ross (1924), v. 2, 496
246 [Alex.], 809.28-30
If we consider the first factor in each multiplication series to be one or more units of the elementary substance of the being and the other factors to show how many times we have to take the first factor in order to have the being, then many difficulties occur. Aristotle claims that the numbers involved in multiplication are the same in genus. Therefore, in the series ΑΒΓ, i.e. 1x2x3, A, B and Γ must be homogeneous. And, similarly, in the series ΔΕΖ, i.e. 4x5x7, Δ, E and Z should not differ in genus. Therefore, how A and Δ can be units of a certain sort of substance, while B, Γ and E, Z are merely numbers displaying how many times we take A and Δ respectively? If A is a portion of a material substance and B and Γ express the number of times we have to take this portion for the being to be formed, then it turns out that A differs in genus from B and Γ. If they were to be homogeneous then all three of them should either be portions of the same substance or be numbers showing the amounts we would take of a substance so as to have the being. In the first case, multiplication would not be possible, given that 1 portion of water cannot be multiplied with 2 portions of water and then with 3 portions of water. They can only be added to each other. In the second case -which seems to be the most reasonable- Aristotle would have chosen a certain series of multiplied numbers to describe the quantity of matter that comprises each particular being. But, again, each being should be created out of one and the same element, since it is not possible that the one factor of the multiplication series multiplies element “x”, while another factor of the same series multiplies element “y”. If this is so, then there is no point in saying that one element is multiplied 1x2x3 times and not saying directly that it is multiplied 6 times.

On account of the aforementioned difficulties, I would disagree with Ross and Annas in that the series ΑΒΓ and ΔΕΖ correspond to series of numbers. It seems much more reasonable that the letters of the alphabet act as arbitrary symbols here. Crubellier suggests that the series ΑΒΓ and ΔΕΖ should be considered to graphically represent series of three tangible elements (or else three concrete and not unitary-numbers), of which the first element (i.e. A in case of ΑΒΓ and Δ in case of ΔΕΖ) acts

247 Crubellier (1994), 581-582
as a “unit of measurement” - in the sense of an original model - that is replicated by the other two elements that belong in the same series. As Syrianus\textsuperscript{248} notes, it is true that in multiplication there are numbers that act as units of measurement for other numbers. For instance, in the multiplication table, the number nine is counted by the number three and the number twelve is counted by the number three and the number four. If the number nine and the number twelve are considered as geometrical objects, then the number three and the number four will play the role of the “side” by which the geometrical objects are counted. Therefore, it seems that the letters in the series ΑΒΓ and ΔΕΖ are symbols that represent the geometrical actualization of beings.

1093a3-13: In this section, Aristotle uses some examples in order to prove that, if all things shared in number, then a great deal of difficulty would occur. In particular:

a) If we accept that there is a certain number that describes the motion of the sun and of the moon and also the age of life of each living thing,
b) then what prevents some of these numbers from being square numbers, others from being cubes,
c) and also others from being equal, while others from being double?
d) Aristotle claims that there is nothing to prevent it.
e) Moreover, it is unavoidable that different things fall under the same number.
f) Therefore, if the same number belongs to different things, then these things will be the same as each other. For example, the sun and the moon must necessarily be the same, if they fall under the same number.

Aristotle formulates two claims here: firstly, at 1093a3-9, he claims that, if all the beings are expressible in certain numbers, then it reasonably follows that all the beings can be expressed in numbers that are either square or cube numbers or equal or double to one another. Secondly, at 1093a9-13, Aristotle says that, if all existing things share in number, then it could happen that two or more things fall under the same number. But, if two or more different things fall under the same number, they

\textsuperscript{248} Syr., 189.34-38
will necessarily be identified with each other and therefore they will be thought of as one and the same thing, which is absurd. Aristotle gives here the example of the spheres involved in the locomotion of sun and moon, which are both said by Eudoxus (cf. *Metaphysics*, Α.8, 1073b17-38) to fall under number five.

Aristotle lays emphasis on the fact that nothing prevents the beings from being expressible in square or cube numbers and also in numbers that are equal or double to one another. It is likely that he refers especially to square and cube numbers in order to reinforce his claim that the identification of the beings that fall under the same number with each other is obligatory. According to Alexander, all the cube numbers were considered to be substantially the same as each other. They were treated as different from each other only in terms of size. For example, number eight (i.e. $2^3$) and number twenty-seven (i.e. $3^3$) were thought of as different not in essence (for, they are both cubes), but only in size, in the same way in which a child would be thought of as different from an adult (they are both human beings, but the adult is larger in size than the child). The same would be true of the square numbers and also of the numbers that are equal or double to one another. Therefore, given that the universe can be described by means of physical measurements, it would unavoidably turn out that many phenomena involve the same number.

1093a13-19: In this section, Aristotle claims that, if we accept that each thing falls under a certain number, then there is no reason why we should treat this number as the cause of the thing. In particular:

a) There are seven vowels, the scale has seven strings, there are seven Pleiads, some of the animals lose their teeth at seven and there were seven heroes against Thebes.

b) But, the fact that the heroes against Thebes were seven or that the Pleiad consists of seven stars is not due to the fact that number seven is of a specific sort.

[249] [Alex.], 811.1-4
c) Furthermore, according to Aristotle, it is noteworthy that, while we count the Pleiad as seven and the Bear as twelve, other people count more stars in these constellations depending on where they live.

Aristotle uses the aforementioned examples in order to show that number seven is not the number of vowels, chords, age of losing teeth and heroes against Thebes in a sacred sense which would rule out the possibility of all these things falling under any other number. It is totally accidental that the heroes against Thebes were seven and not five. For, if Thebes had five gates instead of seven, then the leaders of the army against Thebes would also be five instead of seven.\footnote{\[Alex.,\] 811.21-22}

Similarly, the reason why the constellation of the Pleiad consists of seven stars is not that the number seven has certain qualities that make it typical of this specific constellation. In fact, the number of stars one can see in the Pleiad depends on the place where they live and from where they can see the constellation. According to Alexander\footnote{\[Alex.,\] 811.22-27} and Syrianus\footnote{Syr., 191.21-24}, the Greeks would count seven stars in the Pleiad and twelve stars in the Bear; while the Chaldeans would count more stars in each constellation due to their looking at them from a different point of the earth.

Therefore, given that numbers of the beings are accidentally and not causally related to them, there is no reason why we should treat number as causes of the beings, according to Aristotle.

\textbf{1093a20-1093b1:} In this section, Aristotle claims that the number of double letters, which is the same as the number of concords (i.e. three), is arbitrary. In particular:

a) According to Aristotle, it is said that just as there are three double consonants, i.e. \(\Xi, \Psi\) and \(\Zeta\), there are also three concords, i.e. the fourth, the fifth and the octave.

b) However, there might be thousands of such letters; for instance, there might be one symbol for \(\Gamma\) and \(\Pi\) together.
c) But, the reason why the double consonants can be only three and no more is that there are three areas in the mouth and only one letter is combined with Σ in each area.

d) Therefore, the fact that the double consonants are three is not due to the fact that there are three concords, especially given that the concords are more than three.

e) Aristotle likens the people who have such beliefs with the old Homeric scholars, who see the small similarities, but overlook the important ones.

f) Some of them say that there are many such cases. For instance, the middle strings are the one nine and the other eight, and the epic verse has seventeen syllables, which is equal to the total of middle strings in number; and the scansion is nine syllables in the right part of the line and eight syllables in the left part of the line.

Continuing the discussion about the accidental (and not causal) relation of numbers to beings, Aristotle uses the example of double consonants and concords, both of which are said to be three in number. The double consonants are Ξ, Ψ and Ζ, while the concords are the fourth, the fifth and the octave. Alexander\textsuperscript{253} says that Ζ was said to correspond to the fourth, Ξ to the fifth and Ψ to the octave.

As far as the double letters are concerned, Archinus\textsuperscript{254}, who introduced the Ionian orthography in the second half of the fifth century, gave the following explanation for their being three in number: there are three areas in the mouth, i.e. the lips, the teeth and the palate, against which the tongue can be placed. Therefore, the consonant Ψ is spelled by the edge of tongue, when it is placed against the lips and it is the combination of the labial Π with Σ. Similarly, Ζ is spelled when the broad part of tongue is placed against the teeth and it is produced from the consonants Δ and Σ. Finally, Ξ is the combination of Κ and Σ and it is spelled when the uttermost part of tongue is placed against the palate. Aristotle claims that there could be many such double letters each of which could be expressed by the use of one symbol. However,

\textsuperscript{253} [Alex.], 811.30-31
\textsuperscript{254} Syr., 191.16-22
according to him, those who believe in the existence of only three double consonants would argue in favor of their view by saying that only Ξ, Ψ and Ζ are consonant combinations each of which takes twice as long to pronounce as a single consonant. Aristotle reasonably refutes this argument by attributing the fact of double consonants being three to the existence of only one letter that is applied to Σ in each of the three areas of mouth. Therefore, the reason why the double consonants are three is not that the concords are also three.

Furthermore, Aristotle says that there are more than three concords that can be produced from a combination of the primary concords, i.e. the fourth, the fifth and the octave. However, the double consonants cannot be combined in the same way so as to create new consonants.

Finally, he makes reference to the ancient interpreters of Homer, such as Pherecydes of Syros, Theagenes of Rhegium, Metrodorus of Lampsacus, Anaxagoras and Democritus. These Homeric scholars said that the chief intermediate notes between the keynote and the octave were the fourth and the fifth, which answer to the ratios 8:6 (4:3) and 9:6 (3:2) respectively. Therefore, it can be said that the one intermediate note is expressed by number nine and the other is expressed by number eight.

The ancient interpreters of Homer are said by Aristotle to have recognized as epic the verse composed of five dactyls (¬UU) and only one spondee (--) at the sixth foot. Such a verse necessarily contains seventeen syllables. According to Alexander255, if the epic verse separates into two parts, each of which consists of three feet, the first half of the verse (which is thought of by Aristotle as the right half) has nine syllables, while the second half of the verse (which is thought of by Aristotle as the left part) has eight syllables.

It seems quite strange that the first three dactyls, which contain nine syllables, are presented as the right part of the epic verse, while the last three feet, which have eight syllables, are presented as the left part. For, judging from how we read the verse, the left half precedes the right one. Alexander explains that “δεξιόν” is a

255 [Alex.], 813.21-26
technical term for the first part of a lyric system, while "ἀρχετέρων" is a term for the second part. It seems reasonable that the ancient thought identified the beginning part with δεξιών, given that most people tend to do things with a direction to the right. Aristotle himself elsewhere identifies δεξιών with the ἀρχή of movement (cf. De Caelo, 2, 285b16: "Δεξίον γὰρ ἐκαστον λέγουμεν, ὅθεν ἢ ἀρχὴ της κατὰ τότον κινήσεως.").

However, as it is claimed by Aristotle here, it is absurd to believe that the epic verse consists of seventeen syllables because there are two middle strings the one of which is described by number eight, while the other is described by number nine. The only common feature in these two cases is number seventeen. Nevertheless, the reason why seventeen is the number of syllables in the epic verse is totally different from the reason why it is the number of the intermediate notes. Therefore, how could these two cases be causally related to each other? Aristotle compares here the Platonists with the ancient Homeric scholars in order to show how frivolous the correspondences and comparisons made by the Platonists between numbers and beings are.

1093b1-11: In this section, Aristotle gives further examples to reinforce his claim that the lauded characteristics of numbers are not causes of things. In particular:

a) According to Aristotle, it is claimed that the distance in letters from A to Ω is equal to that from the lowest to the highest note of the flute, of which the number is equal to the whole system of the universe.

b) However, such analogies occur not only among the eternals, but also among the perishable things.

c) In general, it seems that none of the lauded mathematical properties is a cause in any of the senses of "cause" we have distinguished with reference to the first principles.

Aristotle uses here one further example in order to show that number and its properties cannot be treated as causes in any sense of "cause". According to him, it is
believed that there is a causal relation among number twenty-four of the letters, number twenty-four of the notes on the flute (from the deepest note, i.e. “ἀπὸ τοῦ βόμβυκος”, to the highest one, i.e. “ἐπὶ τὴν ὄξυτατην”) and number twenty-four of the universe (cf. Alexander\(^{256}\), who suggests that the number of the universe is twenty-four, because there are twelve signs of the zodiac, eight spheres and four elements). However, such numerical analogies can be found anywhere, either among the eternal things or among the perishable ones. It is absurd, according to Aristotle, to assume that the number of stars in the constellation of Cancer\(^{257}\) or the number of time needed for the sun and the moon to return to their place in the heavens\(^{258}\) is this specific number due to number’s being so much and such. On the contrary, the number that describes stars in Cancer and the number that describes solar or lunar position are determined by how many stars there are in constellation and how much time is needed for the sun or the moon to return to their initial place. Therefore, the mathematical theorems and properties cannot be thought of as causes in any of the senses that have already been discussed by Aristotle (cf. *Metaphysics*, Δ.1-2).

1093b11-21: In this section, Aristotle seems to accept that there are some cases in which the numbers are related to things, but only due to correspondence and not due to causality. In particular:

a) The odd, the straight, the equal-by-equal, and the powers of certain numbers belong to the series of “good” things.

b) It becomes evident here that the numbers act more as properties and coincidences.

c) Moreover, in each category of being there is an analogous term; as the straight line is in length, so is the level in surface, the odd in number, and the white in color.

\(^{256}\) [Alex.], 814.3-4

\(^{257}\) [Alex.], 814.2

\(^{258}\) Syr., 193.21-24
Aristotle makes reference to Pythagorean views here. The Pythagoreans believed in all likelihood that the quality of being good belongs to certain numbers, i.e. the odd ones, while the quality of being evil belongs to some other numbers, i.e. the even ones. Moreover, as it has already been said elsewhere in the *Metaphysics* (cf. A.5, 986a22-986b8), the Pythagoreans argued that, just as the odd numbers belong to the column of good thing, while the even numbers belong to the column of evil things, there are also things of other categories that belong either to the one or to the other column. For instance, the straight and the even belong to the column of good, while their opposite qualities, namely the curved and the uneven belong to the column of evil. Aristotle claims that all the things that, according to the Pythagoreans, belong to the category of good things are one thing by analogy. In other words, they can all be subsumed under one single category as being good things and therefore be related to each other in the sense of corresponding to each other. For example, the straight in length corresponds to the even in breadth. And both of them correspond to the odd in number. For, each of them is thought of by the Pythagoreans as good and beautiful concerning the category of things to which it belongs.

It is noteworthy that the quality of even ("ἵσον") is not referred to by Aristotle among the things that are said by the Pythagoreans to belong in the series of good. Alexander\(^\text{259}\) adopts the term "ῥαγίς ἱσον" instead of "ἵσον". This seems preferable, given that ῥαγίς ἱσον is in all likelihood a genuine Pythagorean phrase, that also describes "τετράγωνον (i.e. square)", which is included in the Pythagorean group of good things, according to Aristotle.

Making a concession to the Pythagoreans here, Aristotle seems to admit that there really are mathematical structures that are reflected in nature. However, the correspondences between generically different fields are nothing but coincidence\(^\text{260}\). Therefore, it is once more pointed out by Aristotle that numbers and beings are not related to each other by causality, but only by analogy.

\(^{259}\) [Alex.], 814.21
\(^{260}\) Annas (1976), 219
In this section, Aristotle draws his final conclusions concerning the numbers as causes and the existence of Forms. In particular:

a) Aristotle claims that it is not the Form numbers that are responsible for the harmonic relations and the like;

b) for these numbers differ from each other in form, even though they are equal, given that their units also differ.

c) Therefore, we should not make numbers Forms.

d) In conclusion, the fact that so much trouble is encountered with the generation of Form numbers and there is no way in which they could be brought into harmony, would seem to indicate that the objects of mathematics are not separable from sensible things, as some say, and that they are not the first principles of all beings.

Even if numbers were to be thought of as causes of other things, they would not be Form numbers. For, the units out of which Form numbers are composed are different in kind and therefore they are non-combinable. Hence, it is mathematical numbers that are involved in ratios, harmonic relations etc. For, mathematical numbers are repeatable. But, again, mathematical numbers cannot be thought of as causes of beings. Hence, it turns out than numbers are not the first principles of beings.
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